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Advanced Models of Neural Networks

Nonlinear dynamics and stochasticity in biological neurons

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To Elektra

Foreword

This book, provides a complete study on neural structures exhibiting nonlinear and stochastic dynamics. The book elaborates on neural dynamics by introducing advanced models of neural networks. It overviews the main findings in the modelling of neural dynamics in terms of electrical circuits and examines their stability properties with the use of dynamical systems theory. Such electric circuit models are characterized by attractors and fixed points while in certain cases they exhibit bifurcations and chaotic dynamics. Moreover, solutions of the neural dynamics in the form of travelling waves equations are derived. The dynamics of interconnected neurons is analyzed with the use of forced oscillator and coupled oscillator models. It is shown that by introducing stochastic uncertainty and variations in the previous deterministic coupled oscillator model, stochastic coupled oscillator models can be derived. Next, going into a more refined analysis level it is shown how neural dynamics can be interpreted with the use of quantum and stochastic mechanics. It is proven that the model of interacting coupled neurons becomes equivalent to the model of interacting Brownian particles that is associated to the equations and dynamics of quantum mechanics. It is shown that such neural networks with dynamics compatible to quantum mechanics principles exhibit stochastic attractors. Furthermore, the spectral characteristics of such neural networks are analyzed. Additionally, a stochastic mechanics approach to stabilization of particle systems with quantum dynamics is presented. It also shown that the eigenstates of the quantum harmonic oscillator can be used as activation functions in neural networks. Moreover, a gradient-based approach to stabilization of particle systems with quantum dynamics is provided. There are remarkable new results in the book concerned with: (ii) nonlinear synchronizing control of coupled neural oscillators (ii) neural structures based on stochastic mechanics or quantum mechanics principles, (ii) nonlinear estimation of the wave-type dynamics of neurons, (iii) neural and wavelet networks with basis functions localized both in space and frequency, (iv) stochastic attractors in neural structures, and (v) engineering applications of advanced neural network models.

Industrial Systems Institute, October 2013 *Dr. Gerasimos G. Rigatos*

Preface

This book aims at analyzing advanced models of neural networks, starting with methods from dynamical systems theory and advancing progressively to stochasticity-based models and models compatible with principles of quantum mechanics. Advanced models of neural networks enable on the one side to understand patterns of neuronal activity seen in experiments in the area of neuroscience and on the other side to develop neurocomputing methods in the area of information sciences that remain consistent with physics principles governing biological neural structures. The first chapters of the book (Chapters 1 to 6) make use of dynamical systems theory to explain the functioning of neural models. Dynamical systems and computational methods are widely used in research to study activity in a variety of neuronal systems. The dynamics of the neural structures are described with the use of linear or nonlinear ordinary differential equations or with the use of partial differential equations. This approach to neural modelling focuses on the following issues: (i) Modelling neural networks in terms of electrical circuits (The Hodgin-Huxley equations. The FitzHugh-Nagumo equations. ion channels) (ii) Elements of dynamical systems theory: The phase plane. Stability and fixed points. Attractors, Oscillations. Bifurcations, Chaotic dynamics (iii) Chaotic dynamics in neural excitation. Travelling waves solutions to models of neural dynamics (iv) Neural oscillators (forced oscillators and coupled oscillator models).

The latter chapters of the book (Chapters 7 to 13) analyze the significance of noise and stochasticity in modelling of neuronal dynamics. The dynamics of neural structures are no longer described by linear or nonlinear ordinary differential equations or by partial differential equations but are formulated in terms of stochastic differential equations. Neural computation based on principles of quantum mechanics can provide improved models of memory processes and brain functioning and is of primary importance for the realization of quantum computing machines. To this end, the book studies neural structures with weights that follow the model of the quantum harmonic oscillator. The proposed neural networks have stochastic weights which are calculated from the solution of Schrödinger's equation under the assumption of a parabolic (harmonic) potential. These weights correspond to diffusing particles, which interact to each other as the theory of Brownian motion (Wiener process) predicts. The learning of the stochastic weights (convergence of the diffusing particles to an equilibrium) is analyzed. In the case of associative memories the proposed neural model results in an exponential increase of patterns storage capacity (number of attractors). It is also shown that conventional neural networks and learning algorithms based on error gradient can be conceived as a subset of the proposed quantum neural structures. Thus, the complementarity between classical and quantum physics can be also validated in the field of neural computation. Furthermore, in continuation to modelling of neural networks as interacting Brownian particles it is shown how stabilization can be succeeded either for particle systems which are modelled as coupled stochastic oscillators and for particle systems with quantum dynamics. Finally, engineering applications of the previously described advanced models of neural dynamics are provided.

The book's chapters are organized as follows: (1) Modelling biological neurons in terms of electrical circuits, (2) Systems theory for the analysis of biological neuron dynamics, (3) Bifurcations and limit cycles in biological neurons (4) Oscillatory dynamics in Biological Neurons, (5) Synchronization of circadian neurons and protein synthesis control, (6) Wave dynamics in the transmission of neural signals, (7) Stochastic models of biological neuron dynamics, (8) Synchronization of stochastic neural oscillators using Lyapunov methods, (9) Synchronization in coupled stochastic neurons using differential flatness theory, (10) Attractors in associative memories with stochastic weights, (11) Spectral analysis of neural models with stochastic weights, (12) Neural networks based on the eigenstates of the quantum harmonic oscillator and engineering applications, (13) Gradient-based feedback control for dynamical systems following the model of the quantum harmonic oscillator.

A summary of the chapters' content is given in the following:

In Chapter 1, it is shown that the functioning of the cells membrane can be represented as an electric circuit. To this end: (1) the ion channels are represented as resistors, (2) the gradients of the ions concentration are represented as voltage sources, (3) the capability of the membrane for charge storage is represented as a capacitor. It is considered that the neurons have the shape of a long cylinder, or of a cable with specific radius.The Hodgkin-Huxley model is obtained from a modification of the cables PDE, which describes the change of the voltage along dendrites axis. Cables equation comprises as inputs the currents which are developed in the ions channels. Other models of reduced dimensionality that describe voltage variations along the neuron's membrane are the FitzHugh-Nagumo model and the Morris-Lecar model. Cable's equation is also shown to be suitable for describing voltage variations along dendrites. Finally, the various types of ionic channels across the neurons' membrane are analyzed.

In Chapter 2, the main elements of systems theory are overviewed, thus providing the basis for modeling of biological neurons dynamics. To understand oscillatory phenomena and consequently the behavior of biological neurons benchmark examples of oscillators are given. Moreover, using a low order mathematical model of biological neurons the following properties are analyzed: phase diagram, isoclines, attractors, local stability, fixed points bifurcations and chaos properties.

In Chapter 3, a systematic method is proposed for fixed point bifurcations analysis in biological neurons using interval polynomials theory. The stages for performing fixed point bifurcation analysis in biological neurons comprise (i) the computation of fixed points as functions of the bifurcation parameter and (ii) the evaluation of the type of stability for each fixed point through the computation of the eigenvalues of the Jacobian matrix that is associated with the system's nonlinear dynamics model. Stage (ii) requires the computation of the roots of the characteristic polynomial of the Jacobian matrix. This problem is nontrivial since the coefficients of the characteristic polynomial are functions of the bifurcation parameter and the latter varies within intervals. To obtain a clear view about the values of the roots of the characteristic polynomial and about the stability features they provide to the system, the use of interval polynomials theory and particularly of Kharitonov's stability theorem is proposed. In this approach the study of the stability of a characteristic polynomial with coefficients that vary in intervals is equivalent to the study of the stability of four polynomials with crisp coefficients computed from the boundaries of the aforementioned intervals. The efficiency of the proposed approach for the analysis of fixed points bifurcations in nonlinear models of biological neurons is tested through numerical and simulation experiments.

In Chapter 4, it is shown that the voltage of the neurons membrane exhibits oscillatory variations after receiving suitable external excitation either when the neuron is independent from neighboring neural cells or when the neuron is coupled to neighboring neural cells through synapses or gap junctions. In the latter case it is significant to analyze conditions under which synchronization between coupled neural oscillators takes place, which means that the neurons generate the same voltage variation pattern possibly subject to a phase difference. The loss of synchronism between neurons can cause several neuro-degenerative disorders. Moreover, it can affect several basis functions of the body such as gait, respiration and hearts rhythm. For this reason synchronization of coupled neural oscillators has become a topic of significant research during the last years. It is also noted that the associated results have been used in several engineering applications, such as biomedical engineering and robotics. For example, synchronization between neural cells can result in a rhythm generator that controls joints motion in quadruped, multi-legged and biped robots.

In Chapter 5, a new method is proposed for synchronization of coupled circadian cells and for nonlinear control of the associated protein synthesis process using differential flatness theory and the derivative-free nonlinear Kalman Filter. By proving that the dynamic model of the synthesis of the FRQ protein (protein extracted from the frq gene) is a differentially flat one its transformation to the linear canonical

(Brunovsky) form becomes possible. For the transformed model one can find a state feedback control input that makes the oscillatory characteristics in the concentration of the FRQ protein vary according to desirable setpoints. To estimate the nonmeasurable elements of the state vector, a new filtering method named *Derivative-free nonlinear Kalman Filter* is used. The Derivative-free nonlinear Kalman Filter consists of the standard Kalman Filter recursion on the linearized equivalent model of the coupled circadian cells and on computation of state and disturbance estimates using the diffeomorphism (relations about state variables transformation) provided by differential flatness theory. Moreover, to cope with parametric uncertainties in the model of the FRQ protein synthesis and with stochastic disturbances in measurements, the Derivative-free nonlinear Kalman Filter is redesigned in the form of a disturbance observer. The efficiency of the proposed Kalman Filter-based control scheme is tested through simulation experiments.

In Chapter 6, an analysis is given on wave-type partial differential equations that describe the transmission of neural signals and proposes filtering for estimating the spatiotemporal variations of voltage in the neurons' membrane. It is shown that in specific neuron models the spatiotemporal variations of the membrane's voltage follow partial differential equations (PDEs) of the wave type while in other models such variations are associated with the propagation of solitary waves in the membrane. To compute the dynamics of the membrane PDE model without knowledge of boundary conditions and through the processing of noisy measurements, the Derivative-free nonlinear Kalman Filter is proposed. The PDE of the membrane is decomposed into a set of nonlinear ordinary differential equations with respect to time. Next, each one of the local models associated with the ordinary differential equations is transformed into a model of the linear canonical (Brunovsky) form through a change of coordinates (diffeomorphism) which is based on differential flatness theory. This transformation provides an extended model of the nonlinear dynamics of the membrane for which state estimation is possible by applying the standard Kalman Filter recursion. The proposed filtering method is tested through numerical simulation tests.

In Chapter 7, neural networks are examined in which the synaptic weights correspond to diffusing particles and are associated with a Wiener process. Each diffusing particle (stochastic weight) is subject to the following forces: (i) a spring force (drift) which is the result of the harmonic potential and tries to drive the particle to an equilibrium and (ii) a random force (noise) which is the result of the interaction with neighboring particles. This interaction can be in the form of collisions or repulsive forces. It is shown that the diffusive motion of the stochastic particles (weights update) can be described by Fokker-Planck's, Ornstein-Uhlenbeck or Langevins equation which under specific assumptions are equivalent to Schrödinger's diffusion equation. It is proven that Langevins equation is a generalization of the conventional gradient algorithms.

In Chapter 8, a neural network with weights described by the interaction of Brownian particles is considered again. Each weight is taken to correspond to a Brownian particle. In such a case, neural learning aims at leading a set of M weights (Brownian particles) with different initial values on the 2-D phase plane, to a desirable final position. A Lyapunov function describes the evolution of the phase diagram towards the equilibrium Convergence to the goal state is assured for each particle through the negative definiteness of the associated Lyapunov function. The update of each neural weight (trajectory in the phase diagram) is affected by (i) a drift force due to the harmonic potential, and (ii) the interaction with neighboring weights (particles). Using Lyapunov methods it is shown that the mean of the particles converges to the equilibrium while using LaSalle's theorem it is proven that the individual particles remain within a small area encircling the equilibrium.

In Chapter 9, examples of chaotic neuronal dynamics are presented first and control of chaotic neuron model with the use of differential flatness theory is explained. Moreover, a synchronizing control method is presented for neurons described again as particle systems and modeled as coupled stochastic oscillators. The proposed synchronization approach is also flatness-based control. The kinematic model of the particles is associated with the model of the quantum harmonic oscillator and stands for a differentially-flat system. It is also shown that after applying flatness-based control the mean of the particle system can be steered along a desirable path with infinite accuracy, while each individual particle can track the trajectory within acceptable accuracy levels.

In Chapter 10, neural associative memories are considered in which the elements of the weight matrix are taken to be stochastic variables. The probability density function of each weight is given by the solution of Schrödinger's diffusion equation. The weights of the proposed associative memories are updated with the use of a learning algorithm that satisfies quantum mechanics postulates. The learning rule is proven to satisfy two basic postulates of quantum mechanics: (i) existence in superimposing states, (ii) evolution between the superimposing states with the use of unitary operators. Therefore it can be considered as a quantum learning algorithm. Taking the elements of the weight matrix of the associative memory to be stochastic variables means that the initial weight matrix can be decomposed into a superposition of associative memories. This is equivalent to mapping the fundamental memories (attractors) of the associative memory into the vector spaces which are spanned by the eigenvectors of the superimposing matrices and which are related to each other through unitary rotations. In this way, it can be shown that the storage capacity of the associative memories with stochastic weights increases exponentially with respect to the storage capacity of conventional associative memories.

In Chapter 11, spectral analysis of neural networks with stochastic weights (stemming from the solution of Schrödinger's diffusion equation) is performed. It is shown that: (i)The Gaussian basis functions of the weights express the distribution of the energy with respect to the weights' value. The smaller the spread of the basis functions is, the larger becomes the spectral (energy) content that can be captured therein. Narrow spread of the basis functions, results in wide range of frequencies of a Fourier-transformed pulse, (ii) The stochastic weights satisfy an equation which is analogous to the principle of uncertainty.

In Chapter 12, feedforward neural of networks with orthogonal activation functions (Hermite polynomials) which come from the solution of Schrödinger's diffusion equation are considered. These neural networks have significant properties: (i) the basis functions are invariant under the Fourier transform, subject only to a change of scale, (ii) the basis functions are the eigenstates of the quantum harmonic oscillator (QHO), and stem from the solution of Schrödinger's harmonic equation. The proposed neural networks have performance that is equivalent to wavelet networks and belong to the general category of nonparametric estimators. They can be used for function approximation, image processing and system fault diagnosis. The considered basis functions are also analyzed with respect to uncertainties principles and the Balian-Low theorem.

In Chapter 13, the interest is in control and manipulation of processes at molecular scale, as the ones taking place in several biological and neuronal systems. To this end, a gradient method is proposed for feedback control and stabilization of particle systems using Schrödinger's and Lindblad's descriptions. The eigenstates of the quantum system are defined by the spin model. First, a gradient-based control law is computed using Schrödinger's description. Next, an estimate of state of the quantum system is obtained using Lindblad's differential equation. In the latter case, by applying Lyapunov's stability theory and LaSalle's invariance principle one can compute a gradient control law which assures that the quantum system's state will track the desirable state within acceptable accuracy levels. The performance of the control loop is studied through simulation experiments for the case of a two-qubit quantum system.

The book contains teaching material that can be used in several undergraduate or post-graduate courses in university schools of engineering, computer science, mathematics, physics and biology. The book can be primarily addressed to final year undergraduate students and to first years postgraduate students pursuing studies in electrical engineering, computer science as well as in physics, mathematics and biology. The book can be used as a main reference in upper level courses on machine learning, artificial intelligence and computational intelligence as well as a complementary reference in upper level courses on dynamical systems theory. Moreover, engineers and researchers in the areas of nonlinear dynamical systems, artificial and computational intelligence and machine learning can get profit from this book which analyzes advanced models of neural networks and which goes beyond the common information representation schemes with the use of artificial neural networks.

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Contents