

# **Nonlinear Optimal and Flatness-based Control Methods and Applications for Complex Dynamical Systems**

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**In memory of my mother Diamantina Rigatou (1939-2018)**  
*Gerasimos Rigatos*

**To my wife Elham, and my sons, Arman and Ario**  
*Masoud Abbaszadeh*

**To my family**  
*Pierluigi Siano*

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# Overview

Control of nonlinear dynamical systems has been often based on the concept of diffeomorphisms. This requires state-variables and state-space description transformations that bring the system into an equivalent linear form. In the new state-space description, a solution for both the control and state estimation problem is enabled. However, far from being an ideal solution, the use of such transformations is not always a straightforward procedure since the controlled system should previously satisfy feedback linearizability conditions. Besides, in this approach the control inputs are computed for the linearized equivalent model and it is necessary to apply inverse transformations so as to find the control signals that should be finally used in the initial nonlinear state-space description of the system. Thus one may come against singularity issues which signify that the existence of state-space regions where the inverse transformations cannot be performed because of generating non-bounded (infinite) control inputs.

Taking into account the above, in the present monograph novel solutions to the control problem of complex nonlinear dynamical systems are developed and tested. These are (i) a nonlinear optimal (H-infinity) control approach, (ii) a flatness-based control approach implemented in successive loops. The new control methods are not constrained by the aforementioned shortcomings of global linearization-based control schemes (complicated changes of state variables, forward and backwards state-space transformations, singularities). It is shown that such methods can be used in a wide class of nonlinear dynamical systems without needing to transform the systems' state-space model into equivalent linearized forms. It is also shown that the new control methods can be implemented in a computationally simple manner and are also followed by global stability proofs.

The monograph's nonlinear optimal (H-infinity) control scheme is a novel contribution to the area of nonlinear control. The method is based on an approximate linearization of the dynamics of the controlled system taking place at each sampling instance around a temporary operating point which is defined by the present value of the system's state vector and by the last sampled value of the control inputs vector. The linearization process relies on first-order Taylor series expansion and on the computation of the associated Jacobian matrices. For the approximately linearized model of the system a stabilizing H-infinity feedback controller is designed. The controller's feedback gains are computed by solving repetitively (at each time-step of the control algorithm) an algebraic Riccati equation. The global stability properties of the nonlinear optimal control scheme are proven through Lyapunov analysis. The method of nonlinear optimal (H-infinity) control is of proven global stability and remains computationally tractable. It retains the advantages of linear optimal control, that is fast and accurate tracking of reference setpoints under moderate variations of the control inputs. The nonlinear optimal control method is applicable to a wider class of dynamical systems than approaches based on the solution of State Dependent Riccati Equations (SDRE). The SDRE approaches can be applied only to dynamical systems which can be transformed to the Linear Parameter Varying (LPV) form. Besides the novel nonlinear optimal (H-infinity) control method performs better than optimal control approaches which are based on approximations of the solution of the Hamilton-Jacobi-Bellman equation by Galerkin series expansion (the stability properties of the Galerkin series-based optimal control schemes are still unproven).

The monograph introduces also a second solution to the problem of nonlinear control of complex dynamical systems without the need to apply changes of state variables (diffeomorphisms) and complicated state-space transformations. The new solution is a flatness-based control approach implemented in successive loops. In this method the dynamic model of the nonlinear system is separated into subsystems which are connected in a cascading manner. This control method is directly applicable (but not limited) to dynamical systems of the triangular form and to nonlinear systems which can be transformed into such a form. The state-space model of the initial nonlinear system is decomposed into cascading subsystems which satisfy differential flatness properties. For each subsystem of the state-space model a virtual control input is computed, capable of inverting the subsystem's dynamics and of eliminating the subsystem's tracking error. The control input which is actually applied to the initial nonlinear system is computed from the last row of the state-space description. This control input incorporates in a recursive manner all virtual control inputs which were computed from the individual subsystems included in the initial state-space equation. The control input that should be applied to the nonlinear system so as to assure that all state vector elements will converge to the desirable set-points is obtained at each iteration of the control algorithm by tracing backwards the subsystems of the state-space model.

Concisely, in the present monograph novel solutions to the control problem of complex nonlinear dynamical systems are developed and tested. These are (i) a nonlinear optimal (H-infinity) control approach, (ii) a flatness-based control approach implemented in successive loops. The new control methods are free of shortcomings met in control schemes which are based diffeomorphisms and global linearization (complicated changes of state variables, forward and backwards state-space transformations, singularities). It is shown that such methods can be used in a wide class of nonlinear dynamical systems without needing to transform the systems' state-space model into equivalent linearized forms. It is also shown that the new control methods can be implemented in a computationally simple manner and are also followed by global stability proofs. Potential application domains of the nonlinear optimal and flatness-based control concepts for complex dynamical systems are: (i) robotic systems (ii) robotic cranes and pendulums (iii) space robots. aerospace systems and satellites (v) mechatronic systems (iv) power electronics (v) biosystems (vi) financial systems.

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# Glossary

AC/DC: alternating current / direct current  
ADCS: Attitude Determination and Control Subsystem  
AMB: Active Magnetic Bearing  
ARE: Algebraic Riccati Equation  
CDPR: Cable Driven Parallel Robot  
CLT: Central Limit Theorem  
CNC: Computer Numerical Control  
DC/DC: direct current / direct current  
DFIG: Doubly-Fed Induction Generator  
DOF: Degrees of Freedom  
DKF: Derivative-free Nonlinear Kalman Filter  
EKF: Extended Kalman Filter  
EMF: Electromagnetic Force  
FSRM: Free-Floating Space Robotic Manipulator  
HKF: H-infinity Kalman Filter  
 $H_\infty$  control: H-infinity Control  
 $H_\infty$  Kalman Filter: H-infinity Kalman Filter  
HVDC: High Voltage Direct Current  
IM: Induction Motor  
KF: Kalman Filter  
LMI: Linear Matrix Inequality  
LPV: Linear Parameter Varying system  
LQR: Linear Quadratic Regulator  
LQG: Linear Quadratic Gaussian  
MIMO: Multi-input multi-output  
MPC: Model Predictive Control  
NES: Normalized Error Square  
NMPC: Nonlinear Model Predictive Control  
ODE: Ordinary Differential Equation  
PDE: Partial Differential Equation  
PID: Proportional Derivative Integral  
PMSM: Permanent Magnet Synchronous Motor  
PMLSM: Permanent Magnet Linear Synchronous Motor  
PMLSG: Permanent Magnet Linear Synchronous Generator  
PMSG: Permanent Magnet Synchronous Generator  
PMSM: Permanent Magnet Synchronous Motor

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PWM: Pulse Width Modulation  
RSV: Reentry Space Vehicle  
RMSE: Root Mean Square Error  
SCARA: Selective Compliance Articulated Robot Arm  
SDRE: State-Dependent Riccati Equation  
SISO: Single-input single-output  
SMC: Sliding Mode Control  
VSC: Voltage Source Converter  
VSI: Voltage Source Inverter  
WEC: Wave Energy Converter

# Preface

A main approach in the control of nonlinear dynamical systems has been so far the concept of diffeomorphisms, that is of state-variables and state-space description transformations that allow for bringing the system into an equivalent linear form where a solution for both the control and state estimation problem becomes feasible. However, the application of such transformations is not always a straightforward procedure since the controlled system should previously satisfy feedback linearizability conditions. Besides, once the control inputs have been computed for the linearized equivalent model it is necessary to apply inverse transformations so as to find the control signals that should be finally used in the initial nonlinear state-space description of the system. This process often comes against singularity issues which signify that there may exist certain state-space regions where the inverse transformations cannot be performed because of generating non-bounded (infinite) control inputs.

Taking into account the above, in the present monograph novel solutions to the control problem of complex nonlinear dynamical systems are developed and tested. These are (i) a nonlinear optimal (H-infinity) control approach, (ii) a flatness-based control approach implemented in successive loops. The new control methods are not constrained by the aforementioned shortcomings of global linearization-based control schemes (complicated changes of state variables, forward and backwards state-space transformations, singularities). It is shown that such methods can be used in a wide class of nonlinear dynamical systems without needing to transform the systems' state-space model into equivalent linearized forms. It is also shown that the new control methods can be implemented in a computationally simple manner and are also followed by global stability proofs.

The monograph's nonlinear optimal (H-infinity) control scheme is a novel contribution to the area of nonlinear control. The method is based on an approximate linearization of the dynamics of the controlled system taking place at each sampling instance around a temporary operating point which is defined by the present value of the system's state vector and by the last sampled value of the control inputs vector. The linearization process relies on first-order Taylor series expansion and on the computation of the associated Jacobian matrices. For the approximately linearized model of the system a stabilizing H-infinity feedback controller is designed. The controller's feedback gains are computed by solving repetitively (at each time-step of the control algorithm) an algebraic Riccati equation. The global stability properties of the nonlinear optimal control scheme are proven through Lyapunov analysis. The method of nonlinear optimal (H-infinity) control is of proven global stability and remains computationally tractable. It retains the advantages of linear optimal control, that is fast and accurate tracking of reference setpoints under moderate variations of the control inputs. The nonlinear optimal control method is applicable to a wider class of dynamical systems than approaches based on the solution of State Dependent Riccati Equations (SDRE). The SDRE approaches can be applied only to dynamical systems which can be transformed to the Linear Parameter Varying (LPV) form. Besides the novel nonlinear optimal (H-infinity) control method performs better than optimal control approaches which are based on approximations of the solution of the Hamilton-Jacobi-Bellman equation by Galerkin series expansion (the stability properties of the Galerkin series-based optimal control schemes are still unproven).



A comparison of the nonlinear optimal (H-infinity) control method against other linear and nonlinear control schemes for complex dynamical systems has shown the following: (1) unlike Lie algebra-based control, the new nonlinear optimal control method does not rely on complicated transformations (diffeomorphisms) of the system's state variables. The control inputs that the nonlinear optimal control method computes can be applied directly to the initial nonlinear dynamics of the system and are not used on its transformed equivalent description. The inverse transformations which are met in global linearization-based control are avoided and consequently one does not come against singularity issues (2) unlike Model Predictive Control and Nonlinear Model Predictive Control the nonlinear optimal control method is of proven global stability. It is known that Model Predictive Control is a linear control method which if applied to systems with complex nonlinear dynamics the stability of the control loop will be lost. Besides, in Nonlinear Model Predictive Control the convergence of the iterative search for an optimum depends on initialization and parameter values selection and consequently the global stability properties of the NMPC method cannot be ensured either (2) unlike sliding-mode and backstepping control the nonlinear optimal control method does not require the state-space description of the system to be found in a specific form. About sliding-mode control it is known that when the controlled system is not found in the input-output linearized form, the definition of the sliding surface can be an intuitive procedure. About backstepping control it is known that it cannot be directly applied to a dynamical system if the related state-space model is not found in the strict-feedback (backstepping integral) form (4) unlike PID control, the nonlinear optimal control method is of proven global stability, the selection of the controller's parameters does not rely on a heuristics-based tuning procedure and the stability of the control loop is ensured in case of changes of operating points (5) unlike multiple local models-based control, the nonlinear optimal control method uses only one linearization point and needs the solution of only one Riccati equation so as to compute the stabilizing feedback gains of the controller. Equivalently, the nonlinear optimal control method does not require the solution of complicated Linear Matrix Inequalities. Consequently, in terms of computation load the nonlinear optimal control method is much more efficient.

The monograph introduces also a second solution to the problem of nonlinear control of complex dynamical systems without the need to apply changes of state variables (diffeomorphisms) and complicated state-space transformations. The new solution is a flatness-based control approach implemented in successive loops. In this method the dynamic model of the nonlinear system is separated into subsystems which are connected in a cascading manner. This control method is directly applicable (but not limited) to dynamical systems of the triangular form and to nonlinear systems which can be transformed into such a form. The state-space model of the initial nonlinear system is decomposed into cascading subsystems which satisfy differential flatness properties. For each subsystem of the state-space model a virtual control input is computed, capable of inverting the subsystem's dynamics and of eliminating the subsystem's tracking error. The control input which is actually applied to the initial nonlinear system is computed from the last row of the state-space description. This control input incorporates in a recursive manner all virtual control inputs which were computed from the individual subsystems included in the initial state-space equation. The control input that should be applied to the nonlinear system so as to assure that all state vector elements will convergence to the desirable set-points is obtained at each iteration of the control algorithm by tracing backwards the subsystems of the state-space model.

Concisely, in the present monograph, novel solutions to the control problem of complex nonlinear dynamical systems are developed and tested. These are (i) a nonlinear optimal (H-infinity) control approach, (ii) a flatness-based control approach implemented in successive loops. The new control methods are free of shortcomings met in control schemes which are based diffeomorphisms and global linearization (complicated changes of state variables, forward and backwards state-space transformations, singularities). It is shown that such methods can be used in a wide class of nonlinear dynamical systems without needing to transform the systems' state-space model into equivalent linearized forms. It is also shown that the new control methods can be implemented in a computationally simple manner and are also followed by global stability proofs. Potential application domains of the nonlinear optimal and flatness-based control concepts for complex dynamical systems are: (i) robotic systems (ii) robotic cranes and pendulums (iii) space robots. aerospace systems and satellites (v) mechatronic systems (iv) power electronics (v) biosystems (vi) financial systems.



The monograph is structured in the following chapters:

Chapter 1 analyzes nonlinear optimal and flatness-based control methods for robotic systems. First, the chapter treats the nonlinear optimal control problem of the 6-DOF parallel Stewart robotic manipulators. Next, the control problem of 4-DOF SCARA robots is treated. Additionally the problem of nonlinear control of dual-arm robotic manipulators is solved, with the use of a nonlinear optimal control method. Finally, flatness-based control in successive loops is applied to the dynamic model of electropneumatic robots and particularly to exoskeletons with electropneumatic actuation.

Chapter 2 analyzes further nonlinear optimal and flatness-based control methods for robotic systems. First the problem of multi-loop flatness-based control for dual-arm robotic manipulators is solved. Next, a multi-loop flatness-based control method is developed for the dynamic model of the an electrohydraulic robotic manipulator. Additionally, a multi-loop flatness-based control approach is proposed for the dynamic model of a multi-DOF flexible-joint robotic manipulator. Moreover, a multi-loop flatness-based control scheme is applied to the model of a three-link redundant robotic manipulator. Finally, the chapter analyzes a method for flatness-based disturbance observer and control with application to robotic mining excavators

Chapter 3 analyzes nonlinear optimal and flatness-based control methods for robotic cranes and underactuated pendulums. First the problem of nonlinear optimal control of the 4-DOF robotic tower crane is solved. Next, the problem of nonlinear optimal control of the dual PMLSM-driven gantry crane is treated. Moreover, a nonlinear optimal control scheme is developed for the 2-cable-driven parallel robot which functions as a crane. Finally, nonlinear optimal control is applied to the dynamic model of the 4 cable driven parallel robot which functions as a crane.

Chapter 4 analyzes further nonlinear optimal and flatness-based control methods for robotic cranes and underactuated pendulums. First, a multi-loop flatness-based control method is proposed for the control and stabilization problem of the dual PMLSM-driven gantry crane. Next, the problem of nonlinear optimal control of parallel inverted pendulums is treated. Additionally, a nonlinear optimal control method is proposed for the control and stabilization problem of the rotary double inverted pendulum (double Furuta pendulum). Moreover, a nonlinear optimal control scheme is developed for the dynamic model of the three-link biped robot

Chapter 5 analyzes nonlinear optimal and flatness-based control for aerospace systems. First, it solves the nonlinear optimal control problem for the 4-DOF dynamic model of a reentry space vehicle. Next it applies nonlinear optimal control and multi-loop flatness-based control to the 6-DOF attitude dynamics of a reentry space vehicles. Furthermore it applies multi-loop flatness-based control to the 3-DOF dynamic model of the attitude of microsatellites. Finally, it applies multi-loop flatness-based control to the dynamic model of the 3-DOF free-floating space robotic manipulator.

Chapter 6 analyzes further nonlinear optimal and flatness-based control methods for aerospace systems. First it solves the nonlinear optimal control for the dynamic model of the 3-DOF free-floating space robotic manipulator. Besides, it analyzes nonlinear optimal control for the dynamic model of the underactuated tethered satellite. Next, it generalizes towards nonlinear optimal control for the dynamic model of a tethered multi-satellite system. Finally it presents results on nonlinear optimal control for the dynamic model of the wing rock effect that appears in aircrafts and reentry space vehicles.

Chapter 7 analyzes nonlinear optimal control and multi-loop flatness-based control for mechatronic systems and for power electronics. First it solves the nonlinear optimal and flatness-based control problem for the dynamic model of an induction motor driven desalination units. Moreover, it develops a multi-loop flatness-based control method for centrifugal gas compressors driven by induction motors and permanent magnet synchronous motors. Besides, it introduces

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nonlinear optimal control for wave energy conversion units which consist of a permanent magnet linear synchronous generator connected to a three-phase AC/DC converter. Additionally, it analyzes nonlinear optimal control and multi-loop flatness-based control for the dynamic model of 5-DOF active magnetic bearings. Next, a differential flatness and Kalman Filter-based disturbance observer is developed for the fault diagnosis problem of 5-DOF active magnetic bearings. Finally, a differential flatness and Kalman Filter-based disturbance observer is proposed for the fault diagnosis problem of the VSC-HVDC transmission system in the electric power grid.

Chapter 8 analyzes nonlinear optimal control and multi-loop flatness-based control for dynamical models of biological systems and financial systems. Nonlinear optimal control is applied for the treatment of bacterial infections with resistance to antibiotics. Additionally, nonlinear optimal control is proposed for the optimized infusion of chemotherapy and the treatment of leukemia. Moreover, nonlinear optimal control and flatness-based control is proposed for treatment of tumor growth. Furthermore, nonlinear optimal control and multi-loop flatness-based control is introduced for the stabilization of population dynamics described by the Lotka-Volterra model. Finally, nonlinear optimal control is used for the optimized management of multi-echelon supply chain networks under time-delays.

The monograph is completed with two Appendices which elaborate on the theoretical background of the nonlinear optimal and flatness-based control methods for complex dynamical systems:

Appendix A analyses nonlinear optimal control and Lie algebra-based control. In the first section of this appendix control and state estimation based of approximate linearization of the dynamic models of nonlinear systems is explained. Linearization is performed sequentially through Taylor series expansion and the solution of a Riccati equation. The H-infinity Kalman Filter is developed as a robust state estimator. In the second section of the chapter global linearization-based control and state estimation is developed for nonlinear systems. To this end input-output and input-state linearization conditions are analyzed. Results from Lie algebra-based control and Lie algebra-based state observers are explained. The concept of dynamic extension is also outlined.

Appendix B analyzes differential flatness theory and flatness-based control methods. In the first section of the Appendix global linearization-based control with the use of differential flatness theory is analyzed, comprising issues such as differential flatness for finite dimensional systems, equivalence and differential flatness, feedback control and equivalence flatness-based control and state feedback for systems with model uncertainties and classification of types of differentially flat systems. In the second section of this appendix the concept of flatness-based control of nonlinear dynamical systems in cascading loops is analysed. This comprises issues such as decomposition of the state-space model into cascading differentially flat subsystems, tracking error dynamics for flatness-based control in successive loops and comparison to backstepping control for nonlinear systems.

By providing solutions to the control problem of complex dynamical systems the monograph has contributed to the advancement of the subject area. Based on these findings and by further exploiting the new nonlinear control methods that the monograph has analyzed it is expected that additional progress in this technological domain will be achieved in the forthcoming years.

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