Lecture on

New approaches to nonlinear control of electric power systems:

Global linearization methods

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1. Outline

- Nonlinear control based on **global linearization** of the system's dynamics is proposed for **distributed dynamical systems**

- Moreover, **global linearization transformations** based on **differential flatness theory** are used for implementing a **new nonlinear filtering method**, that solves in a conditionally optimal manner the state estimation problem in distributed dynamical systems

- The system's model is first subjected to a **linearization transformation** that is based on **differential flatness theory** and next state estimation is performed by applying the standard **Kalman Filter recursion** to the linearized model.

- The proposed **derivative-free nonlinear Kalman Filter** is redesigned as a disturbance observer. This enables to estimate at the same time the **non-measurable elements of each generator's state vector**, the unknown input power (torque) and the **disturbance terms** induced by interarea oscillations.

- The **real-time estimation** of the **aggregate disturbance** that affects each local generator makes also possible to maintain the power system on its nominal operating conditions.

2. The model of the distributed synchronous generators

The **dynamic model of the distributed power generation units** is assumed to be that of synchronous generators. The modelling approach is also applicable to PMSGs (permanent magnet synchronous generators) which are a special case of synchronous electric machines.

$$\dot{\delta} = \omega$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \frac{\omega_0}{2J}(P_m - P_e) \quad (1)$$

δ : turn angle of the rotor

ω : turn speed of the rotor

ω_0 : synchronous speed

J : moment of inertia of the rotor

P_e : active electrical power of the machine

P_m : mechanical power of the machine

D : damping coefficient

T_e : electromagnetic torque

The **generator's electrical dynamics** is:

$$\dot{E}_q' = \frac{1}{T_{d_o}}(E_f - E_q) \quad (2)$$

E_q' is the quadrature-axis transient voltage (a variable related to the magnetic flux)

E_q is quadrature axis voltage of the generator

T_{d_o} is the direct axis open-circuit transient time constant

E_f

2. The model of the distributed synchronous generators

The **synchronous generator's model** is complemented by a set of algebraic equations:

$$E_q = \frac{x_{d_\Sigma}}{x'_{d_\Sigma}} E'_q - (x_d - x'_d)\frac{V_s}{x'_{d_\Sigma}} cos(\Delta\delta)$$
$$I_q = \frac{V_s}{x'_{d_\Sigma}} sin(\Delta\delta)$$
$$I_d = \frac{E'_q}{x'_{d_\Sigma}} - \frac{V_s}{x'_{d_\Sigma}} cos(\Delta\delta)$$
$$P_e = \frac{V_s E'_q}{x'_{d_\Sigma}} sin(\Delta\delta)$$
$$Q_e = \frac{V_s E'_q}{x'_{d_\Sigma}} cos(\Delta\delta) - \frac{V_s^2}{x_{d_\Sigma}}$$
$$V_t = \sqrt{(E'_q - X'_d I_d)^2 + (X'_d I_q)^2} \qquad (3)$$

where: $x_{d_\Sigma} = x_d + x_T + x_L$ $\quad$ $x'_{d_\Sigma} = x'_d + x_T + x_L$

x_d : direct-axis synchronous reactance

x_T : reactance of the transformer

x'_d : direct-axis transient reactance

x_L : transmission line reactance

I_d and I_q : direct and quadrature axis currents

V_s : infinite bus voltage

Q_e : reactive power of the generator

V_t : terminal voltage of the generator

2. The model of the distributed synchronous generators

From Eq. 1 and Eq. 2 one obtains the **dynamic model of the synchronous generator**:

$$\dot{\delta} = \omega - \omega_0$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E_q'}{x_{d_\Sigma}'} sin(\Delta\delta)$$
$$\dot{E}_q' = -\frac{1}{T_d'} E_q' + \frac{1}{T_{d_o}} \frac{x_d - x_d'}{x_{d_\Sigma}'} V_s cos(\Delta\delta) + \frac{1}{T_{d_o}} E_f$$

Moreover, the generator can be written in a **state-space form**:

$$\dot{x} = f(x) + g(x)u$$

where the state vector is $x = (\Delta\delta \quad \Delta\omega \quad E_q')^T$ and

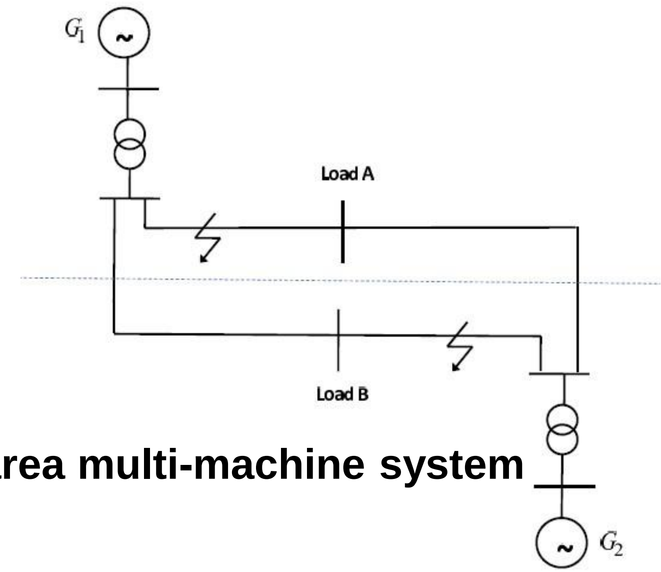
$$f(x) = \begin{pmatrix} \omega - \omega_0 \\ -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E_q'}{x_{d\Sigma}'} sin(\Delta\delta) \\ -\frac{1}{T_d'} E_q' + \frac{1}{T_{do}} \frac{x_d - x_d'}{x_{d\Sigma}'} V_s cos(\Delta\delta) \end{pmatrix}$$

$$g(x) = (0 \quad 0 \quad \frac{1}{T_{do}})^T$$

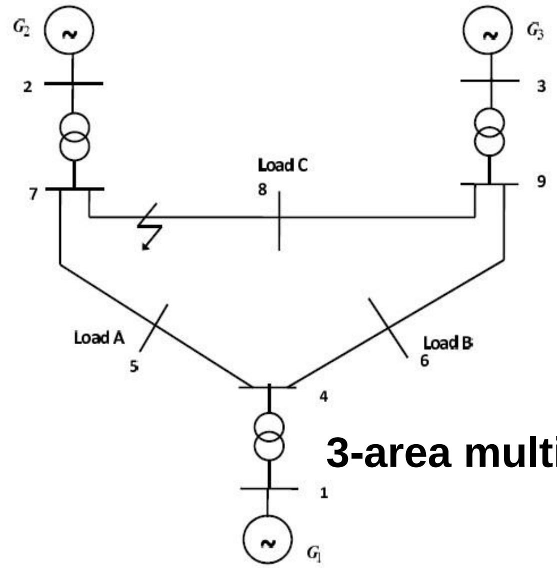
while the system's output is $y = h(x) = \delta - \delta_0$

2. The model of the distributed synchronous generators

The **interconnection between distributed power generators** results into a **multi-area multi-machine power system** model



2-area multi-machine system



3-area multi-machine system

The dynamic model of a power system that comprises **n-interconnected power generators** is

$$\dot{\delta}_i = \omega_i - \omega_0$$

$$\dot{\omega}_i = -\frac{D_i}{2J_i}(\omega_i - \omega_0) + \omega_0\frac{P_{m_i}}{2J_i} -$$

$$-\omega_0\frac{1}{2J_i}[G_{ii}E_{qi}^{'2} + E'_{qi}\sum_{j=1,j\neq i}^{n}E'_{qj}G_{ij}sin(\delta_i - \delta_j - \alpha_{ij})]$$

$$\dot{E}'_{q_i} = -\frac{1}{T'_{d_i}}E'_{q_i} + \frac{1}{T_{do_i}}\frac{x_{d_i} - x'_{d_i}}{x'_{d\Sigma i}}V_{s_i}cos(\Delta\delta_i) + \frac{1}{T_{do_i}}E_{f_i}$$

2. The model of the distributed synchronous generators

For the case of the three interconnected generators one has the state-space equations:

$$\dot{x}_1 = x_2 - \omega_0$$

$$\dot{x}_2 = -\frac{D_1}{2J_1}(x_2 - \omega_0) + \omega_0 \frac{P_{m_1}}{2J_1} -$$
$$-\frac{\omega_0}{2J_1}\{G_{11}x_3^2 + x_3[x_6 G_{12} sin(x_1 - x_4 - a_{12}) + x_9 G_{13} sin(x_1 - x_7 - a_{13})]$$

$$\dot{x}_3 = -\frac{1}{T'_{d_1}}x_3 + \frac{1}{T_{d_{o1}}}\frac{x_{d_1} - x'_{d_1}}{x'_{d_{\Sigma_1}}}V_s cos(x_1) + \frac{1}{T_{d_{o1}}}u_1$$

$$\dot{x}_4 = x_5 - \omega_0$$

$$\dot{x}_5 = -\frac{D_2}{2J_2}(x_5 - \omega_0) + \omega_0 \frac{P_{m_2}}{2J_2} -$$
$$-\frac{\omega_0}{2J_2}\{G_{22}x_6^2 + x_6[x_3 G_{21} sin(x_4 - x_1 - a_{21}) + x_9 G_{23} sin(x_4 - x_7 - a_{23})]$$

$$\dot{x}_6 = -\frac{1}{T'_{d_2}}x_6 + \frac{1}{T_{d_{o2}}}\frac{x_{d_2} - x'_{d_2}}{x'_{d_{\Sigma_2}}}V_s cos(x_4) + \frac{1}{T_{d_{o2}}}u_2$$

$$\dot{x}_7 = x_8 - \omega_0$$

$$\dot{x}_8 = -\frac{D_2}{2J_3}(x_8 - \omega_0) + \omega_0 \frac{P_{m_3}}{2J_3} -$$
$$-\frac{\omega_0}{2J_3}\{G_{32}x_9^2 + x_9[x_3 G_{31} sin(x_7 - x_1 - a_{31}) + x_6 G_{32} sin(x_7 - x_4 - a_{32})]$$

$$\dot{x}_9 = -\frac{1}{T'_{d_3}}x_9 + \frac{1}{T_{d_{o3}}}\frac{x_{d_3} - x'_{d_3}}{x'_{d_{\Sigma_3}}}V_s cos(x_7) + \frac{1}{T_{d_{o3}}}u_3$$

2. The model of the distributed synchronous generators

The system of the interconnected generators is also written in the matrix form:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \\ f_6(x) \\ f_7(x) \\ f_8(x) \\ f_9(x) \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_1(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & g_2(x) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_3(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

where

$$f_1(x) = x_2 - \omega_0$$

$$f_2(x) = -\frac{D_1}{2J_1}(x_2 - \omega_0) + \omega_0 \frac{P_{m_1}}{2J_1} -$$
$$-\frac{\omega_0}{2J_1}\{G_{11}x_3^2 + x_3[x_6 G_{12} sin(x_1 - x_4 - a_{12}) + x_9 G_{13} sin(x_1 - x_7 - a_{13})]$$

$$f_3(x) = -\frac{1}{T'_{d_1}}x_3 + \frac{1}{T_{d_{o_1}}}\frac{x_{d_1} - x'_{d_1}}{x'_{d_{\Sigma_1}}} V_s cos(x_1)$$

2. The model of the distributed synchronous generators

and also

$$f_4(x) = x_5 - \omega_0$$

$$f_5(x) = -\frac{D_2}{2J_2}(x_5 - \omega_0) + \omega_0\frac{P_{m2}}{2J_2} -$$
$$-\frac{\omega_0}{2J_2}\{G_{22}x_6^2 + x_6[x_3G_{21}sin(x_4 - x_1 - a_{21}) + x_9G_{23}sin(x_4 - x_7 - a_{23})]$$

$$f_6(x) = -\frac{1}{T'_{d_2}}x_6 + \frac{1}{T_{d_{o2}}}\frac{x_{d_2} - x'_{d_2}}{x'_{d_{\Sigma_2}}}V_s cos(x_4)$$

$$f_7(x) = x_8 - \omega_0$$

$$f_8(x) = -\frac{D_2}{2J_3}(x_8 - \omega_0) + \omega_0\frac{P_{m3}}{2J_3} -$$
$$-\frac{\omega_0}{2J_3}\{G_{32}x_9^2 + x_9[x_3G_{31}sin(x_7 - x_1 - a_{31}) + x_6G_{32}sin(x_7 - x_4 - a_{32})]$$

$$f_9(x) = -\frac{1}{T'_{d_3}}x_9 + \frac{1}{T_{d_{o3}}}\frac{x_{d_3} - x'_{d_3}}{x'_{d_{\Sigma_3}}}V_s cos(x_7)$$

$$g_1 = \frac{1}{T_{d_{o1}}} \quad g_2 = \frac{1}{T_{d_{o2}}} \quad g_3 = \frac{1}{T_{d_{o3}}}u_3$$

3. Outline of differential flatness theory

- **Differential flatness theory** has been developed as a **global linearization control method** by M. Fliess (Ecole Polytechnique, France) and co-researchers (Lévine, Rouchon, Mounier, Rudolph, Petit, Martin, Zhu, Sira-Ramirez et. al)

- A dynamical system can be written in the ODE form $S_i(w, \dot{w}, \ddot{w}, \ldots, w^{(i)}),\ \ i = 1,2,\ldots,q$

  where $w^{(i)}$ stands for the i-th derivative of either a state vector element or of a control input

- The system is said to be **differentially flat** with respect to the **flat output**

$$y_i = \phi(w, \dot{w}, \ddot{w}, \ldots, w^{(a)}),\ i = 1,\ldots,m \quad \text{where} \quad y = (y_1, y_2, \ldots, y_m)$$

if the following two conditions are satisfied

(i) There does not exist any differential relation of the form

$$R(y, \dot{y}, \ddot{y}, \ldots, y^{(\beta)}) = 0$$

which means that **the flat output and its derivatives are linearly independent**

(ii) All system variables are **functions of the flat output and its derivatives**

$$w^{(i)} = \psi(y, \dot{y}, \ddot{y}, \ldots, y^{(\gamma_i)})$$

3. Outline of differential flatness theory

The proposed Lyapunov theory-based control method is based on the **transformation** of the nonlinear system's model into the **linear canonical form**, and this transformation is succeeded by exploiting the system's differential flatness properties

- **All single input nonlinear systems** are differentially flat and can be transformed into the linear canonical form

One has to define also which are the **MIMO nonlinear systems** which are differentially flat.

- Differential flatness holds for **MIMO nonlinear systems** that admit **static feedback linearization.**and which can be transformed into the linear canonical form through a change of variables (diffeomorphism) and feedback of the state vector.

- Differential flatness holds for **MIMO nonlinear models** that admit **dynamic feedback linearization**, This **is the case of specific underactuated robotic models.** In the latter case the state vector of the system is extended by considering as additional flat outputs some of the control inputs and their derivatives

- Finally, a more rare case is the so-called **Liouvillian systems**. These are systems for which differential flatness properties hold for part of their state vector (constituting a flat subsystem) while the non-flat state variables can be obtained by integration of the elements of the flat subsystem.

4. Differential flatness of the distributed generators

The **active power** associated with the i-th power generator is given by:

$$P_{e_i} = G_{ii}E_{qi}^{'\,2} + E_{qi}'\sum_{j=1,j\neq i}^{n} E_{qj}' G_{ij} sin(\delta_i - \delta_j - \alpha_{ij})$$

The state vector of the distributed power system is given by $x = [x^1, x^2, \cdots, x^n]^T$

where $x^i = [x_1^i, x_2^i, x_3^i]^T$ with $x_1^i = \Delta\delta_i$ $x_2^i = \Delta\omega_i$ and $x_3^i = E_{qi}'$ $i = 1, 2, \cdots, n$

Next, **differential flatness** is proven for the model of the **stand-alone synchronous generator**.

In state-space form one has:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{D}{2J}x_2 + \omega_0\frac{P_m}{2J} - \frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}x_3 sin(x_1)$$
$$\dot{x}_3 = -\frac{1}{T_d'}x_3 + \frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(x_1) + \frac{1}{T_{do}}u$$

The **flat output** is taken to be $y = x_1$

It holds that $x_1 = y$ $x_2 = \dot{y}$ and for $x_1 \neq \pm n\pi$,

$$x_3 = \frac{\omega_0\frac{P_m}{2J} - \ddot{y} - \frac{D}{2J}\dot{y}}{\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}sin(y)}, \text{ or } x_3 = f_a(y, \dot{y}, \ddot{y})$$

4. Differential flatness of the distributed generators

while for the **generator's control input** one has

$$u = T_{do}[\dot{x}_3 + \frac{1}{T_d'}x_3\frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(x_1)], \text{ or}$$
$$u = f_b(y, \dot{y}, \ddot{y})$$

Consequently, **all state variables** and the **control input** of the synchronous generator are written as **differential functions** of the flat output and thus the differential flatness of the model is confirmed.

By defining the **new state variables** $y_1 = y,\ y_2 = \dot{y},\ y_3 = \ddot{y}$

the generator's model is transformed into the **canonical (Brunovsky) form**:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v$$

with $v = f_c(y, \dot{y}, \ddot{y}) + g_c(y, \dot{y}, \ddot{y})u$ where

$$f_c(y, \dot{y}, \ddot{y}) = (\frac{D}{2J})^2\dot{y} - \omega_0\frac{D}{2J}\frac{P_m}{2J} + \omega_0\frac{D}{(2J)^2}\frac{V_s}{x_{d\Sigma}'}x_3 sin(\dot{y}) +$$
$$+\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}\frac{1}{T_d'}x_3 sin(y) - \frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}\frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(y)sin(y) -$$
$$-\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}x_3 cos(y)\dot{y}$$

and

$$g_c(y, \dot{y}, \ddot{y}) = -\frac{\omega_0}{2J}\frac{1}{T_{do}}\frac{V_s}{x_{d\Sigma}'}sin(y)$$

4. Differential flatness of the distributed generators

Differential flatness can be also proven for the **model of the n-interconnected power generators**

The **flat output** is taken to be the vector of the turn angles of the n-power generators

$$y = [y_1^1, y_1^2, \cdots, y_1^n] \text{ or } y = \Delta\delta^1, \Delta\delta^2, \cdots, \Delta\delta^n$$

For the n-machines power generation system it holds

$$x_1^1 = y^1,\ x_1^2 = y^2,\ x_1^3 = y^3,\ \cdots,\ x_1^n = y^n$$

$$x_2^1 = \Delta\omega^1 = \dot{y}^1,\ x_2^2 = \Delta\omega^2 = \dot{y}^2,\ x_2^3 = \Delta\omega^3 = \dot{y}^3,\ \cdots,\ x_2^n = \Delta\omega^n = \dot{y}^n$$

Moreover, it holds

$$\dot{x}_2^i = -\frac{D_i}{2J_i} x_2^i + \frac{\omega_0}{2J_i} P_{mi} - \\ -\frac{\omega_0}{2J_i}[G_{ii} {x_3^i}^2 + x_3^i \sum_{j=1, j\neq i}^n [x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] \quad (4)$$

or using the flat outputs notation

$$\ddot{y}^i = -\frac{D_i}{2J_i} \dot{y}^i + \frac{\omega_0}{2J_i} P_{mi} - \\ -\frac{\omega_0}{2J_i}[G_{ii} {x_3^i}^2 + x_3^i \sum_{j=1, j\neq i}^n [x_3^j G_{ij} sin(y^i - y^j - \alpha_{ij})]$$

4. Differential flatness of the distributed generators

The **external mechanical torque** P_{mi} is considered to be a piecewise constant variable

From Eq. 4 and for one $i=1,2,\cdots,n$ has a system of n equations which can be solved with respect to the variables $x_3^i, i=1,2,\cdots,n$

Actually, all variables x_3^i, can be expressed as **differential functions of the flat outputs**

$$y^i,\ i=1,2,\cdots,n$$

and thus one has $x_3^i = f_{x_3}(y^1,y^2,\cdots,y^n)$

Moreover, from

$$\dot{E}_{q_i} = -\frac{1}{T_{d_i}}E'_{q_i} + \frac{1}{T_{d_{oi}}}\frac{x_{d_i}-x'_{d_i}}{x_{d_{\Sigma_i}}}V_{s_i}cos(\Delta\delta_i) + \frac{1}{T_{d_{oi}}}E_{f_i}$$

one can demonstrate that the control inputs $u_i = E_{f_i}$ can be expressed as **differential functions of the flat outputs** $y^i,\ i=1,2,\cdots,n$

Consequently, all state variables and the control inputs of the distributed power system can be expressed as differential functions of the flat outputs, and **the system is a differentially flat one.**

4. Differential flatness of the distributed generators

Next, the **external mechanical torque** P_{mi} is considered to be time-varying

The effect of this torque is viewed as a **disturbance** to each power generator

In such a case for a model of *n=3* interconnected generators one obtains the **input-output linearized dynamics**

and $\dot{z}_3^i = a^i(x) + b_1^i(x)g_1u_1 + b_2^i(x)g_2u_2 + b_3^i(x)g_3u_3 + \tilde{d}^i$ where $z_3^i = \ddot{\delta}^i = \dot{\omega}^i$

$$a^i = (\frac{D_i}{2J_i})^2 x_2^i + \frac{D_i\omega_0}{(2J_i)^2}[G_{ii}{x_3^i}^2 + x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] -$$
$$-\frac{\omega_0}{2J_i}[G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i}-x'_{d_i}}{x'_{d\Sigma_i}}V_{s_i}cos(x_1^i))]$$
$$-\frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i}-x'_{d_i}}{x'_{d\Sigma_i}}V_{s_i}cos(x_1^i))$$
$$-\frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij})x_2^i$$
$$+\frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij})x_2^j$$

and

$$b_1^i = -\frac{\omega_0}{2J_i}[2G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})]\frac{1}{T_{doi}}$$

$$b_2^i = -\frac{\omega_0}{2J_i}G_{i2} sin(x_1^i - x_1^2 - \alpha_{i2})\frac{1}{T_{do2}}$$

$$b_3^i = -\frac{\omega_0}{2J_i}G_{i3} sin(x_1^i - x_1^3 - \alpha_{i3})\frac{1}{T_{do3}}$$

4. Differential flatness of the distributed generators

while the external disturbances are:

$$\tilde{d}^i = -\frac{D_i\omega_0}{(2J_i)^2}P_m^i + \frac{\omega_0}{2J_i}\dot{P}_m^i$$

Thus, one has the following description of the dynamics of the *i*-th power generator

$$\begin{aligned}
\dot{z}_1^i &= z_2^i \\
\dot{z}_2^i &= z_3^i \\
\dot{z}_3^i &= a^i(x) + {b_1}^i g1 u_1 + {b_2}^i g_2 u_2 + {b_3}^i g_3 u_3 + \tilde{d}^i
\end{aligned}$$

For the complete system of the 3 generators one has

$$\begin{aligned}
\dot{z}_3^1 &= a^1(x) + {b_1}^1 g_1 u_1 + {b_2}^1 g_2 u_2 + {b_3}^1 g_3 u_3 + \tilde{d}^1 \\
\dot{z}_3^2 &= a^2(x) + {b_1}^2 g_1 u_1 + {b_2}^2 g_2 u_2 + {b_3}^2 g_3 u_3 + \tilde{d}^2 \\
\dot{z}_3^3 &= a^3(x) + {b_1}^3 g_1 u_1 + {b_2}^3 g_2 u_2 + {b_3}^3 g_3 u_3 + \tilde{d}^1
\end{aligned}$$

or in matrix form

$$\begin{pmatrix} \dot{z}_3^1 \\ \dot{z}_3^2 \\ \dot{z}_3^3 \end{pmatrix} = \begin{pmatrix} a^1(x) \\ a^2(x) \\ a^3(x) \end{pmatrix} + \begin{pmatrix} {b_1}^1 g1 & {b_2}^1 g_2 & {b_3}^1 g_3 \\ {b_1}^2 g1 & {b_2}^2 g_2 & {b_3}^2 g_3 \\ {b_1}^3 g1 & {b_2}^3 g_2 & {b_3}^3 g_3 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{d}_3 \end{pmatrix}$$

or, in an equivalent matrix form description one has $\dot{z}_3 = f_a(x) + Mu + \tilde{d}$

5. Stabilizing feedback control

Setting $v = f_a(x) + Mu + \tilde{d},$ one has for the i-th power generator

$$\begin{pmatrix} \dot{z}_1^i \\ \dot{z}_2^i \\ \dot{z}_3^i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^i \\ z_2^i \\ z_3^i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (v^i + \tilde{d}^i)$$

or

$$\dot{z} = Az + B(u^i + \tilde{d}^i)$$

and the stabilizing feedback control is given by:

$$v^i = z_d^{(3)^i} - k_3(\ddot{z}^i - \ddot{z}_d^i) - k_2(\dot{z}^i - \dot{z}_d^i) - k_1(z^i - z_d^i) - \tilde{d}^i$$

For the implementation of the distributed power generation control scheme, the controller at the *i*-th powergenerator makes use of not only its own state vector,

$$X^i = [x_1^i, x_2^i, x_3^i]^T$$

but also of the state vectors of the rest $n-1$ power generators

$$x^j = [x_1^j, x_2^j, x_3^j], \ j \neq i, \ j = 1, 2, \cdots, n$$

For the i-th power generator, written in the linearized canonical form, one can carry out state estimation using the Kalman Filter recursion

5. Stabilizing feedback control

Denoting by A_d, B_d, C_d the discrete-time equivalents of matrices A, B, C one has:

measurement update:

$$K(k) = P^-(k)\tilde{C}_d^T[\tilde{C}_d \cdot P^-(k)\tilde{C}_d^T + R]^{-1}$$
$$\hat{x}(k) = \hat{x}^-(k) + K(k)[z(k) - \tilde{C}_d\hat{x}^-(k)]$$
$$P(k) = P^-(k) - K(k)\tilde{C}_d P^-(k)$$

time update:

$$P^-(k+1) = \tilde{A}_d(k)P(k)\tilde{A}_d^T(k) + Q(k)$$
$$\hat{x}^-(k+1) = \tilde{A}_d(k)\hat{x}(k) + \tilde{B}_d(k)\tilde{v}(k)$$

For the i-th Kalman Filter it holds that: $\hat{x}_1^i = \hat{y}^i \quad \hat{x}_2^i = \hat{\dot{y}}^i$

while for the computation of $\hat{x}_3^i$ for $i = 1, 2, \cdots, n$

one has to solve the system of equations

$$\hat{\ddot{y}}^1 = -\frac{D_1}{2J_1}\hat{\dot{y}}^1 + \frac{\omega_0}{2J_1}P_{m1} - \frac{\omega_0}{2J_1}[G_{11}{x_3^1}^2 + \hat{x}_3^1\sum_{j=1,j\neq 1}^n[\hat{x}_3^j G_{1j} sin(y^1 - y^j - \alpha_{1j})]$$
$$\hat{\ddot{y}}^2 = -\frac{D_i}{2J_2}\hat{\dot{y}}^2 + \frac{\omega_0}{2J_2}P_{m2} - \frac{\omega_0}{2J_2}[G_{22}{x_3^2}^2 + x_3^2\sum_{j=1,j\neq 2}^n[\hat{x}_3^j G_{2j} sin(y^2 - y^j - \alpha_{2j})]$$
$$\cdots$$
$$\hat{\ddot{y}}^i = -\frac{D_i}{2J_i}\hat{\dot{y}}^i + \frac{\omega_0}{2J_i}P_{mi} - \frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2 + \hat{x}_3^i\sum_{j=1,j\neq i}^n[\hat{x}_3^j G_{ij} sin(y^i - y^j - \alpha_{ij})]$$

This is a nonlinear optimization problem which can be solved again with the use of nonlinear programming methods.

6. Estimation of the generators disturbance input using Kalman Filtering

It was shown that using differential flatness theory the initial nonlinear model of the PMSG can be written in the canonical form

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \ddot{y}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \dot{y}_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (v)$$

Taking also into account modelling errors and external perturbations in the dynamics of the i-th generator

$$y^{(3)^i} = v^i + \tilde{d}^i$$

$$y^{(3)^i} = v^i - \omega_0 \frac{D_i}{(2J_i)^2} P_m^i + \frac{\omega_0}{2J_i} \dot{P}_m^i \ \text{or}$$

$$y^{(3)^i} = v^i - T_m^i$$

where $$v^i = a^i(x) + b_1^i g_1 u_1 + b_2^i g_2 u_2 + b_3^i g_3 u_3 + \tilde{d}^i$$

$$T_m = -\omega_0 \frac{D}{(2J)^2} P_m + \frac{\omega_0}{2J} \dot{P}_m$$

Then, in the new state-space description one has

$$z_1 = y_1, \ z_2 = y_2, \ z_3 = y_3$$

$$z_4 = T_m = -\omega_0 \frac{D}{(2J)^2} P_m + \frac{\omega_0}{2J} \dot{P}_m, \ z_5 = \dot{T}_m, \ \text{and} \ z_6 = \ddot{T}_m$$

6. Estimation of the generators disturbance input using Kalman Filtering

The disturbance input is assumed to be described by its 3rd order derivative $\dot{z}_6 = T_m^{(3)}$

Using the previous definition of state variables one has the matrix equations

$$\dot{z}^i = \tilde{A} \cdot z^i + \tilde{B} \cdot \tilde{v}^i$$

where the inputs vector is given by $\tilde{v}^i = \left(v^i \quad {T_m^{(3)}}^i\right)^T$ and

$$\tilde{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \tilde{B} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \tilde{C}^T = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

where the measurable state variable is z_1. Since the dynamics of the disturbance input are taken to be unknown in the design of the associated disturbances' estimator one has the following dynamics

$$\dot{z}_o^i = \tilde{A}_o^i \cdot z^i + \tilde{B}_o^i \cdot \tilde{v}^i + K(C_o^i z^i - C_o^i \hat{z}^i)$$

where $K \in R^{6\times 1}$ is the state estimator's gain and

6. Estimation of the generators disturbance input using Kalman Filtering

$$\tilde{A}_o = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad \tilde{B}_o = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \tilde{C}_o^T = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The disturbance observer model of the PMSG defined above is observable.

Defining as $\tilde{A}_d$, $\tilde{B}_d$, and $\tilde{C}_d$, the discrete-time equivalents of matrices $\tilde{A}_o$, $\tilde{B}_o$ and $\tilde{C}_o$

a Derivative-free nonlinear Kalman Filter can be designed for the aforementioned representation of the system dynamics.

The associated Kalman Filter-based disturbance estimator is given by

measurement update:

$$K(k) = P^-(k)\tilde{C}_d^T[\tilde{C}_d \cdot P^-(k)\tilde{C}_d^T + R]^{-1}$$
$$\hat{x}(k) = \hat{x}^-(k) + K(k)[z(k) - \tilde{C}_d\hat{x}^-(k)]$$
$$P(k) = P^-(k) - K(k)\tilde{C}_d P^-(k)$$

time update:

$$P^-(k+1) = \tilde{A}_d(k)P(k)\tilde{A}_d^T(k) + Q(k)$$
$$\hat{x}^-(k+1) = \tilde{A}_d(k)\hat{x}(k) + \tilde{B}_d(k)\tilde{v}(k)$$

7. Simulation experiments

Different rotation speed setpoints had been assumed. Moreover, different input torques (mechanical input power profiles) have been assumed to affect the dynamic model of each local generator.

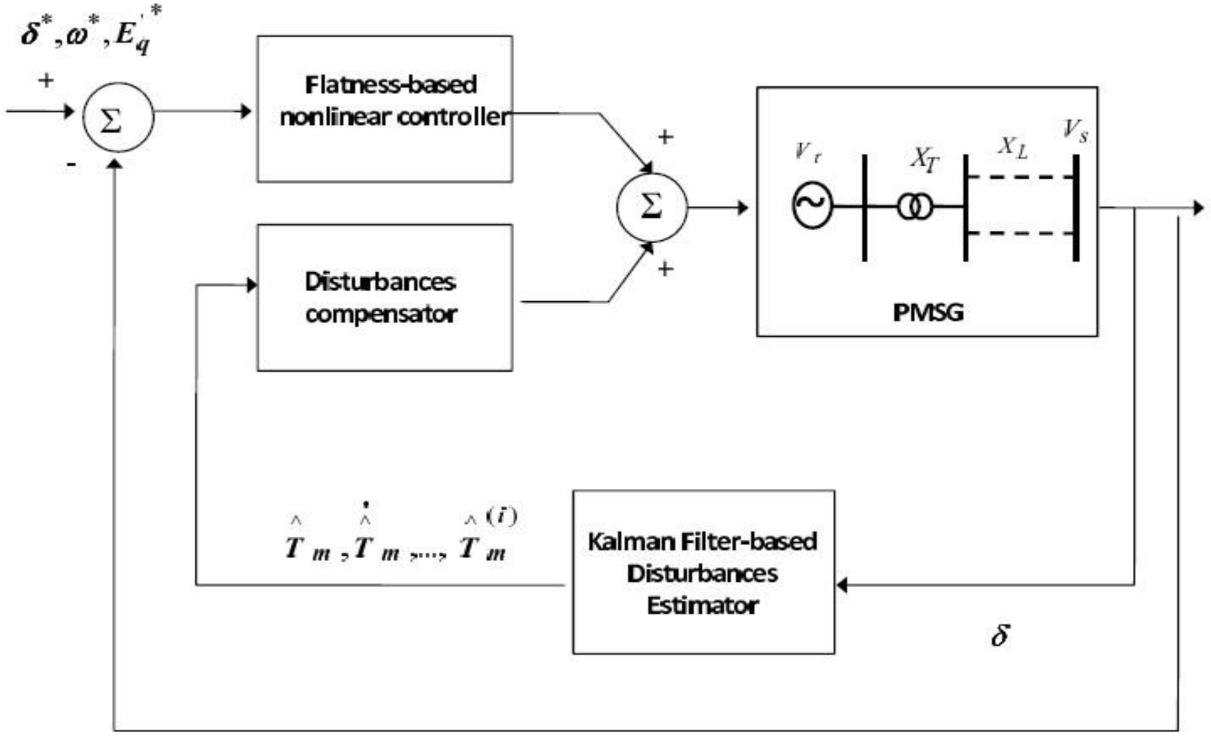
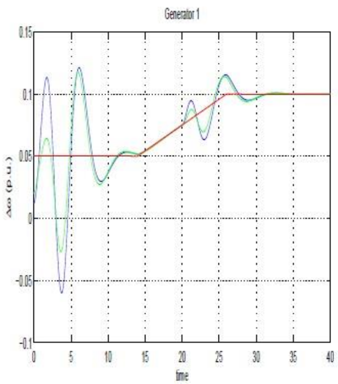
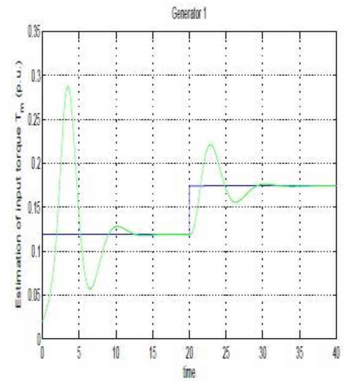


Fig. 2 Control loop for the local PSMG comprising a flatness-based nonlinear controller and a Kalman Filter-based disturbances estimator

Setpoint 1 – Generator 1:

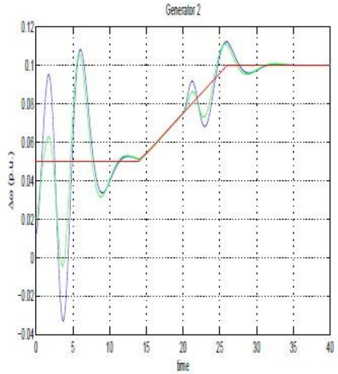


Convergence of the real and estimated values of the angular speed difference $\Delta\omega$

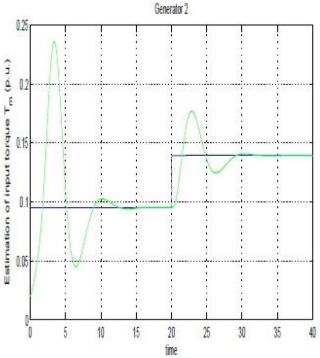


Convergence of the real and estimated values of the external mechanical input torque T_m

Setpoint 1 – Generator 2:

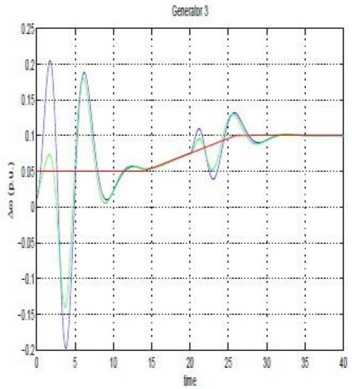


Convergence of the real and estimated values of the angular speed difference $\Delta\omega$

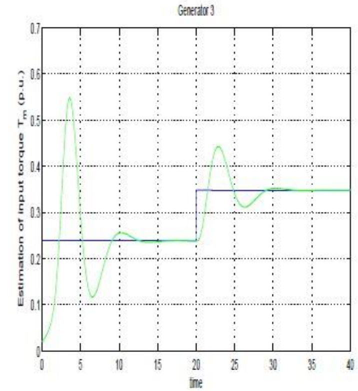


Convergence of the real and estimated values of the external mechanical input torque T_m

Setpoint 1 – Generator 3:

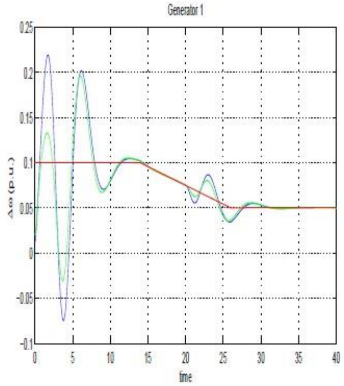


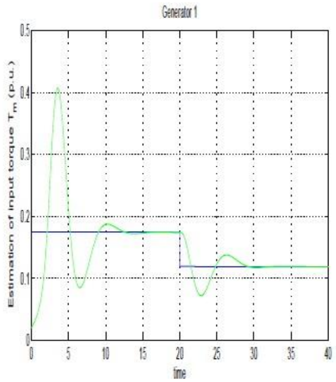
Convergence of the real and estimated values of the angular speed difference Δω



Convergence of the real and estimated values of the external mechanical input torque T_m

Setpoint 2 – Generator 1:

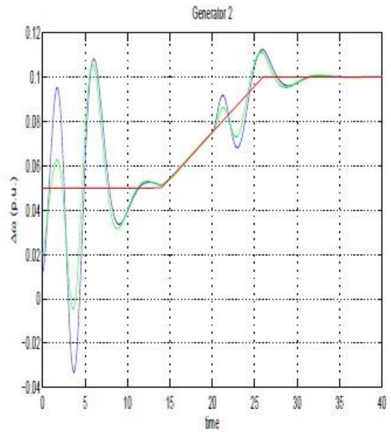




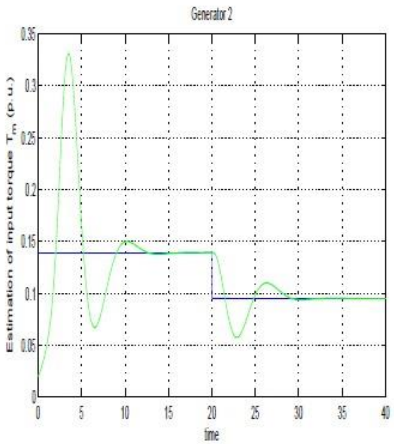
Convergence of the real and estimated values of the angular speed difference Δω

Convergence of the real and estimated values of the external mechanical input torque T_m

Setpoint 2 – Generator 2:

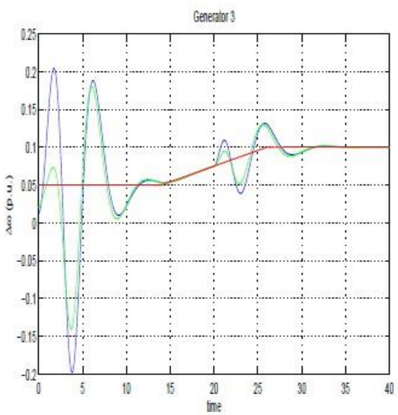


Convergence of the real and estimated values of the angular speed difference $\Delta\omega$

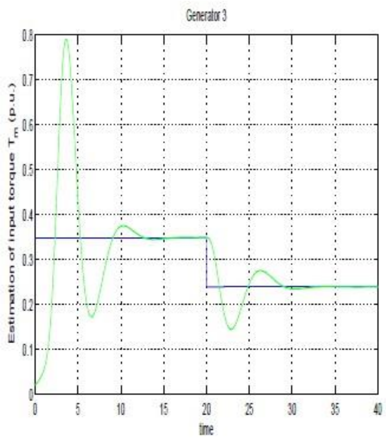


Convergence of the real and estimated values of the external mechanical input torque T_m

Setpoint 2 – Generator 3:



Convergence of the real and estimated values of the angular speed difference $\Delta\omega$



Convergence of the real and estimated values of the external mechanical input torque T_m

8. Conclusions

- The **global linearization-based control** method makes use of the **differential flatness theory** which enables to **transform** the initial nonlinear model of the **interconnected generators** into decoupled local linear models of the **canonical (Brunovsky) form**.

- The **flat output** is chosen to be a vectors with elements the turn angles of the rotors. All elements of the **interconnected power generators** system and the associated control inputs can be written as functions of the elements of the flat output vector and their derivatives.

- This procedure permits to introduce a **change of state variables (diffeomorphism)** and to write the initial nonlinear model of the interconnected generators into the **decoupled local linear models of the canonical form**. This facilitates the design of feedback controllers

- Unlike linearization with the use of Lie-derivatives, the **flatness-based linearization** approach does not require the computation of partial derivatives of the elements of the state vector or the computation of Jacobian matrices.

- To **estimate and compensate for modelling uncertainty** and external perturbation terms the **Derivative-free nonlinear Kalman Filter** has been used as a **disturbance observer.**