

# **Control and Estimation of Dynamical Nonlinear and Partial Differential Equation Systems:**

## **Theory and Applications**



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By  
Gerasimos Rigatos, M. Abbaszadeh and P. Siano

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This book first published 2021

IET Publications

Michael Faraday House, Six Hills Way, Stevenage, Hertfordshire, SG1 2AY, UK

British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

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ISBN (10):



In memory of my mother Diamantina Rigatou  
1939-2018



# Contents

<b>Glossary</b>	<b>6</b>
<b>1 Principles of nonlinear control</b>	<b>8</b>
1.1 Control based on approximate linearization . . . . .	8
1.1.1 Overview of the optimal control concept . . . . .	8
1.1.2 Design of an H-infinity nonlinear optimal controller . . . . .	11
1.1.3 Optimal state estimation with the H-infinity Kalman Filter . . . . .	17
1.2 Global linearization-based control concepts . . . . .	20
1.2.1 Foundations of global linearization-based control . . . . .	20
1.2.2 Elaborating on input-output linearization . . . . .	25
1.2.3 Input-state linearization . . . . .	29
1.2.4 Stages in the implementation of input-state linearization . . . . .	33
1.2.5 Input-output and input-state linearization for MIMO systems . . . . .	34
1.2.6 Dynamic extension . . . . .	34
1.3 Global linearization-based control with the use of differential flatness theory . . . . .	35
1.3.1 The background of differential flatness theory . . . . .	35
1.3.2 Differential flatness for finite dimensional systems . . . . .	36
1.3.3 Equivalence and differential flatness . . . . .	37
1.3.4 Differential flatness and trajectory planning . . . . .	41
1.3.5 Differential flatness, feedback control and equivalence . . . . .	43
1.3.6 Flatness-based control and state feedback for systems with model uncertainties . . . . .	46
1.3.7 Classification of types of differentially flat systems . . . . .	48
1.4 Control of PDE dynamical systems . . . . .	52
1.4.1 Distributed parameter systems and their transformation into the canonical form . . . . .	52
1.4.2 State-space description of a heat diffusion dynamics . . . . .	52
1.4.3 Differential flatness of the nonlinear heat-diffusion PDE . . . . .	54
1.4.4 Computation of a boundary conditions-based feedback control law . . . . .	56
1.4.5 Closed loop dynamics . . . . .	57
<b>2 Control and estimation based on approximate linearization for robotic systems</b>	<b>59</b>
2.1 Nonlinear control of the cart and double-pendulum overhead crane . . . . .	59
2.1.1 Outline . . . . .	59
2.1.2 Dynamic model of the double-pendulum overhead crane . . . . .	60
2.1.3 Approximate linearization of the double-pendulum overhead crane . . . . .	63
2.1.4 Computation of the feedback control gains . . . . .	67
2.1.5 Simulation tests . . . . .	67
2.2 Nonlinear control of the underactuated offshore crane . . . . .	68
2.2.1 Outline . . . . .	68
2.2.2 Dynamic model of the boom crane . . . . .	77
2.2.3 Approximate linearization of the dynamic model of boom cranes . . . . .	79

2.2.4	Computation of the feedback control gains . . . . .	83
2.2.5	Simulation tests . . . . .	84
2.3	Nonlinear control of the inertia wheel and pendulum system . . . . .	84
2.3.1	Outline . . . . .	84
2.3.2	Dynamic model of the inertia wheel inverted pendulum . . . . .	91
2.3.3	Approximate linearization of the inertia wheel inverted pendulum . . . . .	93
2.3.4	Computation of the feedback control gains . . . . .	93
2.3.5	Simulation tests . . . . .	94
2.4	Nonlinear control of the torsional oscillator with rotational actuator . . . . .	94
2.4.1	Outline . . . . .	94
2.4.2	Dynamic model of the translational oscillator with rotating actuator . . . . .	99
2.4.3	Approximate linearization of the state-space model . . . . .	101
2.4.4	Computation of the feedback control gains . . . . .	102
2.4.5	Simulation tests . . . . .	103
2.5	Nonlinear control of robotic exoskeletons . . . . .	108
2.5.1	Outline . . . . .	108
2.5.2	Dynamic model of the 2-DOF lower-limb exoskeleton . . . . .	108
2.5.3	Approximate linearization of the exoskeleton's dynamic model . . . . .	113
2.5.4	Simulation tests . . . . .	115
2.6	Nonlinear control of brachiation robots . . . . .	116
2.6.1	Outline . . . . .	116
2.6.2	Dynamic model of the multi-DOF brachiation robot . . . . .	119
2.6.3	Approximate linearization of the dynamic model of the brachiation robot . . . . .	124
2.6.4	Simulation tests . . . . .	130
2.7	Nonlinear control of power line inspection robots . . . . .	130
2.7.1	Dynamic model of the power line inspection robot . . . . .	139
2.7.2	Approximate linearization of the power line inspection robot . . . . .	143
2.7.3	Simulation tests . . . . .	145
2.8	Nonlinear control of robots with electrohydraulic actuators . . . . .	149
2.8.1	Outline . . . . .	149
2.8.2	Dynamic model of the multi-DOF electrohydraulic manipulator . . . . .	150
2.8.3	Differential flatness properties of the hydraulic robotic manipulator . . . . .	152
2.8.4	Approximate linearization of the model of the electro-hydraulic manipulator . . . . .	154
2.8.5	Simulation tests . . . . .	157
2.9	Nonlinear control of robots with electropneumatic actuators . . . . .	157
2.9.1	Outline . . . . .	157
2.9.2	Dynamic model of a robotic manipulator with electropneumatic actuators . . . . .	166
2.9.3	Approximate linearization of the robot with electropneumatic actuation . . . . .	170
2.9.4	Differential flatness of the robot with electropneumatic actuation . . . . .	178
2.9.5	Simulation tests . . . . .	180
2.10	Nonlinear control of flexible joint robots . . . . .	180
2.10.1	Outline . . . . .	180
2.10.2	Dynamic model of a multi-DOF robotic manipulator with flexible joints . . . . .	189
2.10.3	Approximate linearization of the model of the flexible-joints robot . . . . .	193
2.10.4	Jacobian matrices of the linearized model . . . . .	193
2.10.5	Simulation tests . . . . .	197
2.11	Nonlinear control of redundant robotic manipulators . . . . .	198
2.11.1	Outline . . . . .	198
2.11.2	Kinematic and dynamic model of the redundant manipulator . . . . .	207
2.11.3	Approximate linearization of the model of the redundant manipulator . . . . .	212
2.11.4	Simulation tests . . . . .	218

2.12	Nonlinear control of parallel closed-chain robotic manipulators . . . . .	218
2.12.1	Outline . . . . .	218
2.12.2	Dynamic model of the five-link parallel robot . . . . .	228
2.12.3	Approximate linearization of the five-link parallel robot . . . . .	233
2.12.4	Simulation tests . . . . .	240
<b>3</b>	<b>Control and estimation based on approximate linearization for autonomous vehicles</b>	<b>249</b>
3.1	Nonlinear control of tracked autonomous vehicles . . . . .	249
3.1.1	Outline . . . . .	249
3.1.2	Kinematic model of the tracked mobile robot . . . . .	251
3.1.3	Approximate linearization of the model of the tracked vehicle . . . . .	253
3.1.4	Simulation tests . . . . .	255
3.2	Nonlinear control of the autonomous articulated fire-truck . . . . .	260
3.2.1	Outline . . . . .	260
3.2.2	Kinematic model of the autonomous fire-truck robot . . . . .	261
3.2.3	Approximate linearization of the state-space model of the autonomous fire-truck . . . . .	263
3.2.4	The nonlinear H-infinity control . . . . .	264
3.2.5	Simulation tests . . . . .	265
3.3	Nonlinear control of the truck and N-trailer system . . . . .	272
3.3.1	Outline . . . . .	272
3.3.2	Kinematic model of the truck and N trailer robotic system . . . . .	273
3.3.3	Approximate linearization of the truck and N-trailer robotic system . . . . .	275
3.3.4	Simulation tests . . . . .	279
3.4	Nonlinear control of the ball-bot autonomous robot . . . . .	279
3.4.1	Outline . . . . .	279
3.4.2	Dynamic model of the ballbot . . . . .	285
3.4.3	Approximate linearization of the ballbot's state-space model . . . . .	289
3.4.4	Computation of the feedback control gains . . . . .	291
3.4.5	Simulation tests . . . . .	291
3.5	Nonlinear control of the ball-and-plate dynamical system . . . . .	292
3.5.1	Outline . . . . .	292
3.5.2	Dynamic model of the ball and plate system . . . . .	297
3.5.3	Approximate linearization of the model of the ball and plate system . . . . .	299
3.5.4	Simulation tests . . . . .	302
3.6	Nonlinear control of 3-DOF unmanned surface vessels . . . . .	303
3.6.1	Outline . . . . .	303
3.6.2	Dynamic model of the Unmanned Surface Vessel . . . . .	312
3.6.3	Approximate linearization of the USV state-space model . . . . .	316
3.6.4	Simulation tests . . . . .	321
3.7	Nonlinear control of the 3-DOF autonomous underwater vessel . . . . .	330
3.7.1	Outline . . . . .	330
3.7.2	Kinematic and dynamic model of the AUV . . . . .	331
3.7.3	Differential flatness properties of the AUV's model . . . . .	332
3.7.4	Approximate linearization of the state-space model of the AUV . . . . .	332
3.7.5	Simulation tests . . . . .	336
3.8	Nonlinear control of the Vertical Take-off and Landing Aircraft . . . . .	336
3.8.1	Outline . . . . .	336
3.8.2	Dynamic model of the vertical take-off and landing aircraft . . . . .	345
3.8.3	Differential flatness properties of the VTOL aircraft . . . . .	346
3.8.4	Approximate linearization of the VTOL aircraft dynamic model . . . . .	346
3.8.5	H-infinity feedback control . . . . .	347

3.8.6	Simulation tests	348
3.9	Nonlinear control of aerial manipulators	348
3.9.1	Outline	348
3.9.2	Dynamic model of the aerial robotic manipulator	355
3.9.3	Approximate linearization of the model of the aerial robotic maipulator	360
3.9.4	Differential flatness properties of the aerial robotic manipulator	363
3.9.5	Computation of the feedback control gains	364
3.9.6	Simulation tests	365
3.10	Nonlinear control of the 6-DOF autonomous octocopter	365
3.10.1	Outline	365
3.10.2	Dynamic model of the octorotor	373
3.10.3	Approximate linearization of the octorotor's model	376
3.10.4	Simulation tests	377
3.11	Nonlinear control of hypersonic aerial vehicles	378
3.11.1	Outline	378
3.11.2	Dynamic model of the autonomous hypersonic aerial vehicle	385
3.11.3	Differential flatness properties of the hypersonic vehicle	388
3.11.4	Approximate linearization for the dynamic model of the hypersonic vehicle	390
3.11.5	Computation of the feedback control gains	392
3.11.6	Simulation tests	392
<b>4</b>	<b>Control and estimation based on approximate linearization for energy conversion systems</b>	<b>398</b>
4.1	Nonlinear control of the VSI-fed three-phase PMSM	398
4.1.1	Outline	398
4.1.2	Dynamic model of the VSI-PMSM system	399
4.1.3	Approximate linearization of the inverter-PMSM dynamics	403
4.1.4	Simulation tests	406
4.2	Nonlinear control of VSI fed six-phase PMSMs	406
4.2.1	Outline	406
4.2.2	Dynamic model of the VSI-fed six-phase PMSM	414
4.2.3	Differential flatness properties of the model of the VSI-fed six-phase PMSM	421
4.2.4	Approximate linearization of the model of the VSI-fed six-phase PMSM	425
4.2.5	Simulation tests	428
4.3	Nonlinear control of the DC electric microgrids	429
4.3.1	Outline	429
4.3.2	Dynamic model of the DC microgrid	438
4.3.3	Approximate linearization of the state-space model of the DC microgrid	439
4.3.4	Computation of the feedback control gains	441
4.3.5	Simulation tests	441
4.4	Nonlinear control of distributed marine-turbine power generation units	442
4.4.1	Outline	442
4.4.2	Dynamic model of the distributed marine turbine power generation units	448
4.4.3	The dynamic model of the distributed power system	448
4.4.4	Differential flatness of the distributed marine power generation units	452
4.4.5	Approximate linearization of the distributed marine power generators	453
4.4.6	Computation of the feedback control gains	455
4.4.7	Simulation tests	456
4.5	Nonlinear control of PMLSGs in wave energy conversion systems	464
4.5.1	Outline	464
4.5.2	Dynamic model of the tubular permanent magnet linear synchronous generators	465
4.5.3	Approximate linearization of the model of the tubular PMLSG	467

4.5.4	Computation of the feedback control gains . . . . .	468
4.5.5	Simulation tests . . . . .	468
4.6	Nonlinear control of Permanent Magnet Brushless DC motors . . . . .	469
4.6.1	Outline . . . . .	469
4.6.2	Dynamic model of the PMSM motor . . . . .	473
4.6.3	Differential flatness of the model of the motor with non-sinusoidal back EMF . . . . .	477
4.6.4	Computation of the feedback control gains . . . . .	478
4.6.5	Simulation tests . . . . .	478
4.7	Nonlinear optimal control of Hybrid Electric Vehicles powertrains . . . . .	479
4.7.1	Outline . . . . .	479
4.7.2	Dynamic model of the HEV power supply / traction system . . . . .	483
4.7.3	Approximate linearization of the model of the HEV's powertrain . . . . .	486
4.7.4	Differential flatness properties of the HEV's powertrain . . . . .	487
4.7.5	Computation of the feedback control gains . . . . .	489
4.7.6	Simulation tests . . . . .	489
4.8	Nonlinear control of shipboard AC/DC microgrids . . . . .	498
4.8.1	Outline . . . . .	498
4.8.2	Dynamic model of the shipboard AC/DC microgrid . . . . .	500
4.8.3	Computation of the feedback control gains . . . . .	503
4.8.4	Simulation tests . . . . .	507
4.9	Nonlinear control of power generation in hybrid AC/DC microgrids . . . . .	507
4.9.1	Outline . . . . .	507
4.9.2	Dynamic model of the hybrid distributed microgrid . . . . .	516
4.9.3	Approximate linearization of the dynamic model of the hybrid microgrid . . . . .	520
4.9.4	Computation of the feedback control gains . . . . .	523
4.9.5	Differential flatness properties of the dynamic model of the microgrid . . . . .	523
4.9.6	Simulation tests . . . . .	529
<b>5</b>	<b>Control and estimation based on approximate linearization for mechatronic systems</b>	<b>533</b>
5.1	Nonlinear control of electrohydraulic actuators . . . . .	533
5.1.1	Dynamic model of the electrohydraulic actuator . . . . .	534
5.1.2	Approximate linearization of the electrohydraulic actuator's model . . . . .	537
5.1.3	Simulation tests . . . . .	538
5.2	Nonlinear control of electropneumatic actuators . . . . .	539
5.2.1	Outline . . . . .	539
5.2.2	Dynamic model of the electropneumatic actuator . . . . .	543
5.2.3	Approximate linearization of the model of the electropneumatic actuator . . . . .	545
5.2.4	Differential flatness properties of the electropneumatic actuator . . . . .	549
5.2.5	Simulation tests . . . . .	552
5.3	Nonlinear control of hot-steel rolling mills . . . . .	552
5.3.1	Outline . . . . .	552
5.3.2	Dynamic model of the hot-steel rolling mill . . . . .	556
5.3.3	Approximate linearization of the hot-steel rolling mill dynamics . . . . .	559
5.3.4	Computation of the feedback control gains . . . . .	560
5.3.5	Simulation tests . . . . .	560
5.4	Nonlinear control of paper mills . . . . .	561
5.4.1	Outline . . . . .	561
5.4.2	Dynamic model of the mechanical pulping process in paper mills . . . . .	566
5.4.3	Approximate linearization of the state-space model of the pulping process . . . . .	570
5.4.4	Stabilizing feedback control . . . . .	571
5.4.5	Simulation tests . . . . .	572

5.5	Nonlinear control of the injection moulding machine . . . . .	577
5.5.1	Outline . . . . .	577
5.5.2	Dynamic model of the injection moulding process . . . . .	578
5.5.3	Stable feedback control of the injection moulding process . . . . .	582
5.5.4	Simulation tests . . . . .	584
5.6	Nonlinear control of the slosh-container system dynamics . . . . .	585
5.6.1	Outline . . . . .	585
5.6.2	Dynamic model of the slosh-container system . . . . .	589
5.6.3	Approximate linearization of the model of the slosh-container system . . . . .	592
5.6.4	Simulation tests . . . . .	594
5.7	Nonlinear control of micro-satellites' attitude dynamics . . . . .	599
5.7.1	Introduction . . . . .	599
5.7.2	Dynamic model of the micro-satellite attitude system . . . . .	600
5.7.3	Approximate linearization of the satellite's state-space model . . . . .	602
5.7.4	Simulation tests . . . . .	605
5.8	Nonlinear control of the industrial crystallization process . . . . .	606
5.8.1	Outline . . . . .	606
5.8.2	Dynamic model of the industrial crystallization process . . . . .	614
5.8.3	Approximate linearization of the dynamics of the crystallization process . . . . .	616
5.8.4	Simulation tests . . . . .	618
<b>6</b>	<b>Control and estimation based on global linearization for industrial and PDE systems</b>	<b>623</b>
6.1	Control of a robotic exoskeleton subject to time-delays . . . . .	623
6.1.1	Outline . . . . .	623
6.1.2	Dynamic model of the robotic exoskeleton . . . . .	625
6.1.3	Estimation of perturbations with the use of a disturbance observer . . . . .	629
6.1.4	Simulation tests . . . . .	631
6.2	Adaptive control of synchronous reluctance machines . . . . .	636
6.2.1	Outline . . . . .	636
6.2.2	Dynamic model of the Synchronous Reluctance Machines . . . . .	637
6.2.3	Differential flatness of the synchronous reluctance machine . . . . .	638
6.2.4	Flatness-based adaptive neurofuzzy control . . . . .	640
6.2.5	Application of flatness-based adaptive neurofuzzy control to the SRM . . . . .	645
6.2.6	Lyapunov stability analysis . . . . .	649
6.2.7	Simulation tests . . . . .	653
6.3	Control of a mobile robotic manipulator . . . . .	653
6.3.1	Outline . . . . .	653
6.3.2	Dynamic model of the mobile manipulator . . . . .	656
6.3.3	Differential flatness properties of the model of the mobile manipulator . . . . .	661
6.3.4	Design of a flatness-based controller for the mobile manipulator . . . . .	662
6.3.5	Design of a flatness-based disturbances estimator . . . . .	662
6.3.6	Simulation tests . . . . .	664
6.4	State of charge estimation in EVs with a KF-based disturbance observer . . . . .	665
6.4.1	Outline . . . . .	665
6.4.2	Dynamic model of the battery . . . . .	674
6.4.3	Kalman Filter-based disturbance observer . . . . .	676
6.4.4	Simulation tests . . . . .	676
6.5	Control of nonlinear wave PDE dynamics . . . . .	677
6.5.1	Outline . . . . .	677
6.5.2	Transformation of the PDE model into a set of nonlinear ODEs . . . . .	680
6.5.3	Differential flatness of the nonlinear PDE model . . . . .	681



6.5.4	Computation of a boundary conditions-based feedback control law . . . . .	683
6.5.5	Closed loop dynamics . . . . .	684
6.5.6	Simulation tests . . . . .	685
6.6	Control of data-flow PDE for bandwidth allocation in internet routes . . . . .	686
6.6.1	Outline . . . . .	686
6.6.2	PDE of the internet flow per route . . . . .	689
6.6.3	Data flow model . . . . .	691
6.6.4	Differential flatness of the data flow model . . . . .	691
6.6.5	Flatness-based control for the data-flow model . . . . .	692
6.6.6	Stability analysis for the data-flow control loop . . . . .	693
6.6.7	Simulation tests . . . . .	694
6.7	Diffusion PDE control for data flow management in communication networks . . . . .	694
6.7.1	Outline . . . . .	694
6.7.2	Model of diffusion describing data flow in the communication network . . . . .	698
6.7.3	Transformation of the Fokker-Planck PDE into a set of nonlinear ODEs . . . . .	699
6.7.4	Differential flatness of the Fokker-Planck PDE model . . . . .	700
6.7.5	Computation of a boundary conditions-based feedback control law . . . . .	701
6.7.6	Closed loop dynamics . . . . .	703
6.7.7	Simulation tests . . . . .	704
6.8	Control of the diffusion PDE in Li-ion batteries . . . . .	707
6.8.1	Outline . . . . .	707
6.8.2	Diffusion PDE in Li-ion batteries . . . . .	707
6.8.3	Modeling in state-space form of the of the Li-ions diffusion PDE . . . . .	709
6.8.4	Differential flatness of the battery's PDE diffusion model . . . . .	711
6.8.5	Computation of a boundary conditions-based feedback control law . . . . .	712
6.8.6	Closed loop dynamics . . . . .	713
6.8.7	State estimation for the PDE diffusion model . . . . .	714
6.8.8	Simulation tests . . . . .	716
6.9	Control of the diffusion PDE in industrial assets' management . . . . .	719
6.9.1	Outline . . . . .	719
6.9.2	Dynamic model of stock-loans valuation . . . . .	720
6.9.3	Transformation of the stock-loan PDE into a set of nonlinear ODEs . . . . .	721
6.9.4	Differential flatness of the stock-loan PDE model . . . . .	724
6.9.5	Computation of a boundary conditions-based feedback control law . . . . .	726
6.9.6	Closed-loop dynamics . . . . .	727
6.9.7	Simulation tests . . . . .	729
6.10	Estimation of PDE dynamics of the highway traffic . . . . .	732
6.10.1	Outline . . . . .	732
6.10.2	Traffic modeling with the use of PDEs . . . . .	733
6.10.3	Estimation of the Payne-Whitham model using Extended Kalman Filter . . . . .	734
6.10.4	Estimation of Payne-Whitham PDE with the Derivative-free KF . . . . .	737
6.10.5	Derivative-free nonlinear Kalman Filter for the Payne-Whitham PDE . . . . .	739
6.10.6	Simulation tests . . . . .	742
6.11	Estimation of the PDE dynamics of a cable-suspended bridge . . . . .	743
6.11.1	Outline . . . . .	743
6.11.2	Dynamic model of the suspended-bridge and vehicle interaction . . . . .	746
6.11.3	Kalman Filtering for state-estimation in the bridge and vehicle system . . . . .	751
6.11.4	Statistical fault diagnosis using the Kalman Filter . . . . .	754
6.11.5	Simulation tests . . . . .	756

*CONTENTS*

xiv

**References**

**762**

# Acknowledgement

The authors of the monograph appreciate the review remarks of colleagues working in the research area of control of nonlinear and PDE dynamical systems. These comments have enabled to arrive at a concise, meaningful and clear presentation of the analyzed control and estimation methods for nonlinear and PDE dynamical systems and have contributed to developing a research document that will be useful for the engineering and academic community.

# Preface

The monograph presents advances in applied control of nonlinear and PDE dynamical systems, comprising both theoretical analysis of the proposed control methods and case studies about their use in robotics, mechatronics, electric power generation, power electronics, micro-electronics, industrial production processes and cyberphysical systems comprising communication and computer networks. The monograph covers thoroughly the area of automatic control for complex nonlinear dynamical systems, including also applications to distributed parameter systems which are described by partial differential equations. The monograph has a meaningful contribution in the areas of automatic control and systems science. Its results can be classified in the following main approaches for the control of complex nonlinear dynamical systems: (i) control with methods of approximate (local) linearization being associated with the solution of the nonlinear optimal control problem (ii) control with methods of exact (global) linearization comprising also adaptive control methods (iii) control of distributed parameter systems (systems which are described by partial differential equations) and stochastic estimation methods.

With reference to approach (i) that is control methods based on approximate linearization, one can distinguish results towards extending H-infinity control to nonlinear dynamical systems and towards solving the associated nonlinear optimal control problem. The methods which are developed for nonlinear control problems rely on linearization of the systems' dynamics around local operating points while the designed feedback controllers make use of the approximately linearized state-space models. Such controllers are designed to be robust to external perturbations, as well as to modelling errors, and achieve asymptotically (as time advances) the compensation of the nonlinear dynamics of the controlled systems. In this area one can note an important research result which is a new method of H-infinity control. This approach makes use of an approximately linearized model of the system that is obtained through the computation of Jacobian matrices. In contrast to problems of linear control and the method of the linear quadratic regulator, it is far more difficult to achieve a solution of the optimal control problem in the case of nonlinear dynamical systems under model uncertainties and external disturbances. The nonlinear optimal control problem is usually treated with iterative computational methods that are not always of assured convergence to the optimum. Actually, one comes against a differential game where the control signal tries to minimize the system's cost function so as to achieve the convergence of the state vector to the designated reference values, whereas the disturbance inputs try to maximize this cost function. For such problems, the monograph comes to propose a novel H-infinity (optimal) control method. At each time-step of the optimization algorithm approximate linearization takes place around local operating points, with the use of Taylor series expansion and through the computation of Jacobian matrices. The linearization error is considered to be an additional perturbation affecting the system. Next, for the linearized equivalent model of the system, an optimal H-infinity controller can be applied, while to compute this controller's gains an algebraic Riccati equation has to be iteratively solved at each time-step of the control algorithm,. This control scheme is also shown to be sufficiently robust, thus assuring the compensation of modelling errors, parametric uncertainty or external disturbances that affect the control loop. With the use of Lyapunov analysis the global stability properties of the control loop are proven and the convergence of the system's state vector to the designated reference setpoints is demonstrated. The new solutions for the nonlinear optimal (H-infinity) control problem are computationally efficient since they require the solution of only one Riccati equation.

With reference to approach (ii) that is control based on global linearization approaches, the monograph elaborates

on transformations of the initial nonlinear dynamics of the controlled systems into equivalent linear state-space descriptions where finally the design of feedback controllers is performed and the solution of the related stochastic estimation (filtering) problems is also accomplished. In this approach belong the monograph's results on differential flatness theory-based control, which rely on the transformation of the state-space description of the system into the canonical (Brunovsky) form. Through the global linearization-based control approach one can avoid the modelling errors that follow approximate linearization methods and consequently control of high precision and robustness can be achieved. Flatness-based control relies on differential flatness theory and consists of state-variables transformations that finally bring the system's state-space model into an equivalent linear form where the application of the standard linear control and stabilization methods is enabled. The flatness-based method comprises also inverse transformations that allow to compute estimates of the state variables of the initial nonlinear system. In this area one can also classify the monograph's results on a new nonlinear estimation method which is known as Derivative-free nonlinear Kalman Filter and which contributes towards solving the filtering problem for nonlinear dynamical systems in an optimal manner.

In the above-noted area (ii) of global linearization-based control the monograph analyzes also flatness-based adaptive fuzzy control for a wide class of nonlinear dynamical systems. In the flatness-based adaptive fuzzy control approach, one performs first an initial transformation (diffeomorphism) of the system's state-space model into an equivalent linear form. In this new description, the transformed control inputs contain unknown nonlinear functions which can be identified with the use of nonlinear regressors (e.g. neurofuzzy networks, wavelet networks or other networks that comprise nonlinear kernel functions). Learning in such networks takes place with the use of gradient algorithms where the learning rate is regulated through conditions for the minimization of the system's Lyapunov function and for assuring that this cumulative energy function of the system will have always a negative first-order derivative and will be persistently decreasing. At each time-step of the adaptive control algorithm, the estimated values for the nonlinear functions that constitute the system's dynamics are used to compute the feedback control inputs. It is proven that this approach achieves the minimization of the system's Lyapunov function and that the control loop becomes globally asymptotically stable.

With reference to approach (iii), that is control for dynamical systems which are described by nonlinear partial differential equations (PDEs) the monograph advances towards boundary control of the dynamics of the partial differential equations. Actually, in the proposed PDE control methods the control inputs are related only with the boundary conditions of the PDE, thus one arrives at a PDE boundary control problem. The methods are based on semi-discretization of the PDE (only about its spatial dimension) and this allows to substitute the PDE with an equivalent set of ordinary differential equations (ODEs). The solution of the stabilization problem for the PDE with the use of control inputs which are applied through the boundary conditions can be achieved by exploiting differential flatness theory. The design of a stabilizing controller for the PDE is based on the proof that (i) the state-space model of the PDE is a differentially flat system, (ii) each row of the state-space model is also a differentially flat subsystem. Next, for each subsystem (row) being associated with an ordinary differential equation, one can compute a virtual control input which can stabilize the subsystem's dynamics and which can also eliminate the tracking error of the subsystem's output. The virtual control input for the  $i$ -th subsystem becomes a reference setpoint for the  $(i+1)$ -th subsystem. From the last row of the state-space description one can compute the control input (boundary condition) which should be finally applied to the partial differential equation. This control input comprises in a recursive manner all virtual control inputs which were computed for the rows (subsystems) that constitute the PDE's state-space description. Thus, by tracing backwards the rows of the state-space model of the PDE (from the last to the first row) one can finally compute the control input that should be applied in the form of a boundary condition so as all individual state variables to converge to their designated setpoints. The global stability of this PDE control method is proven through Lyapunov analysis. Finally, in the above-noted research area (iii), that is dynamical systems described by PDEs, it is also of worth to include several results on estimation for fault diagnosis. About this topic, one can note results on fault detection and isolation methods which rely on Kalman Filtering and on the statistical properties of the  $\chi^2$  distribution that is followed by the state vector's estimation error (residuals).

Regarding the application part of the control and estimation methods for nonlinear and PDE dynamical systems, the monograph has examined the following:

(a) Control and estimation based on approximate linearization for robotic systems: (i) Nonlinear control of the cart and double-pendulum overhead crane, (ii) Nonlinear control of the underactuated offshore crane, (iii) Nonlinear control of the inertia wheel and pendulum system, (iv) Nonlinear control of the torsional oscillator with rotational actuator, (v) Nonlinear control of robotic exoskeletons, (vi) Nonlinear control of brachiation robots, (vii) Nonlinear control of power line inspection robots, (viii) Nonlinear control of robots with electrohydraulic actuators, (ix) Nonlinear control of robots with electropneumatic actuators, (x) Nonlinear control of flexible joint robots, (xi) Nonlinear control of redundant robotic manipulators, (xii) Nonlinear control of parallel closed-chain robotic manipulators.

(b) Control and estimation based on approximate linearization for autonomous vehicles: (i) Nonlinear control of tracked autonomous vehicles, (ii) Nonlinear control of the autonomous fire-truck, (iii) Nonlinear control of the truck and N-trailer system, (iv) Nonlinear control of the ball-bot autonomous robot, (v) Nonlinear control of the ball-and-plate dynamical system, (vi) Nonlinear control of 3-DOF unmanned surface vessels (vii) Nonlinear control of the 3-DOF autonomous underwater vessel (viii) Nonlinear control of the Vertical Take-off and Landing Aircraft, (ix) Nonlinear control of aerial manipulators (x) Nonlinear control of the 6-DOF autonomous octocopter (xi) Nonlinear control of hypersonic aerial vehicles.

(c) Control and estimation based on approximate linearization for energy conversion systems: (i) Nonlinear control of the VSI-fed three-phase PMSM, (ii) Nonlinear control of the VSI-fed six-phase PMSM (iii) Nonlinear control of the DC electric microgrids (iv) Nonlinear control of distributed marine-turbine power generation units (v) Nonlinear control of PMLSGs (permanent magnet linear synchronous generators) in wave energy conversion systems (vi) Nonlinear control of Permanent Magnet Brushless DC motors (vii) Nonlinear optimal control of Hybrid Electric Vehicles powertrains (viii) Nonlinear control of shipboard AC/DC microgrids (ix) Nonlinear control of power generation in hybrid AC/DC microgrids

(d) Control and estimation based on approximate linearization for mechatronic systems: (i) Nonlinear control of electrohydraulic actuators, (ii) Nonlinear control of electropneumatic actuators (iii) Nonlinear control of hot-steel rolling mills (iv) Nonlinear control of paper mills (v) Nonlinear control of the injection moulding machine (vi) Nonlinear control of the slosh-container system dynamics (vii) Nonlinear control of micro-satellites' attitude dynamics (viii) Nonlinear control of the industrial crystallization process.

(e) Control and estimation based on global linearization for industrial and PDE systems: (i) Control of a robotic exoskeleton subject to time-delays, (ii) Adaptive control of synchronous reluctance machines (iii) Control of a mobile robotic manipulator (iv) SoC (state-of-charge) estimation in Electric Vehicles with a Kalman Filter-based disturbance observer (v) Control of nonlinear wave PDE dynamics (vi) Control of nonlinear wave PDE dynamics, (vii) Control of a data flow PDE for bandwidth allocation in internet routes, (viii) Control of a diffusion PDE describing data flow in communication networks, (ix) Control of the diffusion PDE in Li-ion batteries, (x) Control of a diffusion PDE in industrial assets' management (xi) Estimation of PDE dynamics of the highway traffic, (xii) Estimation of the PDE dynamics of a cable-suspended bridge and use of the obtained estimates for fault diagnosis.

Through the above-noted developments and the methods proposed for control and estimation of nonlinear and PDE dynamical systems this monograph has a useful contribution in the area of nonlinear dynamical systems and control theory. Yet being computationally and algorithmically simple, the presented control schemes assure precise tracking of setpoints and stabilization for complicated nonlinear and PDE dynamical systems. Besides, in several cases they ensure optimal performance of the control loop, as for instance with the solution of the nonlinear optimal control or with the solution of the nonlinear optimal state estimation problem. The application field is wide and comprises primarily what is considered to be industrial systems technology, that is robotic systems (robotic manipulators and autonomous robotic vehicles), several mechatronic systems, electric power generation and power electronics, industrial production processes and cyber-physical systems that include the complex dynamics of communication

and computer networks. These results are undoubtedly of interest for the engineering and academic community and can be used for teaching related courses at the late undergraduate or post-graduate level. Certainly, research in the field control and estimation for nonlinear and PDE dynamical systems has the potential for arriving at many more significant and exploitable findings in the years to come.

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# Glossary

AC/DC: alternating current / direct current  
ADCS: Attitude Determination and Control Subsystem  
AUV: Autonomous Underwater Vessel  
BLDC: Brushless DC motor  
CLT: Central Limit Theorem  
DC/DC: direct current / direct current  
DFIG: Doubly-Fed Induction Generator  
DOF: Degrees of Freedom  
EKF: Extended Kalman Filter  
EMF: Electromagnetic Force  
 $H_\infty$  control: H-infinity Control  
 $H_\infty$  Kalman Filter: H-infinity Kalman Filter  
HESG: Hybrid Excited Synchronous Generator  
HESM: Hybrid Excited Synchronous Machine  
HEV: hybrid electric vehicle  
HSV: Hypersonic Vehicle  
KF: Kalman Filter  
LMI: Linear Matrix Inequality  
LPV: Linear Parameter Varying system  
LQR: Linear Quadratic Regulator  
LQG: Linear Quadratic Gaussian  
MIMO: Multi-input multi-output  
MPC: Model Predictive Control  
NES: Normalized Error Square  
NMPC: Nonlinear Model Predictive Control  
ODE: Ordinary Differential Equation  
PDE: Partial Differential Equation  
PID: Proportional Derivative Integral  
PLI: Power Line Inspection robot  
PMBLDC: Permanent Magnet Brushless Direct Current motor  
PMLSG: Permanent Magnet Linear Synchronous Generator  
PMSG: Permanent Magnet Synchronous Generator  
PMSM: Permanent Magnet Synchronous Motor  
PV: photovoltaic unit  
PWM: Pulse Width Modulation  
RTAC: Rotational-translational actuator  
SDRE: State-Dependent Riccati Equation  
SRG: Synchronous Reluctance Generator  
SRM: Synchronous Reluctance Machine  
SISO: Single-input single-output



## *GLOSSARY*

7

SMC: Sliding Mode Control

SoC: State-of-Charge

TORA: Torsional oscillator with rotational actuator

UGV: Unmanned Ground Vehicle

USV: Unmanned Surface Vessel

VSC: Voltage Source Converter

VSI: Voltage Source Inverter

VTOL: Vertical Take-Off and Landing Aircraft

WSN: Wireless Sensor Networks

UAV: Unmanned Aerial Vehicle