New approaches to nonlinear and optimal control of electric power systems: Applications to electric power generators and power electronics

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1 . Outline

● The efficient functioning of electric power systems relies on the solution of the associated nonlinear control and state estimation problems

● The main approaches followed towards the solution of nonlinear control problem are as follows: (i) **control with global linearization** methods (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods** (adaptive control methods) when the dynamic model of the electric power systems is unknown

● The main approaches followed towards the solution of the nonlinear state estimation problems are as follows: (i) state estimation with methods global linearization (ii) state estimation with methods of approximate (asymptotic) linearization

● Factors of major importance for the control loop of electric power systems are as follows (i) global stability conditions for the related nonlinear control scheme (ii) global stability conditions for the related nonlinear state estimation scheme (iii) global asymptotic stability for the joint control and state estimation scheme

2 . Nonlinear control and state estimation with global linearization

● To this end the differential flatness control theory is used

● The method can be applied to all nonlinear systems which are subject to an input-output linearization and actually such systems posses the property of differential flatness

● The state-space description for the dynamic model of the electric power systems is transformed into a more compact form that is input-output linearized. This is achieved after defining the system's flat outputs

● A system is differentially flat if the following two conditions hold: (i) all state variables and control inputs of the system can be expressed as differential functions of its flat outputs (ii) the flat outputs of the system and their time-derivatives are differentially independent, which means that they are not connected through a relation having the form of an ordinary differential equation

● With the applications of change of variables (diffeomorphisms) that rely on the differential flatness property (i), the state-space description of the electric power system is written into the linear canonical form. For the latter state-space description it is possible to solve both the control and the state estimation problem for the electric power system.

3 . Nonlinear control and state estimation with approximate linearization

● To this end the theory of optimal H-infinity control and the theory of optimal H-infinity state estimation are used

● The nonlinear state-space description of the electric power system undergoes approximate linearization around a temporary operating point which is updated at each iteration of the control and state estimation algorithm

● The linearization relies on first order Taylor series expansion around the temporary operating point and makes use of the computation of the associated Jacobian matrices

● The linearization error which is due to the truncation error of higher-order terms in the Taylor series expansion is considered to be a perturbation that is finally compensated by the robustness of the control algorithm

● For the linearized description of the state-space model an optimal H-infinity controller is designed. For the selection of the controller's feedback gains an algebraic Riccati equation has to be solved at each time step of the control algorithm

● Through Lyapunov stability analysis, the global stability properties of the control method are proven

● For the implementation of the optimal control method through the processing of measurements from a small number of sensors in the electric power system, the H-infinity Kalman Filter is used as a robust state estimator

4 . Nonlinear control and state estimation with Lyapunov methods

● By initially proving the differential flatness properties for the electric power system and by defining its flat outputs a transformation of Its state-space description into an equivalent input-output linearized form is achieved.

● The unknown dynamics of the electric power systems is incorporated into the transformed control inputs of the system, which now appear in its equivalent input-output linearized state-space description

● The control problem for the electric power systems of unknown dynamics in now turned into a problem of indirect adaptive control. The computation of the control inputs of the system is performed simultaneously with the identification of the nonlinear functions which constitute its unknown dynamics.

● The estimation of the unknown dynamics of the electric power system is performed through the adaptation of neurofuzzy approximators. The definition of the learning parameters takes place through gradient algorithms of proven convergence, as demonstrated by Lyapunov stability analysis

● The Lyapunov stability method is the tool for selecting both the gains of the stabilizing feedback controller and the learning rate of the estimator of the unknown system's dynamics

● Equivalently through Lyapunov stability analysis the feedback gains of the state estimators of the electric power system are chosen. Such observers are included in the control loop so as to enable feedback control through the processing of a small number of sensor measurements

Example 1: Nonlinear control and state estimation using global linearization

1. Outline

• **Nonlinear control for wind power generation units that comprise wind turbines, a drivetrain and asynchronous DFIG generators is** developed using **differential flatness theory** and **the Derivative-free nonlinear Kalman Filter**.

• T**he model of the wind power generation unit, is differentially flat** and thus the associated dynamic model can be transformed into a linear canonical form (Brunovsky form) or equivalently into an input-output linearized form

• For the equivalent globally linearized model of the wind power generator a **state feedback controller** can be designed, e.g. using pole placement methods. Such a controller processes measurements of the turn speed of the turbine's rotor, of the generator's rotor as well as of stator and rotor currents of the asynchronous generator

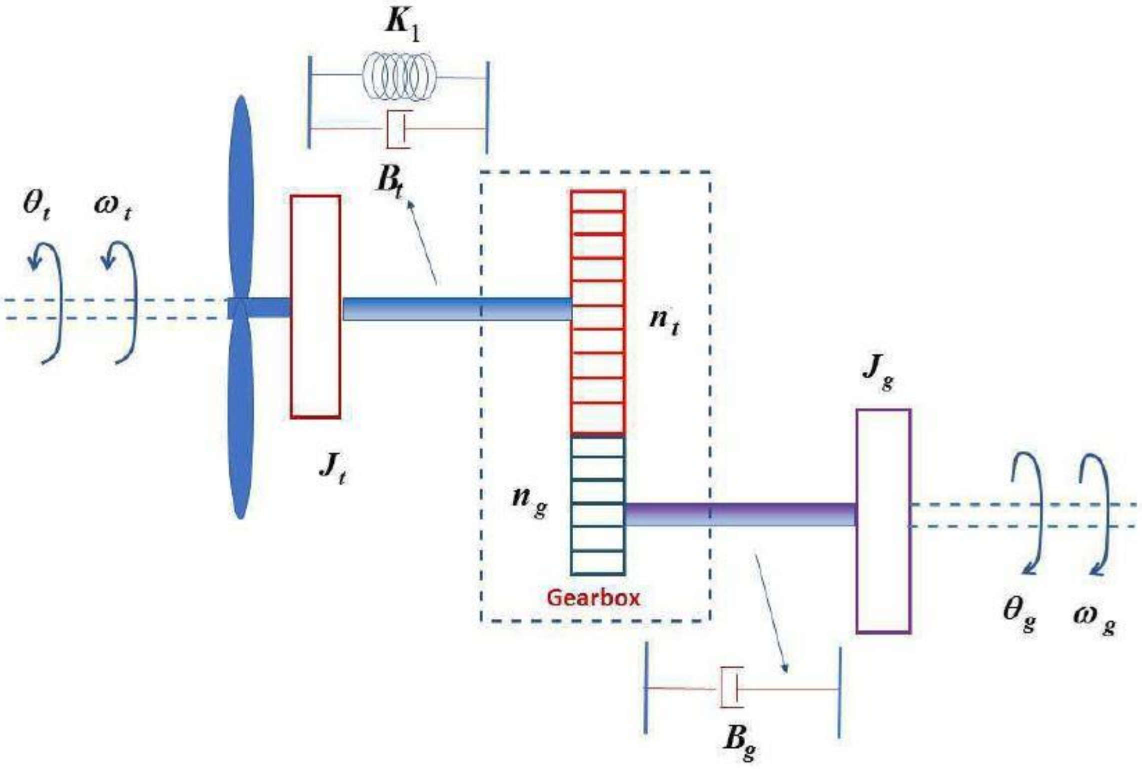
• To **estimate the non-measurable state variables** of the wind power unit, the **Derivative-free nonlinear Kalman Filter** is used. This consists of the Kalman Filter recursion applied to the local linearized model of the wind power unit and of an inverse transformation that is based on differential flatness theory, which enables to compute estimates of the state variables of the initial nonlinear model of the wind power system

• Furthermore, by **redesigning the aforementioned filter as a disturbance observer** it becomes also possible to estimate and compensate for disturbance terms that affect the wind power generation unit

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

The **wind power generation unit** comprising a **multi-mass drive-train** and a **DFIG** is shown in the following diagram



The wind power unit comprises a two-mass drive-train system receiving mechanical torque from a wind turbine, and a DFIG. :

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

The **rotational motion of the wind-turbine** and of the drivetrain system is described by the following differential equation

$$J_t \frac{d\omega_t}{dt} = c_{ba} T_m - T_{ls} - B_t \omega_t \qquad (1)$$

ω_t is the turn speed of the wind turbine

T_m is the mechanical torque provided by the wind

c_{ba} is a coefficient related with the pitch angle of the turbine's blades

The coefficient c_{ba} is a nonlinear function of blades pitch angle, rotor speed and wind speed.

T_{ls} is the torque of the low-speed shaft of the drivetrain (shaft on the side of the wind turbine)

B_t is a damping coefficient (friction) that opposes to the rotational motion of the shaft

Example 1: Nonlinear control and state estimation using global linearization
Dynamic model of the wind-turbine, drive-train and asynchronous generator

The torque of the low-speed shaft is given by

$$T_{ls} = K_1(\theta_t - \theta_g) + D_1(\omega_t - \omega_g) \quad (2)$$

θ_t is the turn angle of the wind turbine

θ_g is the turn angle of the generator's (DFIG) rotor

K_1 is an elasticity coefficient

D_1 is a damping coefficient

Considering that damping is small the torque of the low speed shaft becomes

$$T_{ls} = K_1(\theta_t - \theta_g)$$

The **ratio between the torque of the wind turbine's shaft** (low-speed shaft) and **the torque of the DFIG shaft (high-speed shaft)** is :

$$\frac{T_t}{T_g} = \frac{T_{ls}}{T_{hs}} = \frac{\omega_g}{\omega_t} = \frac{n_t}{n_g} \quad (3)$$

n_t is the number of teeth at the gears placed at the wind turbine's side

n_g is the number of teeth at the gear placed at the generator's side

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

The **rotational motion of the Doubly-Fed Induction Generator** is described by the following differential equation:

$$J_g \frac{d\omega_g}{dt} = T_{hs} - T_e - B_g \cdot \omega_g \quad (4)$$

J_g is the moment of inertia of the generator's rotor

ω_g is the turn speed of the generator's rotor

T_{hs} is the torque exerted at the high-speed shaft of the generator

B_g is a damping coefficient (friction) that opposes to the rotor's motion

T_e is the electromagnetic torque of the generator and is given by

$$T_e = \eta(i_{rq}\psi_{sd} - i_{rd}\psi_{sq}) \quad (5)$$

η is a variable related with the mutual inductance coefficient and with the number of poles

$[i_{rd}, i_{rq}]$ are the vector components of the DFIG's rotor currents in the asynchronously rotating dq reference frame

$[\psi_{sd}, \psi_{sq}]$ are the stator's components of the DFIG's magnetic fluxin the asynchronously rotating dq reference frame

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

The **joint dynamics of the drivetrain and of the DFIG** is expressed by the following wo differential equations

$$J_t \frac{d\omega_t}{dt} = c_{ba} T_m - K_1(\theta_t - \theta_g) - B_t \omega_t \quad (6)$$

$$J_g \frac{d\omega_g}{dt} = K_1(\theta_t - \theta_g)\frac{n_g}{n_t} - T_e - B_g \cdot \omega_g \quad (7)$$

Next, by defining the state variables: $x_1 = \theta_t,\ x_2 = \omega_t,\ x_3 = \theta_g,$ and $x_4 = \omega_g$

one arrives at the following state-space description of the system's dynamics

$$\begin{gathered} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{c_{ba}}{J_t} T_m - \frac{K_1}{J_t}(x_1 - x_3) - B_t x_2 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{K_1}{J_g}(x_1 - x_3)\frac{n_g}{n_t} - T_e - B_g \cdot x_4 \end{gathered} \quad (8)$$

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

About the **dynamics of the electrical part of the wind power generation unit** and using the **asynchronously rotating dq reference frame** one has

$$\frac{d\psi_{s_d}}{dt} = \omega_{dq}\psi_{s_q} - \frac{1}{T_s}\psi_{s_d} + \frac{M}{T_s}i_{r_d} + v_{s_d} \quad (9)$$

$$\frac{d\psi_{s_q}}{dt} = -\frac{1}{T_s}\psi_{s_q} - \omega_{dq}\psi_{s_d} + \frac{M}{T_s}i_{r_q} + v_{s_q} \quad (10)$$

$$\frac{di_{r_d}}{dt} = -\beta\omega_r\psi_{s_q} + \frac{\beta}{T_s}\psi_{s_d} + (\omega_{dq} - \omega_r)i_{r_q} - \gamma_2 i_{r_d} - \beta v_{s_d} + \frac{1}{\sigma L_r}v_{r_d} \quad (11)$$

$$\frac{di_{r_q}}{dt} = \frac{\beta}{T_s}\psi_{s_q} + \beta\omega_r\psi_{s_d} - \gamma_2 i_{r_q} - (\omega_{dq} - \omega_r)i_{r_d} - \beta v_{s_q} + \frac{1}{\sigma L_r}v_{r_q} \quad (12)$$

$\psi_{s_q}, \psi_{s_d}, i_{r_q}, i_{r_d}$ are the stator flux and the rotor currents

$v_{s_q}, v_{s_d}, v_{r_q}, v_{r_d}$ are the stator and rotor voltages

L_s and L_r are the stator and rotor inductances

ω_r is the rotor's angular velocity

M is the mutual inductance

R_s and R_r are the stator and rotor resistances

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

Moreover, the following parameters are defined

$$\sigma = 1 - \frac{M^2}{L_r L_s} \qquad \beta = \frac{1-\sigma}{M\sigma} \qquad \tau_s = \frac{L_s}{R_s}$$
$$\tau_r = \frac{L_r}{R_r} \qquad \gamma_2 = \left(\frac{1-\sigma}{\sigma\tau_s}\right)$$

The **dynamic model of the electric part of the doubly-fed induction generator** can be also written in state-space equations form by defining the following state variables:

$$x_5 = \psi_{s_d},\ x_6 = \psi_{s_q},\ x_7 = i_{r_d} \text{ and } x_8 = i_{r_q}$$

The state-space model of the electrical part is:

$$\dot{x}_5 = -\frac{1}{\tau_s}x_5 + \omega_{dq}x_6 + \frac{M}{\tau_s}x_7 + v_{s_d}$$
$$\dot{x}_6 = -\omega_{dq}x_5 - \frac{1}{\tau_s}x_6 + \frac{M}{\tau_s}x_8 + v_{s_q}$$
$$\dot{x}_7 = -\beta x_4 x_6 + \frac{\beta}{\tau_s}x_5 + (\omega_{dq} - x_4)x_8 - \gamma_2 x_7 + \frac{1}{\sigma L_s}v_{r_d} - \beta v_{s_d}$$
$$\dot{x}_8 = \frac{\beta}{\tau_s}x_6 + \beta x_4 x_5 + (\omega_{dq} - x_4)x_7 - \gamma_2 x_8 + \frac{1}{\sigma L_s}v_{s_d} - \beta v_{s_q}$$

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Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

The **joint dynamics of the mechanical and electrical part of the wind power generation unit**

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{K_1}{J_t}(x_1 - x_3) - B_g x_2 + \frac{u_1}{J_t} T_m$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{K_1}{J_g}(x_1 - x_3)\frac{n_g}{n_t} - \eta(x_8 x_5 - x_7 x_6) - B_g \cdot x_4$$
$$\dot{x}_5 = -\frac{1}{\tau_s} x_5 + \omega_{dq} x_6 + \frac{M}{\tau_s} x_7 + v_{s_d}$$
$$\dot{x}_6 = -\omega_{dq} x_5 - \frac{1}{\tau_s} x_6 + \frac{M}{\tau_s} x_8 + v_{s_q}$$
$$\dot{x}_7 = -\beta x_4 x_6 + \frac{\beta}{\tau_s} x_5 + (\omega_{dq} - x_4)x_8 - \gamma_2 x_7 - \beta v_{s_d} + \frac{1}{\sigma L_s} u_2$$
$$\dot{x}_8 = \frac{\beta}{\tau_s} x_6 + \beta x_4 x_5 + (\omega_{dq} - x_4)x_7 - \gamma_2 x_8 - \beta v_{s_q} + \frac{1}{\sigma L_s} u_3$$

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This is an **affine-in-the-input dynamical system** which is written in the form:

$$\dot{x} = f(x) + g(x)u$$

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where: $x \in R^{8\times 1}$, $f(x) \in R^{8\times 1}$, $g(x) \in R^{8\times 3}$ and $u \in R^{3\times 1}$.

Example 1: Nonlinear control and state estimation using global linearization

Dynamic model of the wind-turbine, drive-train and asynchronous generator

Vector f(x) and matrix G(x) are defined as follows:

$$f(x)=\begin{pmatrix} x_2 \\ -\frac{K_t}{J_t}(x_1-x_3)-B_g x_2 \\ x_4 \\ \frac{K_t}{J_g}(x_1-x_3)\frac{n_g}{n_t}-\eta(x_8x_5-x_7x_6)-B_g\cdot x_4 \\ -\frac{1}{\tau_s}x_5+\omega_{dq}x_6+\frac{M}{\tau_s}x_7+v_{s_d} \\ -\omega_{dq}x_5-\frac{1}{\tau_s}x_6+\frac{M}{\tau_s}x_8+v_{s_q} \\ -\beta x_4x_6+\frac{\beta}{\tau_s}x_5+(\omega_{dq}-x_4)x_8-\gamma_2x_7-\beta v_{s_d} \\ \frac{\beta}{\tau_s}x_6+\beta x_4x_5+(\omega_{dq}-x_4)x_7-\gamma_2x_8-\beta v_{s_q} \end{pmatrix} \quad (16)$$

$$g(x)=\begin{pmatrix} 0 & 0 & 0 \\ \frac{T_m}{J_t} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 \\ 0 & 0 & \frac{1}{\sigma L_s} \end{pmatrix} \quad (17)$$

Example 1: Nonlinear control and state estimation using global linearization

3. Differential flatness of the wind power generation unit

- **Differential flatness theory** has been developed as a **global linearization control method** by M. Fliess (Ecole Polytechnique, France) and co-researchers (Lévine, Rouchon, Mounier, Rudolph, Petit, Martin, Zhu, Sira-Ramirez et. al)

- A dynamical system can be written in the ODE form $S_i(w, \dot{w}, \ddot{w}, \ldots, w^{(i)}),\ \ i = 1,2,\ldots,q$

  where $w^{(i)}$ stands for the i-th derivative of either a state vector element or of a control input

- The system is said to be **differentially flat** with respect to the flat output

$$y_i = \phi(w, \dot{w}, \ddot{w}, \ldots, w^{(a)}),\ i = 1,\ldots,m \quad \text{where} \quad y = (y_1, y_2, \ldots, y_m)$$

if the following two conditions are satisfied

(i) There does not exist any differential relation of the form

$$R(y, \dot{y}, \ddot{y}, \ldots, y^{(\beta)}) = 0$$

which means that **the flat output and its derivatives are linearly independent**

(ii) All system variables are **functions of the flat output and its derivatives**

$$w^{(i)} = \psi(y, \dot{y}, \ddot{y}, \ldots, y^{(\gamma_i)})$$

Example 1: Nonlinear control and state estimation using global linearization

3. Differential flatness of the wind power generation unit

Next, it will be proven that the **wind-power system** which comprises the wind-turbine, the drivetrain and the DFIG is a **differentially flat** one.

The flat outputs vector is taken to be

$$Y = [x_1, x_3, x_5]^T = [\theta_t, \theta_g, \psi_{sd}]^T$$

From the first row of the state-space model one has:

$$x_2 = \dot{x}_1 \Rightarrow x_2 = h_2(Y, \dot{Y}) \tag{18}$$

that is x_2 is a **differential function of the flat outputs** of the system

From the third row of the state-space model one obtains

$$x_4 = \dot{x}_3 \Rightarrow x_4 = h_4(Y, \dot{Y})$$

which signifies that x_4 is also a **differential function of the flat outputs** of the system

From the sixth row of the state-space model one has that due to the assumption of field orientation, that is

$$x_6 = \psi_{sq} = 0, \tag{19}$$

Example 1: Nonlinear control and state estimation using global linearization

3. Differential flatness of the wind power generation unit

From the fifth row of the state-space model and using that

$$x_6 = \psi_{sq} = 0 \text{ and that } v_{sd} = V_s$$

one can solve with respect x_7

$$x_7 = \frac{\tau_s}{M}[\dot{x}_5] + \omega_{dq} x_5 - \frac{1}{\tau_s} x_6 - v_{sq} \Rightarrow x_7 = h_7(Y, \dot{Y}) \quad (19)$$

which means that state variable x_7 is a **differential function of the flat output** of the system

From the fourth row of the state-space model one can solve with respect to x_8. This gives

$$\dot{x}_4 = -\frac{K_1}{J_g}(x_1 - x_3)\frac{n_g}{n_t} + L_m x_7 x_6 + B_g x_4 - L_m x_5 x_8 \Rightarrow$$
$$x_8 = \frac{1}{L_m x_5}\left[\dot{x}_4 - \frac{K_1}{J_g}(x_1 - x_3)\frac{n_g}{n_t} + L_m x_7 x_6 + B_g x_4\right] \Rightarrow \quad (20)$$
$$x_8 = h_8(Y, \dot{Y})$$

Consequently x_8 is also a **function of the differentially flat outputs** of the system.

Example 1: Nonlinear control and state estimation using global linearization

3. Differential flatness of the wind power generation unit

Next, from the second row of the state-space model of the system one can solve with respect to the control input u_1. This gives:

$$u_1 = \frac{J_t}{T_m}[\dot{x}_2 + \frac{K_t}{J_t}(x_1 - x_3) + B_t x_2] \Rightarrow u_1 = q_1(Y, \dot{Y}) \quad (21)$$

which means that the control input u_1 is a **differential function of the flat outputs** of the system.

Equivalently, from the seventh row of the state-space model of the system one can solve with respect to the control input u_2. This gives:

$$u_2 = \sigma L_s\{\dot{x}_7 + \beta x_4 x_6 + \frac{\beta}{\tau_s} x_5 + (\omega_{dq} - x_4)x_8 + \gamma_2 x_7 + \beta v_{sd}\} \Rightarrow u_2 = q_2(Y, \dot{Y}) \quad (22)$$

which shows that the control input u_2 is a **differential function of the flat outputs** of the system

Finally, from the eight row of the state-space model of the system one can solve with respect to the control input u3. This gives:

$$u_3 = \sigma L_s\{\dot{x}_8 + \frac{\beta}{\tau_s} x_6 - \beta x_4 x_5 - (\omega_{dq} - x_4)x_7 + \gamma_2 x_8 + \beta v_{sq}\} \Rightarrow u_3 = q_3(Y, \dot{Y}) \quad (23)$$

Considering that due to field orientation $x_6 = 0$ and v_{sq} = one ha that the control u_3 is also written as a **differential function of the flat outputs** of the power unit.

Example 1: Nonlinear control and state estimation using global linearization

4. Transformation of wind-power system into input-output linearized form

The **flat outputs of the system are differentiated successively**, until **the control inputs reappear**. Thus, from the first row of the state-space model one has:

$$\dot{x}_1 = x_2 \Rightarrow \ddot{x}_1 = \dot{x}_2 \Rightarrow$$
$$\ddot{x}_1 = -\frac{K_1}{J_1}(x_1 - x_3) - B_g x_2 + \frac{T_m}{J_t} u_1 \qquad (24)$$

Equivalently, one can write

$$\dot{x}_1 = f_a(x) + g_{a_1}(x)u_1 + g_{a_2}(x)u_2 + g_{a_3}(x)u_3$$

where

$$f_a = -\frac{K_1}{J_1}(x_1 - x_3) - B_g x_2 \qquad (26)$$

$$g_{a_1}(x) = \frac{T_m}{J_t} \quad g_{a_1}(x) = 0 \quad g_{a_3}(x) = 0 \qquad (27)$$

From the third row of the state-space model and through successive differentiations one gets:

$$\begin{aligned} x_3^{(3)} = {} & \frac{K_1}{J_g}(x_2 - x_4)\frac{n_g}{n_t} - n x_5\left[\frac{\beta}{T_s}x_6 + \beta x_4 x_5 + (\omega_{dq} - x_4)x_7 - \gamma_2 x_8 - \beta v_{s_q}\right] - \\ & -n x_8\left[-\omega_{dq}x_5 - \frac{1}{T_s}x_6 + \frac{M}{T_s}x_8 + v_{s_q}\right] - \\ & -B_g\left[\frac{K_1}{J_g}(x_1 - x_3)\frac{n_g}{n_t} - \eta(x_8 x_5 - x_7 x_6) - B_g \cdot x_4\right] - \\ & -n x_5\left(\frac{1}{\sigma L_s}\right)u_3 \end{aligned} \qquad (28)$$

Example 1: Nonlinear control and state estimation using global linearization

4. Transformation of wind-power system into input-output linearized form

or equivalently

$$x_3^{(3)} = f_b(x) + g_{b_1}(x)u_1 + g_{b_2}(x)u_2 + g_{b_3}(x)u_3 \quad (29)$$

where

$$\begin{aligned} f_b(x) = {} & \frac{K_1}{J_g}(x_2 - x_4)\frac{n_g}{n_t} - n x_5\left[\frac{\beta}{T_s}x_6 + \beta x_4 x_5 + (\omega_{dq} - x_4)x_7 - \gamma_2 x_8 - \beta v_{s_q}\right] - \\ & - n x_8\left[-\omega_{dq}x_5 - \frac{1}{T_s}x_6 + \frac{M}{T_s}x_8 + v_{s_q}\right] - \\ & - B_g\left[\frac{K_1}{J_g}(x_1 - x_3)\frac{n_g}{n_t} - \eta(x_8 x_5 - x_7 x_6) - B_g \cdot x_4\right] \end{aligned} \quad (30)$$

and also

$$g_{b_1}(x) = 0 \qquad g_{b_2}(x) = 0 \qquad g_{b_3}(x) = -n x_5\left(\frac{1}{\sigma L_s}\right) \quad (31)$$

Next, from the fifth row of the state-space model one obtains:

$$\begin{aligned} \ddot{x}_5 = {} & \left\{\left[\frac{1}{T_s^2}x_5 - \frac{M}{T_s^2}x_7\right] + \frac{M}{T_s}\left[\frac{\beta}{T_s}x_5 + (\omega_{dq} - x_4)x_8 - \gamma_2 x_7 - \beta v_{s_d} + \right]\right\} + \\ & + \frac{M}{T_s}\frac{1}{\sigma L_s}u_2 \end{aligned} \quad (32)$$

Example 1: Nonlinear control and state estimation using global linearization

4. Transformation of wind-power system into input-output linearized form

The previous relation can be also written in the concise form:

$$\ddot{x}_5 = f_c(x) + g_{c_1}(x)u_1 + g_{c_2}(x)u_2 + g_{c_3}(x)u_3 \quad (33)$$

where

$$f_c(x) = \{[\frac{1}{T_s^2}x_5 - \frac{M}{T_s^2}x_7] + \frac{M}{T_s}[\frac{\beta}{T_s}x_5 + (\omega_{dq} - x_4)x_8 - \gamma_2 x_7 - \beta v_{s_d} +]\} \quad (34)$$

$$g_{c_1}(x) = 0 \qquad g_{c_2}(x) = \frac{M}{T_s}\frac{1}{\sigma L_s} \qquad g_{c_3}(x) = 0 \quad (35)$$

Using the previous analysis, the **input-output linearized description of the system** becomes:

$$\begin{pmatrix} \ddot{x}_1 \\ x_3^{(3)} \\ \ddot{x}_5 \end{pmatrix} = \begin{pmatrix} f_a(x) \\ f_b(x) \\ f_c(x) \end{pmatrix} + \begin{pmatrix} g_{a_1}(x) & g_{a_2}(x) & g_{a_3}(x) \\ g_{b_1}(x) & g_{b_2}(x) & g_{b_3}(x) \\ g_{c_1}(x) & g_{c_2}(x) & g_{c_3}(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \quad (36)$$

Example 1: Nonlinear control and state estimation using global linearization

4. Transformation of wind-power system into input-output linearized form

or equivalently by defining the vectors $X=[\dot{x}_1,\ddot{x}_3,\dot{x}_5]^T$, and $U=[u_1,u_2,u_3]^T$

one has

$$\dot{X}=F(x)+G(x)U \quad (37)$$

Next one can define the **following virtual control inputs**

$$\begin{aligned} v_1 &= f_a(x)+g_{a_1}(x)u_1+g_{a_2}(x)u_2+g_{a_3}(x)u_3 \\ v_2 &= f_b(x)+g_{b_1}(x)u_1+g_{b_2}(x)u_2+g_{b_3}(x)u_3 \\ v_3 &= f_c(x)+g_{c_1}(x)u_1+g_{c_2}(x)u_2+g_{c_3}(x)u_3 \end{aligned} \quad (38)$$

The **input-output linearized form of the system** is written as:

$$\ddot{x}_1=v_1 \quad x_3^{(3)}=v_2 \quad \ddot{x}_5=v_3 \quad (39)$$

the **stabilizing feedback control** of the wind power generation unit is

$$\begin{aligned} v_1 &= \ddot{x}_{1,d}-k_1^a(\dot{x}_1-\dot{x}_{1,d})-k_2^a(x_1-x_{1,d}) \\ v_2 &= x_{3,d}^{(3)}-k_1^b(\ddot{x}_3-\ddot{x}_{3,d})-k_2^b(\dot{x}_3-\dot{x}_{3,d})-k_3^b(x_3-x_{3,d}) \\ v_3 &= \ddot{x}_{5,d}-k_1^c(\dot{x}_5-\dot{x}_{5,d})-k_2^c(x_5-x_{5,d}) \end{aligned} \quad (40)$$

Example 1: Nonlinear control and state estimation using global linearization

5. State estimation with the Derivative-free nonlinear Kalman Filter

To perform **state estimation for the wind-turbine and DFIG power unit**, there is need to obtain measurements of some elements of the wind power unit's state vector.

First, **the turbine's turn angle can be measured** directly with the use of an encoder.

Due to the fact that the **magnetic flux of the stator cannot be measured directly**, equations that provide **indirect measurements of the flux** (computed through measurements of the stator and rotor currents) will be used,

$$\begin{aligned} \psi_{s_d} &= L_s i_{s_d} + M i_{r_d} \\ \psi_{s_q} &= L_s i_{s_q} + M i_{r_q} \end{aligned} \quad (41)$$

Using **the input-output linearized model**

$$\ddot{x}_1 = v_1 \quad x_3^{(3)} = v_2 \quad \ddot{x}_5 = v_3 \quad (39)$$

the **state estimator for the wind-power generation system** is given by

$$\begin{aligned} \dot{\hat{Z}} &= A_m \hat{Z} + B_m V + K(Z^m - C_m \hat{Z}) \\ \hat{Z}^m &= C_m \hat{Z} \end{aligned}$$

with $Z = [\theta_t, \dot{\theta}_t, \theta_g, \dot{\theta}_g, \psi_{s_d}, \dot{\psi}_{s_d}, \ddot{\psi}_{s_d}]^T$ and $K \in R^{7\times 2}$,

Example 1: Nonlinear control and state estimation using global linearization

5. State estimation with the Derivative-free nonlinear Kalman Filter

About the linearized state-space description of the system (canonical Brunovsky form) it holds that

$$A_m = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad B_m = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad C_m = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

One can measure

(i) The turn angle of the turbine $y_1 = \theta_t$

(ii) The turn angle of the generator $y_2 = \theta_g$

while about the magnetic flux it holds that $y_3 = \psi_s^2 = \psi_{s_d}^2 + \psi_{s_q}^2$,

and due to field orientation one has $y_3 = \psi_s^2 = \psi_{s_d}^2$

Moreover, the following relation allows to compute the magnetic flux from stator currents measurements

$$\psi_{s_d} = L_s i_{s_d} + M i_{r_d}$$
$$\psi_{s_q} = 0$$

Example 1: Nonlinear control and state estimation using global linearization

5. State estimation with the Derivative-free nonlinear Kalman Filter

The **observer's gain K_f** is obtained through the **Kalman Filter recursion**. The application of the Kalman Filter to the linearized model of the system is known as **Derivative-free nonlinear Kalman Filter.**

The estimation process is complemented by **inverse transformations relying on the differential flatness properties of the system**. These allow for identifying the state variables of the initial nonlinear system

Matrices **A_o, B_o and C_o are substituted by their discrete-time equivalents**, through the application of common discretization procedures. These equivalents are denoted as: A_d, B_d and C_d. Thus, finally the **Kalman Filter's recursion** becomes:

measurement update:

$$\begin{array}{c} K_f(k) = P^-(k)C_d^T[C_dP^-(k)C_d^T + R]^{-1} \\ \hat{Z}(k) = \hat{Z}^-(k) + K_f(k)[Z_m - \hat{Z}_m] \\ P(k) = P^-(k) - K_f(k)C_dP^-(k) \end{array} \quad (42)$$

time update:

$$\begin{array}{c} P^-(k+1) = A_dP(k)A_d^T + Q \\ \hat{Z}^-(k+1) = A_d\hat{Z}(k) + B_dv(k) \end{array} \quad (43)$$

Example 1: Nonlinear control and state estimation using global linearization

5. State estimation with the Derivative-free nonlinear Kalman Filter

Next, it is considered that the input-output linearized wind-power generation system is subject to additive input disturbances, having the following form:

$$\ddot{x}_1 = v_1 + d_1 \quad x_3^{(3)} = v_2 + d_2 \quad \ddot{x}_5 = v_3 + d_3$$

Each disturbance input is considered to be described by its 2nd order derivative and the associated initial conditions

The system's state vector is extended by defining as additional state variables the disturbance inputs and their time-derivatives.

Thus one obtains the following state-space description:

$$\dot{Z} = AZ + B\bar{V}$$
$$Z^m = CZ$$

where the extended state vector is defined as

$$Z = [z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}, z_{13}]^T$$

with $z_1 = x_1,\ z_2 = \dot{x}_1,\ z_3 = x_3,\ z_4 = \dot{x}_3,\ z_5 = \ddot{x}_3,\ z_6 = x_5,$

$z_7 = \dot{x}_5 \quad z_8 = d_1,\ z_9 = \dot{d}_1,\ z_{10} = d_2,\ z_{11} = \dot{d}_2,\ z_{12} = d_3,\ z_{13} = \dot{d}_3.$

and $\bar{V} = [v_1, v_2, v_3, f_{d_1}, f_{d_2}, f_{d_3}]^T$

Example 1: Nonlinear control and state estimation using global linearization

5. State estimation with the Derivative-free nonlinear Kalman Filter

About matrices A,B and C appearing in the previous extended state-space description

$$A=\begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix}
\quad
B=\begin{pmatrix}
0&0&0&0&0&0\\
1&0&0&0&0&0\\
0&0&0&0&0&0\\
0&0&0&0&0&0\\
0&1&0&0&0&0\\
0&0&0&0&0&0\\
0&0&1&0&0&0\\
0&0&0&0&0&0\\
0&0&0&1&0&0\\
0&0&0&0&0&0\\
0&0&0&0&1&0\\
0&0&0&0&0&0\\
0&0&0&0&0&1
\end{pmatrix}
\quad
C=\begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0
\end{pmatrix}$$

The extended state-observer for the system is

$$\dot{\hat{Z}} = A_o Z + B_o V + K_f(Z - \hat{Z}^m)$$
$$\hat{Z}^m = C_o \hat{Z}$$

with $A_o = A \in R^{13\times 13},\ C_o = C \in R^{3\times 13},\ V = [v_1, v_2, v_3]^T$

and $$B_o^T = \begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0
\end{pmatrix}$$

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

The performance of the proposed flatness-based control approach for the wind-power generation system that comprises a wind turbine, a drive-train and an asynchronous DFIG generator has been further confirmed through simulation experiments.

Setpoint 1

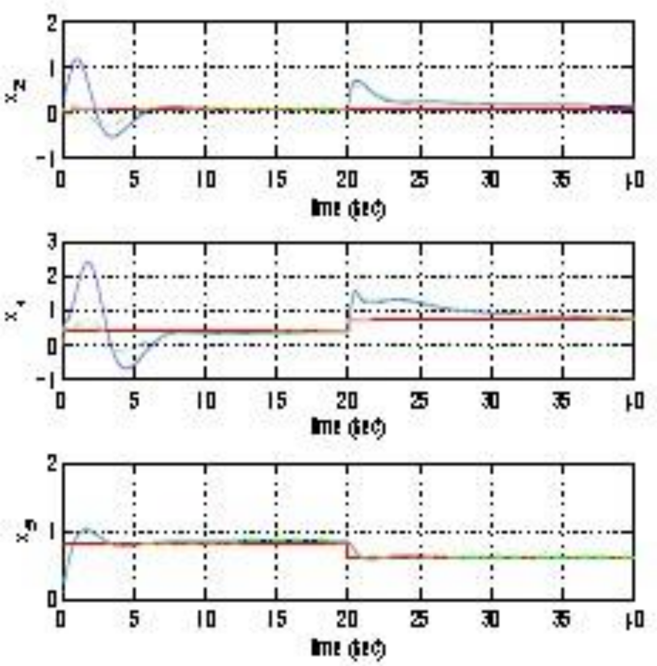

Fig. 2a. Convergence of state variables x_2,x_4,x_5 to reference setpoints

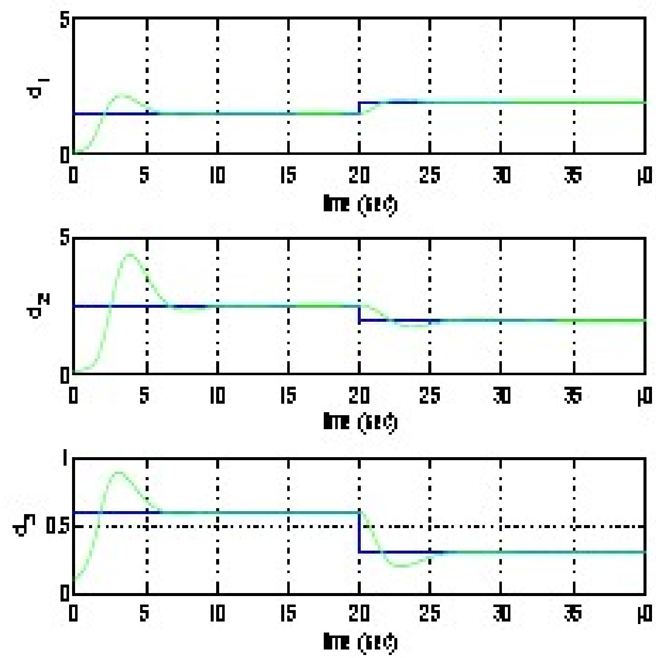

Fig. 2b. Estimation of additive disturbance inputs d_1,d_2,d_3 with the use of the Kalman Filter

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

Setpoint 2

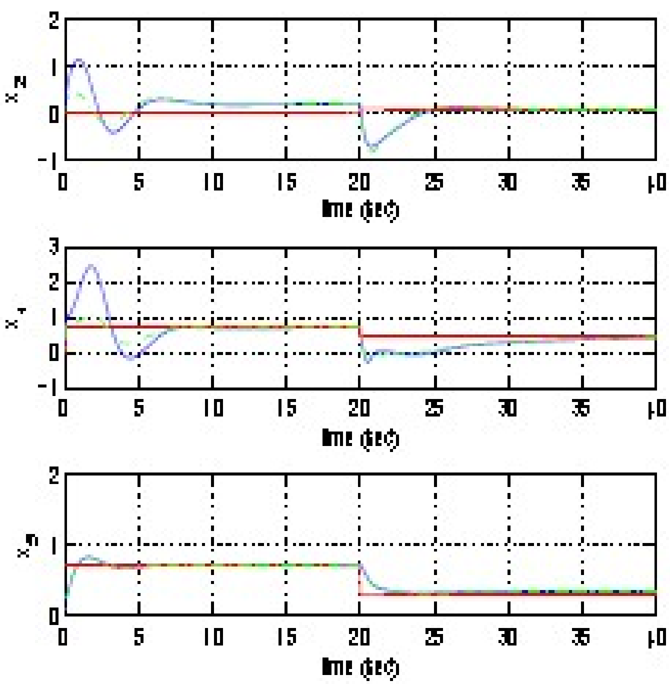

Fig. 3a. Convergence of state variables x_2,x_4,x_5 to reference setpoints

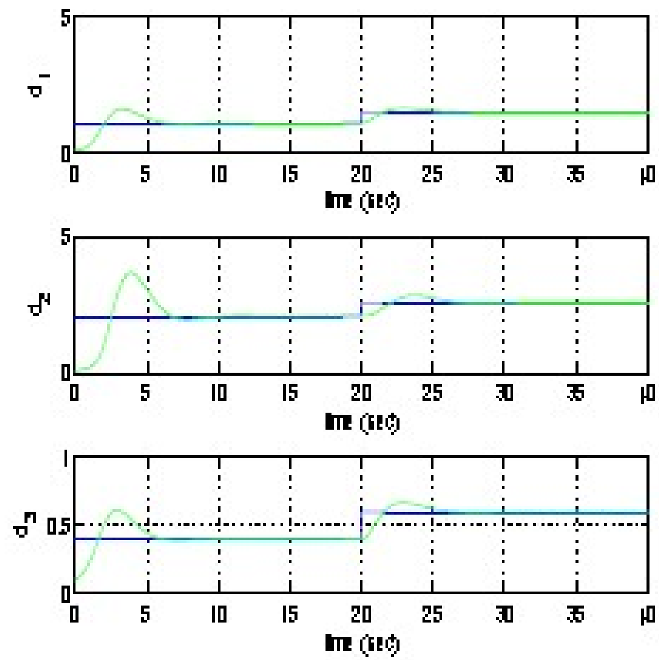

Fig. 3b. Estimation of additive disturbance inputs d_1,d_2,d_3 with the use of the Kalman Filter

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

Setpoint 3

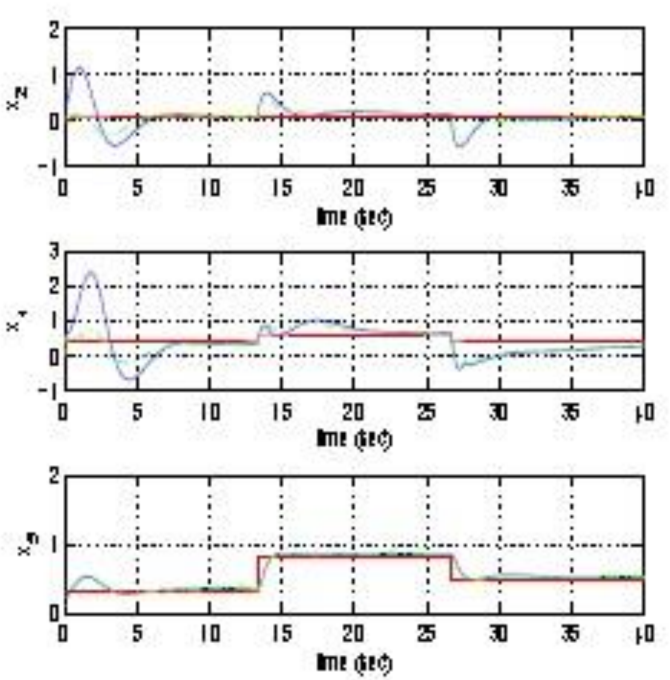

Fig. 4a. Convergence of state variables x_2,x_4,x_5 to reference setpoints

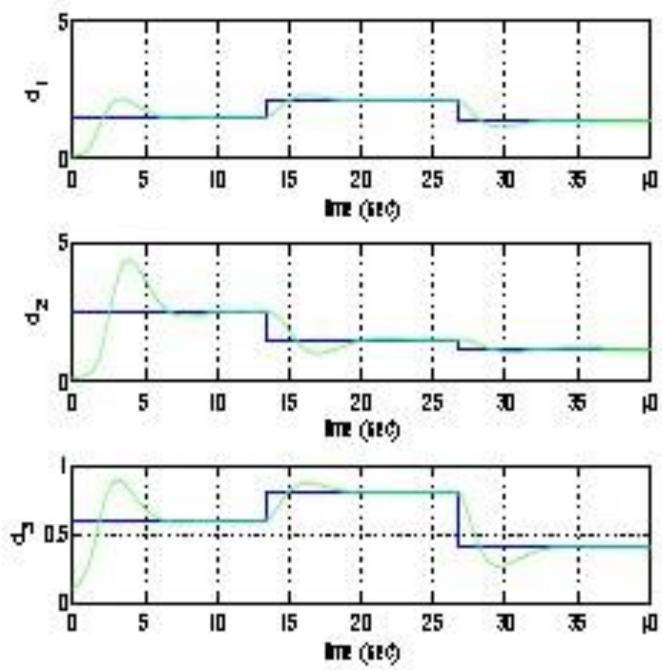

Fig. 4b. Estimation of additive disturbance inputs d_1,d_2,d_3 with the use of the Kalman Filter

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

Setpoint 4

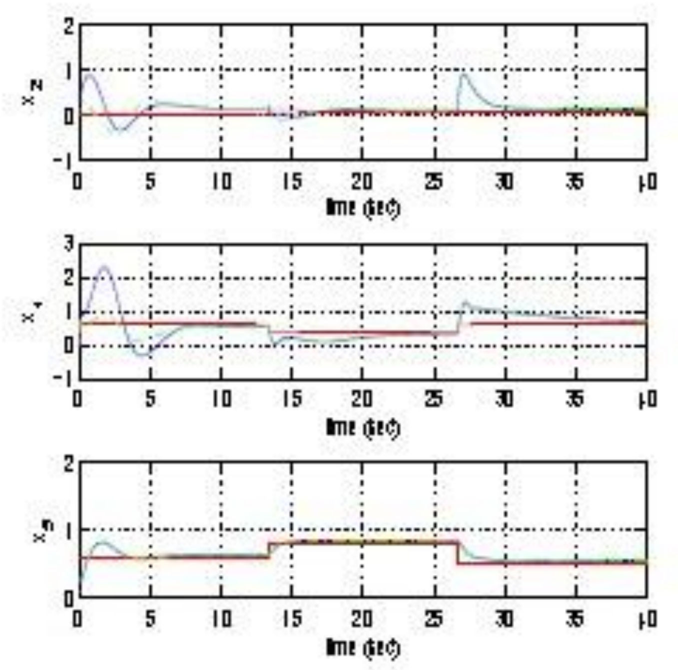

Fig. 5a. Convergence of state variables x_2,x_4,x_5 to reference setpoints

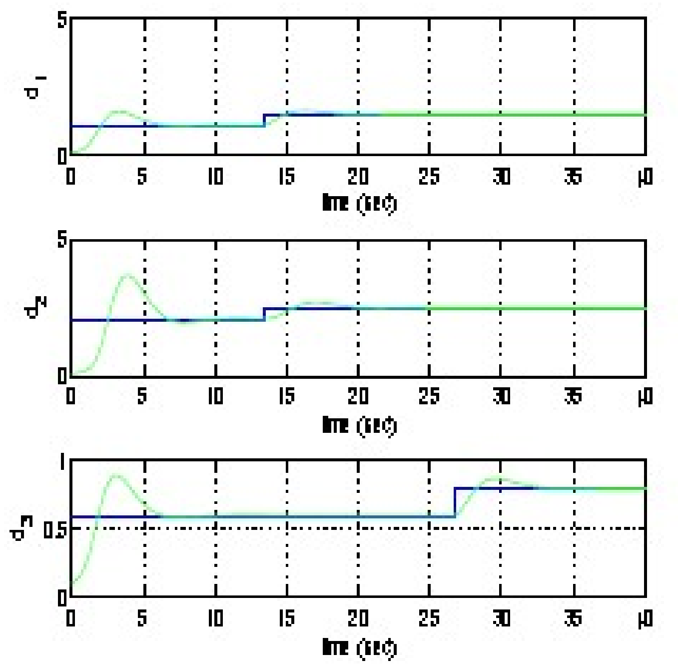

Fig. 5b. Estimation of additive disturbance inputs d_1,d_2,d_3 with the use of the Kalman Filter

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

Setpoint 5

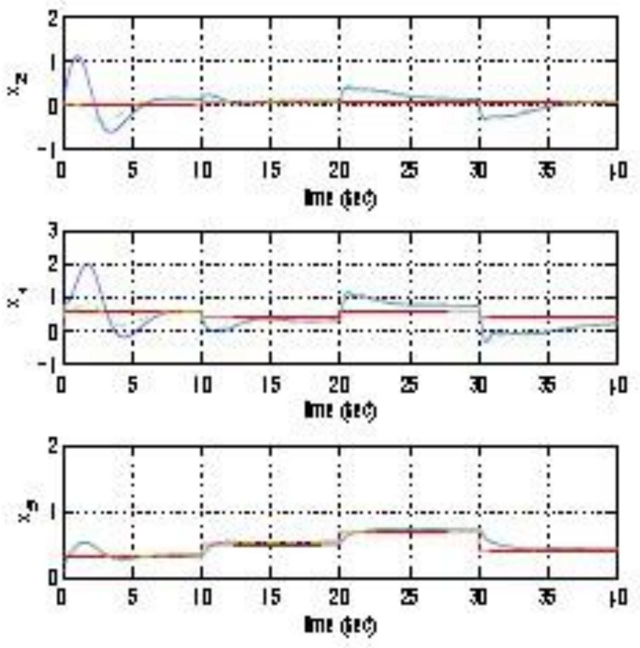

Fig. 6a. Convergence of state variables x_2,x_4,x_5 to reference setpoints

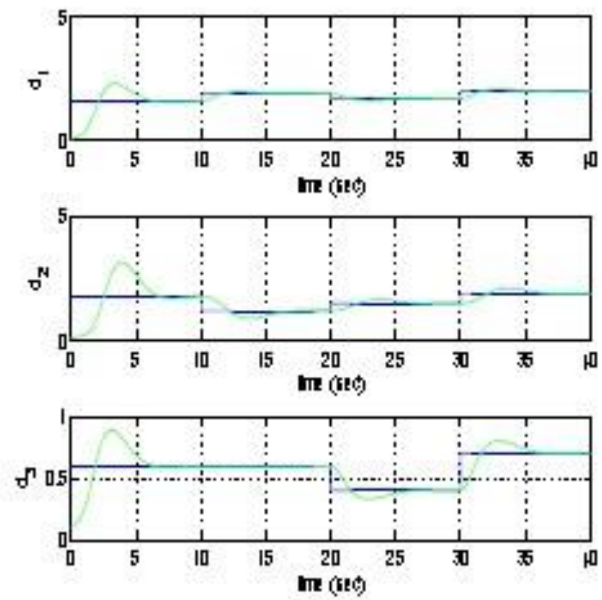

Fig. 6b. Estimation of additive disturbance inputs d_1,d_2,d_3 with the use of the Kalman Filter

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

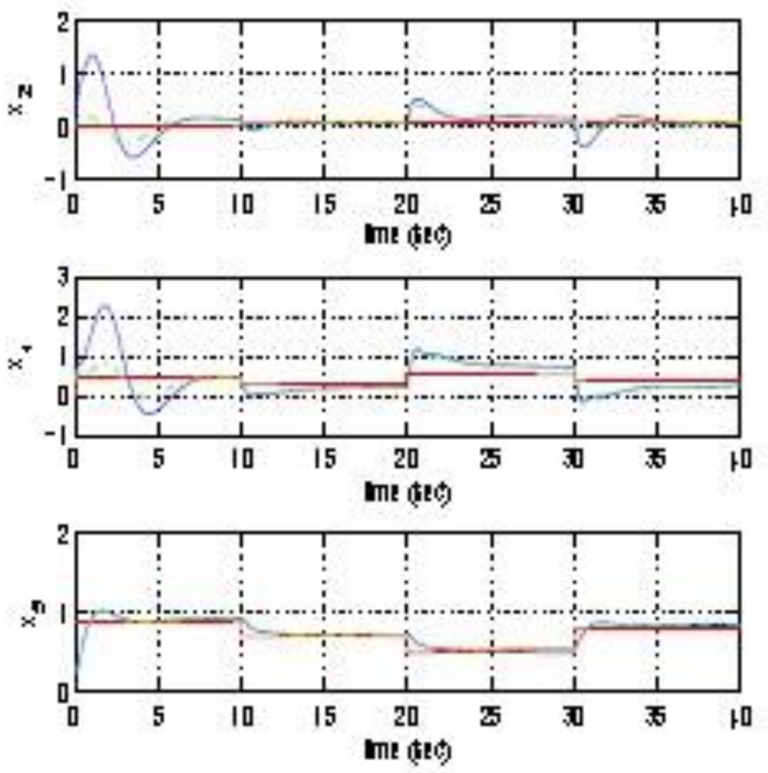

Fig. 7a. Convergence of state variables x_2,x_4,x_5 to reference setpoints

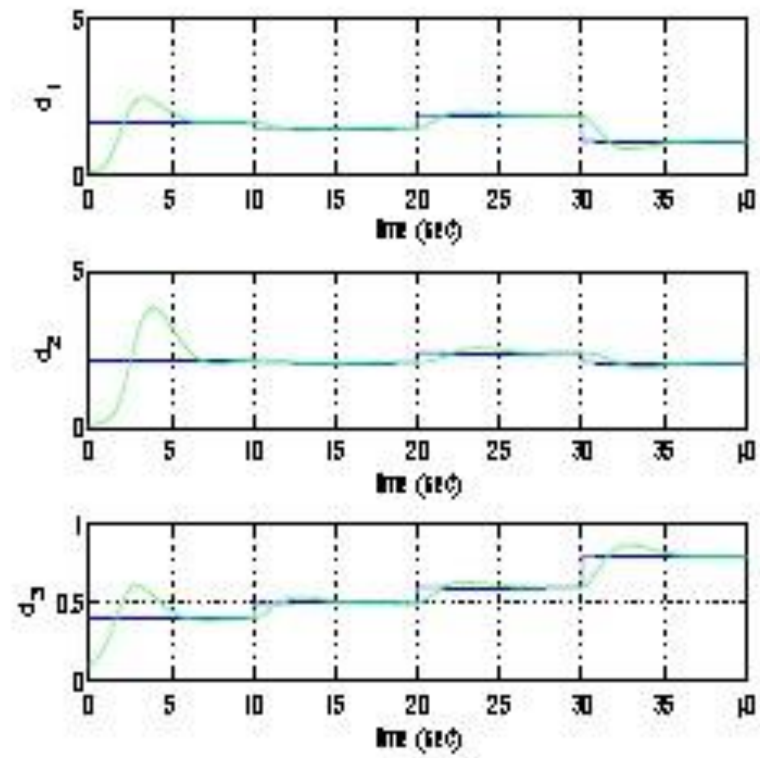

Fig. 7b. Estimation of additive disturbance inputs d_1,d_2,d_3 with the use of the Kalman Filter

Example 1: Nonlinear control and state estimation using global linearization

7. Conclusions

- It has been proven that the state-space model of the wind power system is a differentially flat one

- This signifies that all state variables and the control inputs of the system can be expressed as differential functions of certain state vector elements, the latter known as flat outputs of the system.

- The differential flatness property signifies that the power unit's dynamic model can be transformed into the linear canonical (Brunovsky) form.

- After expressing the system's dynamics into an input-output linearized form, the solution of the associated control and state-estimation problems has become possible.

- To stabilize the asynchronous power generation unit a state feedback controller has been designed with the pole-placement technique.

- To perform state estimation the Derivative-free nonlinear Kalman Filter has been used

- By redesigning the aforementioned Kalman Filter as a disturbance observer it has become possible to identify in real-time additive disturbance inputs that affect the dynamics of thepower unit.

Example 2: Nonlinear control and state estimation using approximate linearization

1. Outline

● A new nonlinear H-infinity control approach is applied to wind power generation units. First, the model of the wind power system undergoes approximate linearization, through Taylor series expansion, round local operating points which are defined at each time instant by the present value of the system's state vector and the last value of the control input exerted on it.

● The linearization procedure requires the computation of Jacobian matrices. The modelling error, which is due to the truncation of higher order terms in the Taylor series expansion is perceived as a perturbation that should be compensated by the robustness of the control loop. Next, for the linearized wind power generation system, an H-infinity feedback control loop is designed.

● This approach, is based on the concept of a differential game that takes place between the control input (which tries to minimize the deviation of the state vector from the reference setpoints) and the disturbance input (that tries to maximize it).

● In such a case, the computation of the optimal control input requires the solution of an algebraic Riccati equation at each iteration of the control algorithm. The known robustness properties of H-infinity control enable compensation of model uncertainty and perturbations

● The stability of the control loop is proven through Lyapunov analysis. Actually, it is shown that H-infinity tracking performance is succeeded, while conditionally the asymptotic stability of the control loop is also demonstrated.

Example 2: Nonlinear control and state estimation using approximate linearization

2. Dynamic model of the drivetrain and DFIG wind power generation unit

The wind power generation unit comprises a wind turbine, a drivetrain and an asynchronous power generator (Doubly-fed induction generator - DFIG)

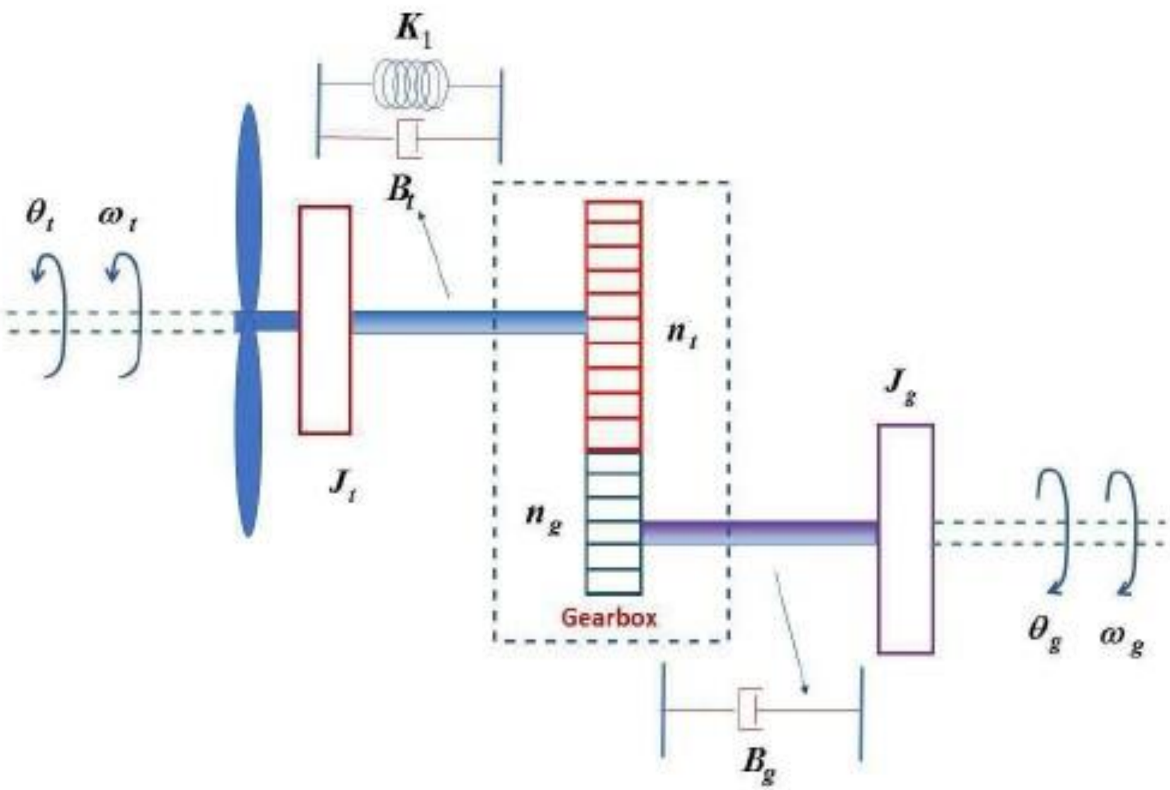

Fig. 1: Wind power generation unit comprising a multi-mass drive-train and a DFIG

Example 2: Nonlinear control and state estimation using approximate linearization

2. Dynamic model of the drivetrain and DFIG wind power generation unit

$$
\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= -\frac{K_1}{J_t}(x_1-x_3) - B_g x_2 + \frac{u_1}{J_t}T_m\\
\dot{x}_3 &= x_4\\
\dot{x}_4 &= \frac{K_1}{J_g}(x_1-x_3)\frac{n_g}{n_t} - \eta(x_8x_5 - x_7x_6) - B_g\cdot x_4\\
\dot{x}_5 &= -\frac{1}{\tau_s}x_5 + \omega_{dq}x_6 + \frac{M}{\tau_s}x_7 + v_{s_d}\\
\dot{x}_6 &= -\omega_{dq}x_5 - \frac{1}{\tau_s}x_6 + \frac{M}{\tau_s}x_8 + v_{s_q}\\
\dot{x}_7 &= -\beta x_4x_6 + \frac{\beta}{\tau_s}x_5 + (\omega_{dq}-x_4)x_8 - \gamma_2x_7 - \beta v_{s_d} + \frac{1}{\sigma L_s}u_2\\
\dot{x}_8 &= \frac{\beta}{\tau_s}x_6 + \beta x_4x_5 + (\omega_{dq}-x_4)x_7 - \gamma_2x_8 - \beta v_{s_q} + \frac{1}{\sigma L_s}u_3
\end{aligned}
$$

1

2

In vector fields form one has $\dot{x} = f(x) + g(x)u$

where $x\in R^{8\times 1}$, $f(x)\in R^{8\times 1}$, $g(x)\in R^{8\times 3}$ and $u\in R^{3\times 1}$

with

$$
f(x) = \begin{pmatrix}
x_2\\
-\frac{K_1}{J_t}(x_1-x_3) - B_g x_2\\
x_4\\
\frac{K_1}{J_g}(x_1-x_3)\frac{n_g}{n_t} - \eta(x_8x_5 - x_7x_6) - B_g\cdot x_4\\
-\frac{1}{\tau_s}x_5 + \omega_{dq}x_6 + \frac{M}{\tau_s}x_7 + v_{s_d}\\
-\omega_{dq}x_5 - \frac{1}{\tau_s}x_6 + \frac{M}{\tau_s}x_8 + v_{s_q}\\
-\beta x_4x_6 + \frac{\beta}{\tau_s}x_5 + (\omega_{dq}-x_4)x_8 - \gamma_2x_7 - \beta v_{s_d}\\
\frac{\beta}{\tau_s}x_6 + \beta x_4x_5 + (\omega_{dq}-x_4)x_7 - \gamma_2x_8 - \beta v_{s_q}
\end{pmatrix}
\qquad
g(x) = \begin{pmatrix}
0 & 0 & 0\\
\frac{T_m}{J_t} & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & \frac{1}{\sigma L_s} & 0\\
0 & 0 & \frac{1}{\sigma L_s}
\end{pmatrix}
$$

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the wind power generation unit

The configuration of the control scheme for the DFIG-based wind power generation system is given in the following diagram:

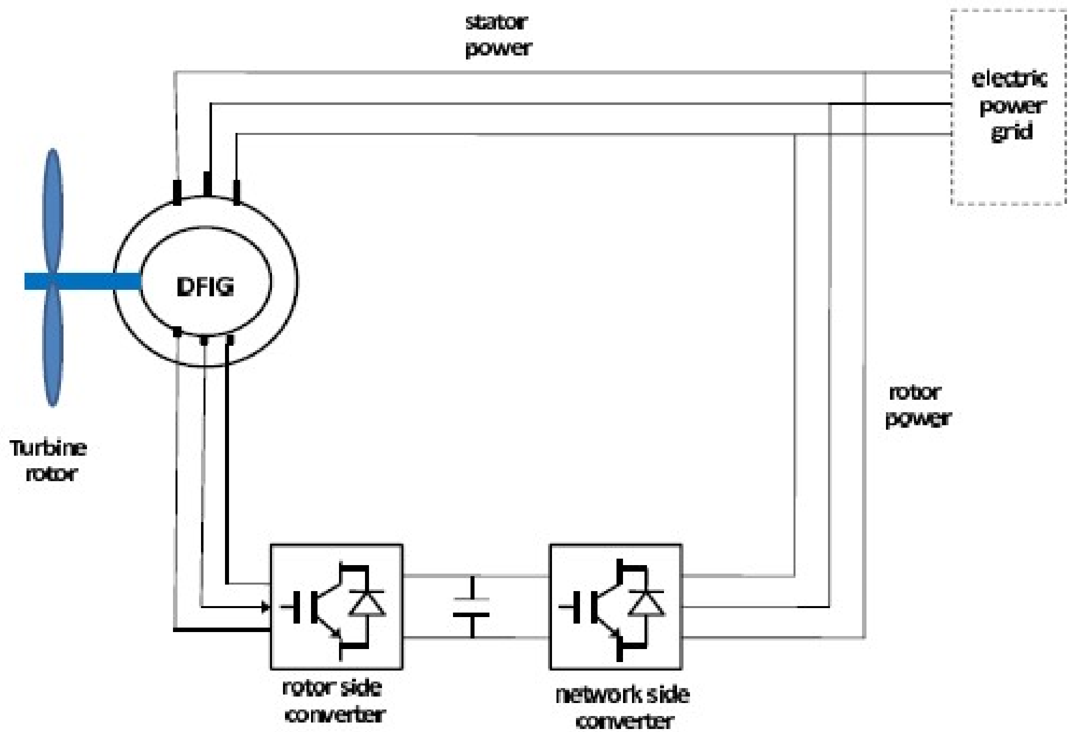

Fig 2: Control of the DFIG-based wind power generation system through a back-to-back voltage source converter

τ_{cp}

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the wind power generation unit

The dynamic model of the wind power generation unit undergoes approximate linearization around a temporary operating point (x*,u*),

x* is the present value of the system's state vector
u* is the last value of the control inputs vector

The operating point is updated at each iteration of the control method. The linearization procedure makes use of first-order Taylor series expansion and relies on the computation of the associated Jacobian matrices.

This gives: $$\dot{x} = Ax + Bu + \bar{d} \quad (3)$$

$\bar{d}$ is the modelling error due to truncation of higher-order terms in Taylor series expansion

Matrices A and B are given

$$A = \nabla_x[f(x) + g(x)u]\,|_{(x^*,u^*)} \Rightarrow A = \nabla_x[f(x)]\,|_{(x^*,u^*)} \quad (4)$$

$$B = \nabla_u[f(x) + g(x)u]\,|_{(x^*,u^*)} \Rightarrow B = g(x)\,|_{(x^*,u^*)} \quad (5)$$

Using the field orientation features of the asynchronous machine $x_s = \psi_{sq} = 0$ and $v_{sq} = 0$

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the wind power generation unit

For the first row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_1(x)}{\partial x_1}=0,\ \frac{\partial f_1(x)}{\partial x_2}=1,\ \frac{\partial f_1(x)}{\partial x_3}=0,\ \frac{\partial f_1(x)}{\partial x_4}=0,\ \frac{\partial f_1(x)}{\partial x_5}=0,\ \frac{\partial f_1(x)}{\partial x_6}=0,$$

$$\frac{\partial f_1(x)}{\partial x_7}=0 \text{ and } \frac{\partial f_1(x)}{\partial x_8}=0$$

For the second row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_2(x)}{\partial x_1}=-\frac{K_1}{J_t},\ \frac{\partial f_2(x)}{\partial x_2}=-B_g,\ \frac{\partial f_2(x)}{\partial x_3}=\frac{K_1}{J_t},\ \frac{\partial f_2(x)}{\partial x_4}=0,\ \frac{\partial f_2(x)}{\partial x_5}=0,$$

$$\frac{\partial f_2(x)}{\partial x_6}=0,\ \frac{\partial f_2(x)}{\partial x_7}=0 \text{ and } \frac{\partial f_2(x)}{\partial x_8}=0$$

For the third row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_3(x)}{\partial x_1}=0,\ \frac{\partial f_3(x)}{\partial x_2}=0,\ \frac{\partial f_3(x)}{\partial x_3}=0,\ \frac{\partial f_3(x)}{\partial x_4}=1,\ \frac{\partial f_3(x)}{\partial x_5}=0,\ \frac{\partial f_3(x)}{\partial x_6}=0,$$

$$\frac{\partial f_3(x)}{\partial x_7}=0 \text{ and } \frac{\partial f_3(x)}{\partial x_8}=0$$

For the fourth row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_4(x)}{\partial x_1}=\frac{K_1}{J_g},\ \frac{\partial f_4(x)}{\partial x_2}=0,\ \frac{\partial f_4(x)}{\partial x_3}=-\frac{K_1}{J_g},\ \frac{\partial f_4(x)}{\partial x_4}=-B_g,\ \frac{\partial f_4(x)}{\partial x_5}=L_m x_8,\ \frac{\partial f_4(x)}{\partial x_6}=-L_m x_7,$$

$$\frac{\partial f_4(x)}{\partial x_7}=-L_m x_6 \text{ and } \frac{\partial f_4(x)}{\partial x_8}=0.$$

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the wind power generation unit

For the fifth row of the Jacobian matrix

$$\frac{\partial f_5(x)}{\partial x_1} = 0,\ \frac{\partial f_5(x)}{\partial x_2} = 0,\ \frac{\partial f_5(x)}{\partial x_3} = 0,\ \frac{\partial f_5(x)}{\partial x_4} = 0,\ \frac{\partial f_5(x)}{\partial x_5} = -\frac{1}{\tau_s},\ \frac{\partial f_5(x)}{\partial x_6} = \omega_{dq},$$

$$\frac{\partial f_5(x)}{\partial x_7} = \frac{M}{} \text{ and } \frac{\partial f_5(x)}{\partial x_8} = 0.$$

For the sixth row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_6(x)}{\partial x_1} = 0,\ \frac{\partial f_6(x)}{\partial x_2} = 0,\ \frac{\partial f_6(x)}{\partial x_3} = 0,\ \frac{\partial f_6(x)}{\partial x_4} = 0,\ \frac{\partial f_6(x)}{\partial x_5} = -\omega_{dq},\ \frac{\partial f_6(x)}{\partial x_6} = -\frac{1}{\tau_s},$$

$$\frac{\partial f_6(x)}{\partial x_7} = 0 \text{ and } \frac{\partial f_6(x)}{\partial x_8} = \frac{M}{\tau_s}$$

For the seventh row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_7(x)}{\partial x_1} = 0,\ \frac{\partial f_7(x)}{\partial x_2} = 0,\ \frac{\partial f_7(x)}{\partial x_3} = 0,\ \frac{\partial f_7(x)}{\partial x_4} = -\beta x_6 - x_8,\ \frac{\partial f_7(x)}{\partial x_5} = \frac{\beta}{\tau_s},\ \frac{\partial f_7(x)}{\partial x_6} = -\beta x_4,$$

$$\frac{\partial f_7(x)}{\partial x_7} = -\gamma_2 \text{ and } \frac{\partial f_7(x)}{\partial x_8} = (\omega_{dq} - x_4).$$

For the eighth row of the Jacobian matrix $\nabla_x f(x)$

$$\frac{\partial f_8(x)}{\partial x_1} = 0,\ \frac{\partial f_8(x)}{\partial x_2} = 0,\ \frac{\partial f_8(x)}{\partial x_3} = 0,\ \frac{\partial f_8(x)}{\partial x_4} = -\beta x_5 - x_7,\ \frac{\partial f_8(x)}{\partial x_5} = \beta x_4,\ \frac{\partial f_8(x)}{\partial x_6} = \frac{\beta}{\tau_s},$$

$$\frac{\partial f_8(x)}{\partial x_7} = \omega_{dq} - x_4 \text{ and } \frac{\partial f_8(x)}{\partial x_8} = -\gamma_2.$$

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the wind power generation unit

After linearization around its current operating point, the model of the wind-turbine and DFIG asynchronous-generator power system is written in the form

$$\dot{x} = Ax + Bu + \bar{d}. \qquad (6)$$

The dynamics of the system of Eq. 6 can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \qquad (7)$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \qquad (8)$$

By subtracting Eq. 8 from Eq. 6 one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \qquad (9)$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\bar{d} = \bar{d}_3 - d_2$ the **tracking error dynamics** becomes

$$\dot{e} = Ae + Bu + \tilde{d} \qquad (10)$$

Example 2: Nonlinear control and state estimation using approximate linearization

4. Design of the H-infinity feedback controller

The initial wind power system is assumed to be in the form

$$\dot{x} = f(x,u) \quad x \in R^n, \ u \in R^m$$

where the **linearization point (temporary equilibrium)** is defined by the present value of the system's state vector and the last value of the control inputs vector exerted on it

$$(x^*, u^*) = (x(t), u(t - T_s)).$$

The **linearized equivalent of the system** is described by

$$\dot{x} = Ax + Bu + Ld \quad x \in R^n, \ u \in R^m, \ d \in R^q$$

where matrices A and B are obtained from the **computation of the Jacobians**

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Bigg|_{(x^*, u^*)} \qquad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Bigg|_{(x^*, u^*)}$$

and vector d denotes disturbance terms due to linearization errors.

The problem of **disturbance rejection** for the linearized model that is described by

$$\begin{aligned} \dot{x} &= Ax + Bu + Ld \\ y &= Cx \end{aligned}$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

The **tracking error dynamics** for the PEM fuel cells system is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \quad (11)$$

where in the **case of the considered wind power system** $L = I \in R^6$ with I being the identity matrix. The following **Lyapunov function** is considered

$$V = \frac{1}{2}e^T P e \quad (12)$$

where $e = x - x_d$ Is the state vector's tracking error

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow$$

$$\dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}e^T A^T P e + \frac{1}{2}u^T B^T P e + \frac{1}{2}\tilde{d}^T L^T P e + \\ \frac{1}{2}e^T P A e + \frac{1}{2}e^T P B u + \frac{1}{2}e^T P L\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(A^TP + PA)e + (\frac{1}{2}u^TB^TPe + \frac{1}{2}e^TPBu) + \\ +(\frac{1}{2}\tilde{d}^TL^TPe + \frac{1}{2}e^TPL\tilde{d})$$

Assumption*:* For given positive definite matrix Q *and coefficients* r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \quad (13)$$

Moreover, the following **feedback control law** is applied to the PEM fuel cells model

$$u = -\frac{1}{r}B^TPe \quad (14)$$

By substituting Eq. (13) and Eq. (14) one obtains

$$\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P]e + \\ +e^TPB(-\frac{1}{r}B^TPe + e^TPL\tilde{d} \Rightarrow$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Continuing with computations one obtains

$$\dot{V} = -\frac{1}{2}e^TQe + \left(\frac{1}{r}PBB^TPe - \frac{1}{2\rho^2}e^TPLL^T\right)Pe$$
$$-\frac{1}{r}e^TPBB^TPe + e^TPL\tilde{d}$$

which next gives

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe + e^TPL\tilde{d}$$

or equivalently

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe + \frac{1}{2}e^TPL\tilde{d} + \frac{1}{2}\tilde{d}^TL^TPe \tag{15}$$

Lemma: The following inequality holds

$$\frac{1}{2}e^TL\tilde{d} + \frac{1}{2}\tilde{d}L^TPe - \frac{1}{2\rho^2}e^TPLL^TPe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Proof : *The binomial* $(\rho a - \frac{1}{\rho} b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2} e^T P L \tilde{d} - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \qquad (16)$$

Eq. 16 is substituted in Eq. 15 and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2} e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \qquad (17)$$

Eq. 17 shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t) dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T ||\bar{d}||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||\bar{d}||^2 dt$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d$$

then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded.

Moreover, V(T) is bounded and from the definition of the Lyapunov function V it becomes clear that **e(t) will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$lim_{t \to \infty} e(t) = 0.$$

This completes the stability proof.

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Example 2: Nonlinear control and state estimation using approximate linearization

6. Robust state estimation with the use of the H-infinity Kalman Filter

- The control loop has to be implemented with the use of information provided by a **small number of measurements** of the state variables of the wind power system

- To reconstruct the missing information about the state vector of the wind power system it is proposed to **use a filter** and based on it to apply state **estimation-based control** .

- The **recursion of the H-infinity Kalman Filter**, for the **wind power system**, can be formulated in terms of a measurement update and a time update part

Measurement update

$$D(k) = [I - \theta W(k)P^{-}(k) + C^{T}(k)R(k)^{-1}C(k)P^{-}(k)]^{-1}$$
$$K(k) = P^{-}(k)D(k)C^{T}(k)R(k)^{-1}$$
$$\hat{x}(k) = \hat{x}^{-}(k) + K(k)[y(k) - C\hat{x}^{-}(k)]$$

Time update

$$\hat{x}^{-}(k+1) = A(k)x(k) + B(k)u(k)$$
$$P^{-}(k+1) = A(k)P^{-}(k)D(k)A^{T}(k) + Q(k)$$

where it is assumed that parameter θ is sufficiently small to assure that the **covariance matrix**

$$P^{-}(k) - \theta W(k) + C^{T}(k)R(k)^{-1}C(k)$$

Is **positive definite**

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

• The performance of the proposed nonlinear **H-nfinity control scheme** for the **wind power generation system** is tested through simulation:

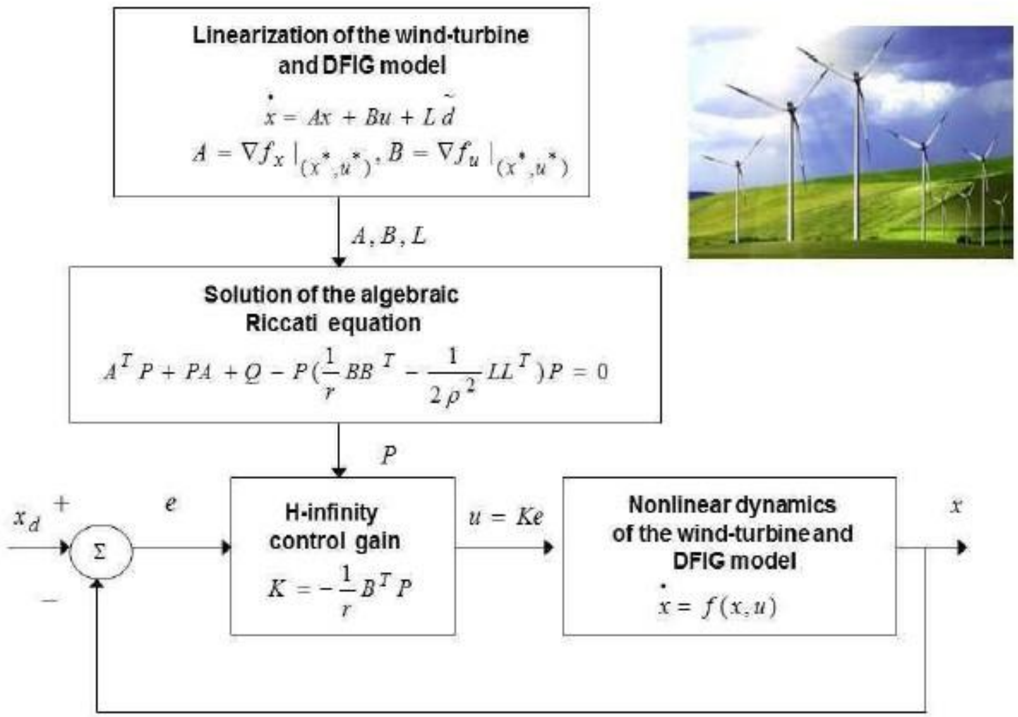


Fig.3 Diagram of the nonlinear optimal control

With the use of the proposed H-infinity control method, fast and accurate tracking of the reference setpoints of the **wind power system's state variables** was achieved

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Setpoint 1

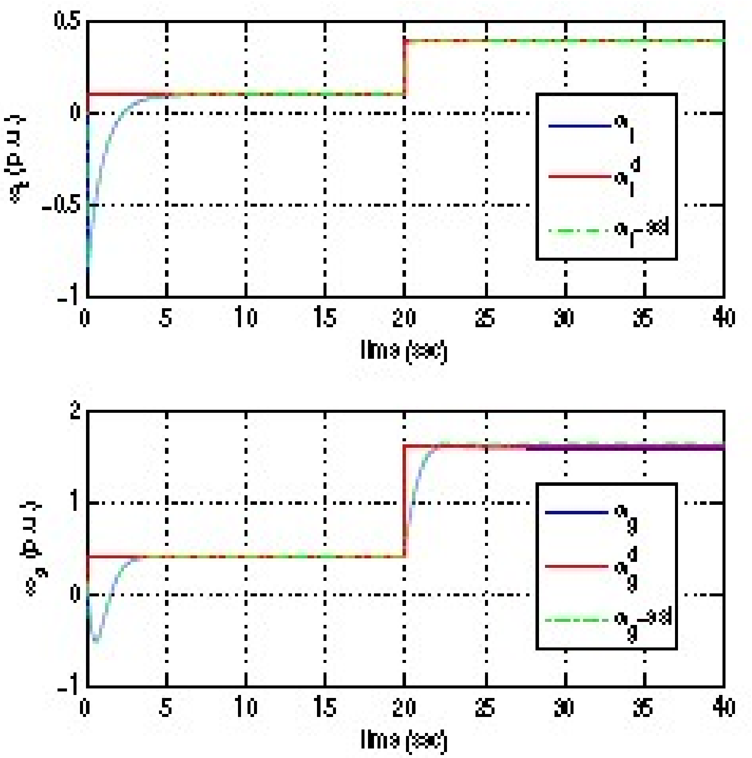


Fig.4a Convergence of state variables x_2 and x_4 to their reference setpoints

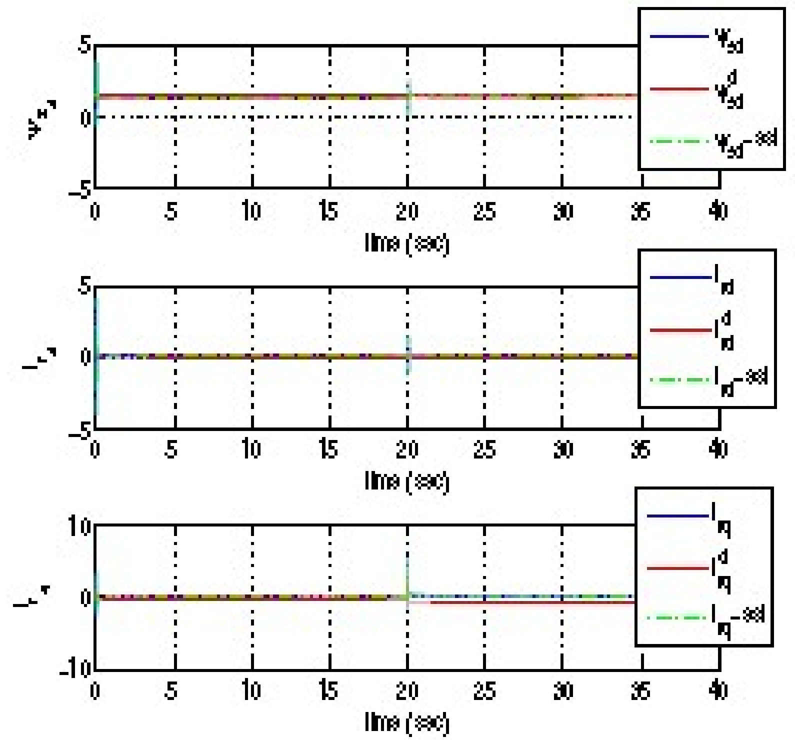


Fig.4b Convergence of state variables x_5, x_7 and x_8 to their reference setpoints

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Setpoint 2

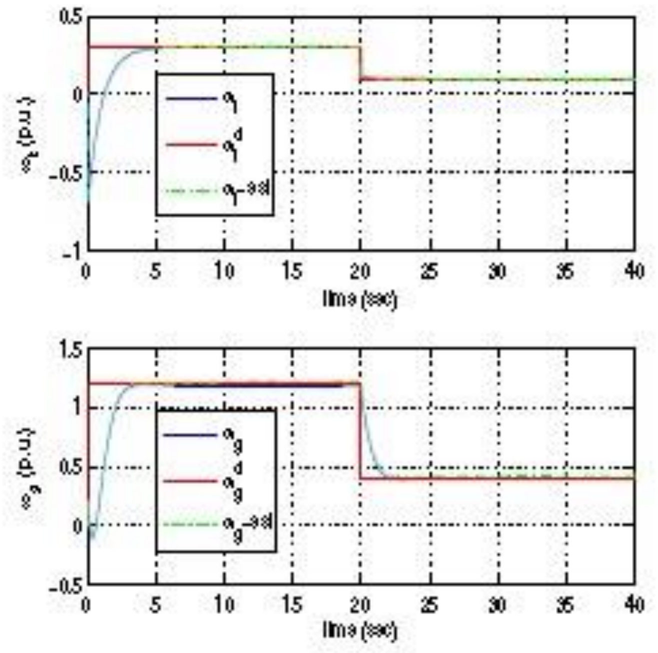


Fig.5a Convergence of state variables x_2 and x_4 to their reference setpoints

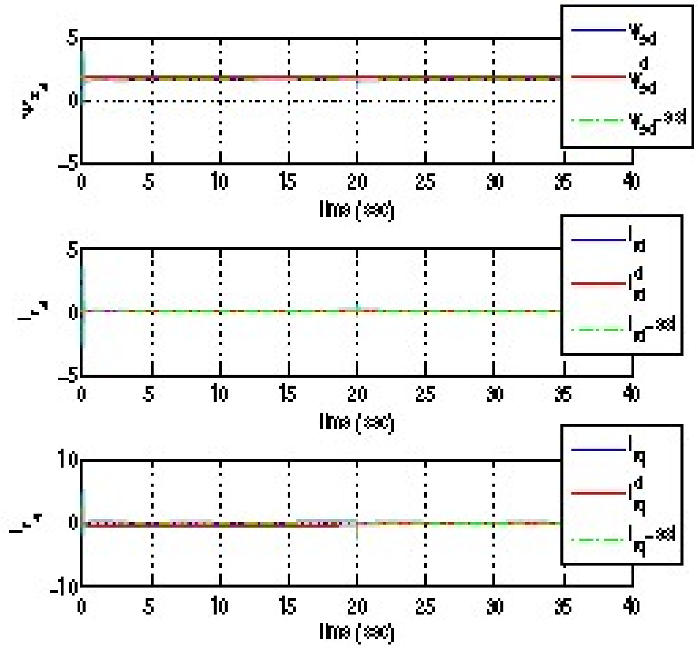


Fig.5b Convergence of state variables x_5, x_7 and x_8 to their reference setpoints

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Setpoint 3

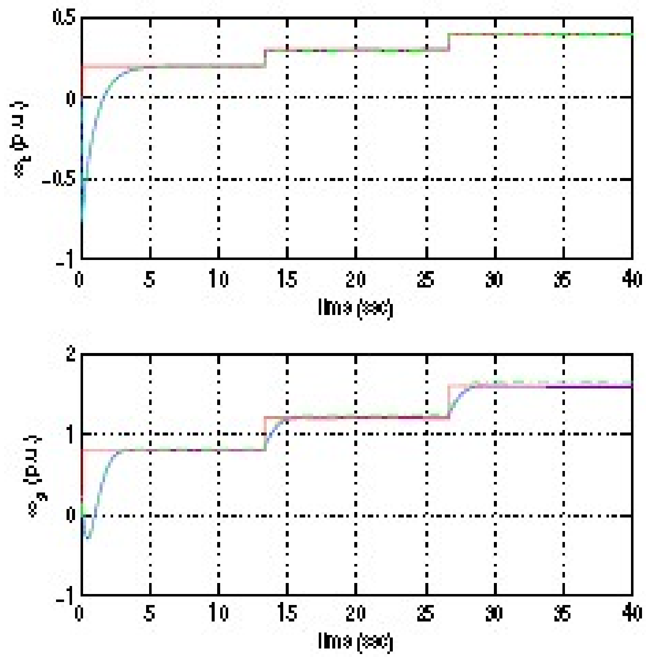


Fig.6a Convergence of state variables x_2 and x_4 to their reference setpoints

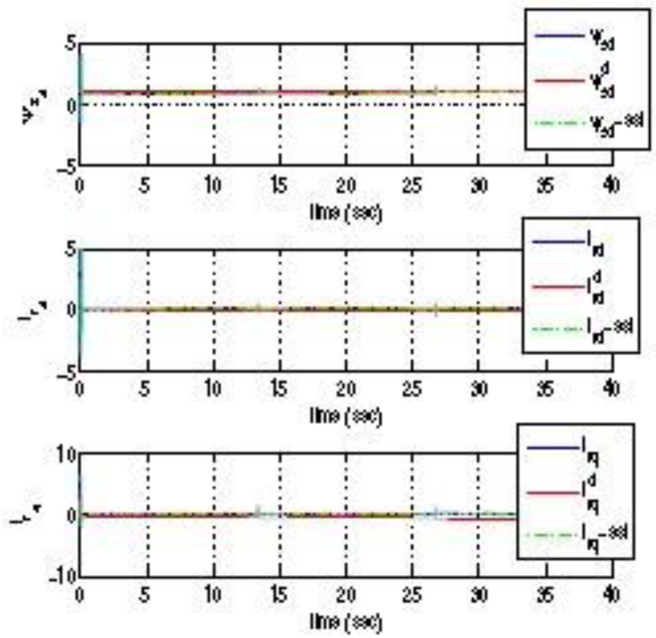


Fig.6b Convergence of state variables $x_{5,}$ x_7 and x_8 to their reference setpoints

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Setpoint 4

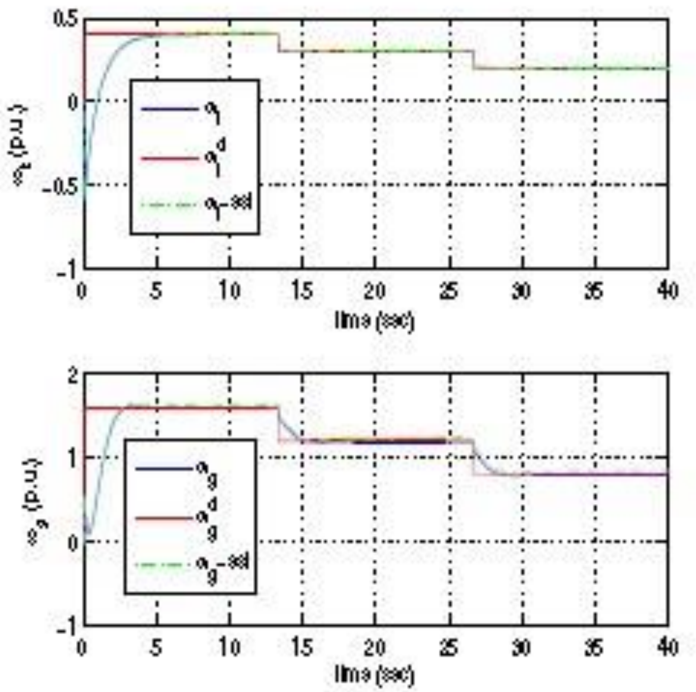


Fig.7a Convergence of state variables x_2 and x_4 to their reference setpoints

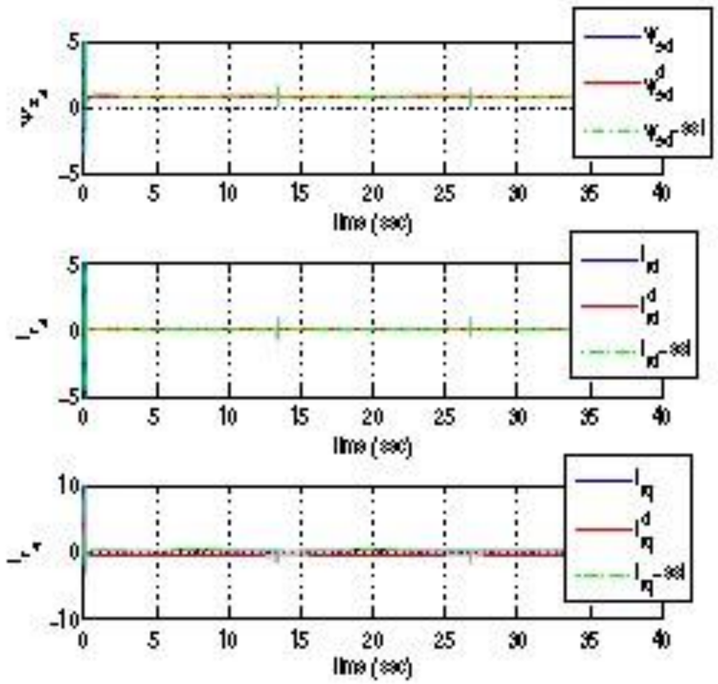


Fig.7b Convergence of state variables $x_{5,}$ x_7 and x_8 to their reference setpoints

Example 2: Nonlinear control and state estimation using approximate linearization

8. Simulation tests

Setpoint 1

Setpoint 2

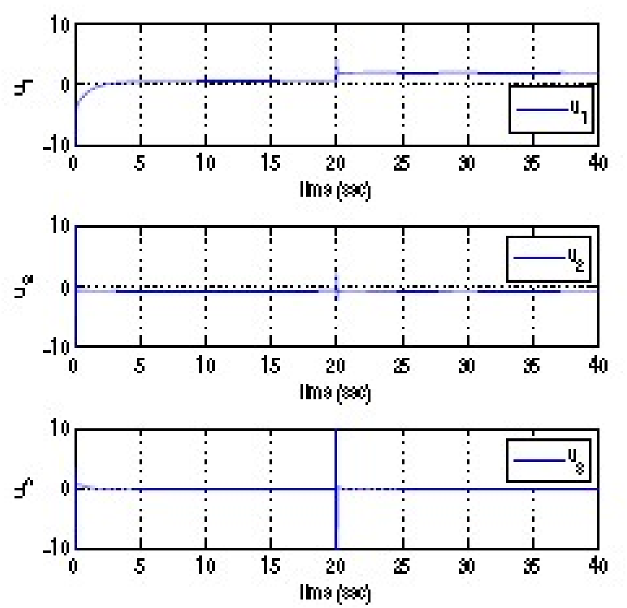


Fig.8a Variation of control inputs u_1, u_2 and u_3 when tracking setpoint 1

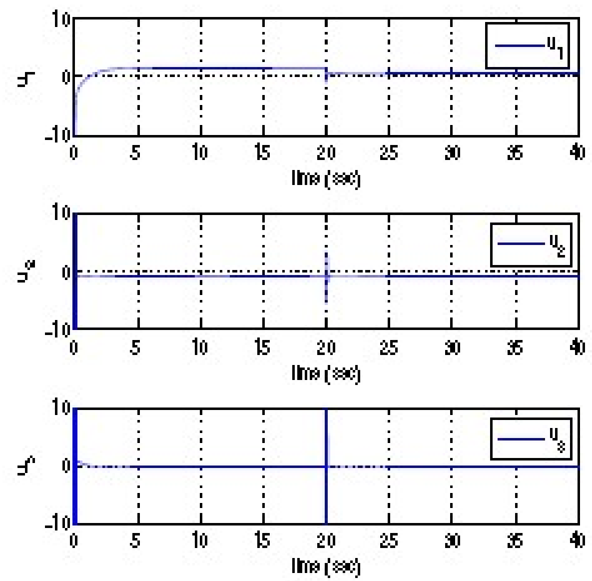


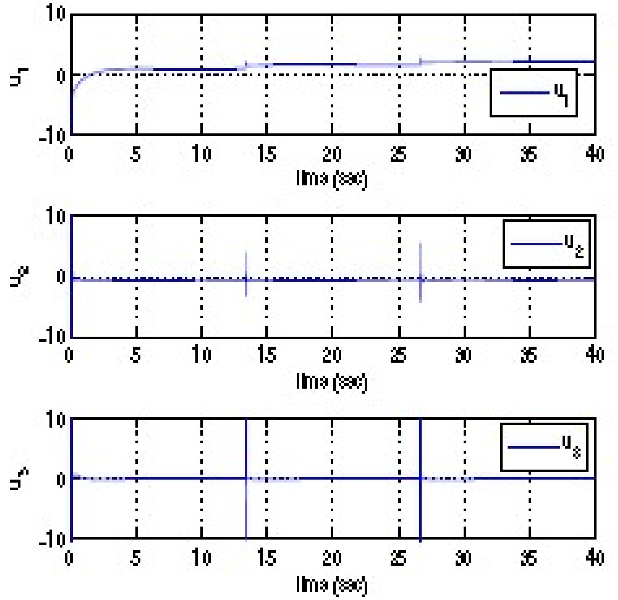
Fig.8b Variation of control inputs u_1, u_2 and u_3 when tracking setpoint 2

Example 2: Nonlinear control and state estimation using approximate linearization

8. Simulation tests

Setpoint 3

Setpoint 4



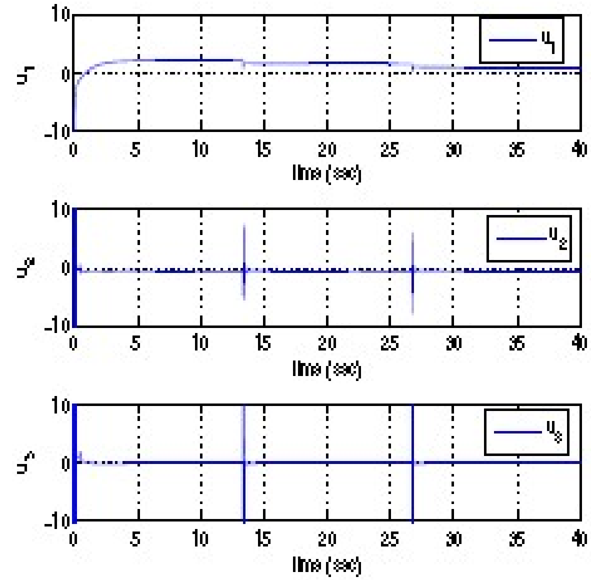


Fig.9a Variation of control inputs u_1, u_2 and u_3 when tracking setpoint 3

Fig.9b Variation of control inputs u_1, u_2 and u_3 when tracking setpoint 4

Example 2: Nonlinear control and state estimation using approximate linearization

8. Simulation tests

Table I: RMSE of state variables in tracking of setpoints

No	RMSEx_2	RMSEx_4	RMSEx_5	RMSEx_7
setpoint 1	0.0001	0.0001	0.0052	0.0038
setpoint 2	0.0001	0.0005	0.0030	0.0059
setpoint 3	0.0001	0.0001	0.0082	0.0068
setpoint 4	0.0001	0.0005	0.0038	0.0045
setpoint 5	0.0001	0.0002	0.0015	0.0036

Table II: RMSE of state variables under model disturbance

% Δa	RMSEx_2	RMSEx_4	RMSEx_5	RMSEx_7
0	0.0001	0.0005	0.0034	0.0059
10	0.0001	0.0005	0.0041	0.0058
20	0.0001	0.0005	0.0049	0.0057
30	0.0001	0.0005	0.0053	0.0056
40	0.0001	0.0005	0.0071	0.0055
50	0.0001	0.0006	0.0086	0.0054
60	0.0001	0.0006	0.0097	0.0052

Example 2: Nonlinear control and state estimation using approximate linearization

9. Conclusions

● A new nonlinear H-infinity control method has been developed for the dynamic model of wind power systems. The first stage for the method's implementation has been the linearization of the fuel cells' dynamic model round local operating points.

● At every time instant, these equilibria consisted of the present value of the system's state vector and of the last value of the control input that was exerted on it.

● For this linearization, Taylor series expansion has been applied to the fuel cells' dynamic model and the associated Jacobian matrices have been computed.

● For the linearized equivalent model of the system H-infinity nonlinear optimal control has been applied.

● The modelling errors which were due to the approximate linearization of the system were perceived as disturbances affecting the wind power system's dynamics and were compensated by the robustness of the H-infinity controller.

● Moreover, conditions which assure the asymptotic stability of the control loop have been formulated. The efficiency of the nonlinear H-infinity control method has been further confirmed through simulation experiments.

Example 3: Nonlinear control and state estimation using Lyapunov methods

1. Outline

● The article proposes an adaptive control approach that is capable of compensating for model uncertainty and parametric changes of the doubly-fed induction generators (DFIGs), as well as for the lack of measurements about the DFRM's state vector elements.

● First it is proven that the DFIG's model is a differentially flat one. By exploiting differential flatness properties it is shown that the DFRM model can be transformed into the linear canonical form.

● For the latter description, the new control inputs comprise unknown nonlinear functions which can be identified with the use of neurofuzzy approximators. The estimated dynamics of the machine is used by a feedback controller thus establishing an indirect adaptive control scheme.

● Moreover, to enforce the robustness of the control loop, a supplementary control term is computed using H-infinity control theory.

● Another problem that has to be dealt with comes from partial measurements of the state vector of the generator. Thus, a state observer is implemented in the control loop.

● The stability of the considered observer-based adaptive control approach is proven using Lyapunov analysis.Moreover, the performance of the control scheme is evaluated through simulation experiments.

Example 3: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the doubly-fed induction generators

The equations of th rotational motion of the rotor of the Doubly-fed induction generator are given by

$$J\dot{\omega} = T_m - K_f\omega - T_e$$

J : moment of inertia of the rotor, T_m : externally applied mechanical torque that makes

T_e : electrical torque due to the machine's currents $k_f\omega$: friction term

The wind generated mechanical torque is $T_m = \frac{1}{2}\rho\pi R^3 C_q(\lambda,\beta)v^2$

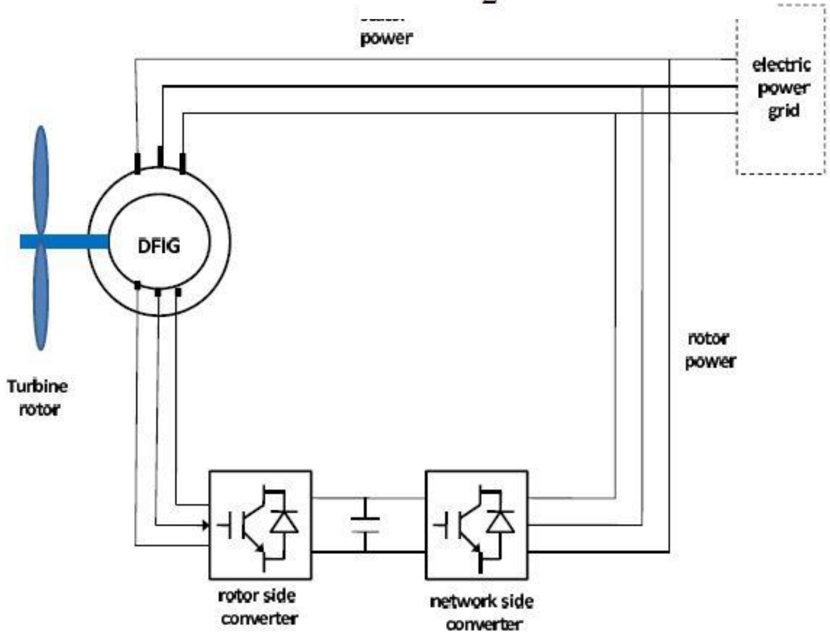


Fig 1: Configuration of a doubly-fed induction generator unit in the power grid

Example 3: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the doubly-fed induction generators

In a compact form, the doubly-fed induction generator can be described by the following set of equations in the d − q reference frame that rotates at an arbitrary speed denoted as ω_{dq}

$$\frac{d\psi_{s_q}}{dt} = -\frac{1}{\tau_s}\psi_{s_q} - \omega_{dq}\psi_{s_d} + \frac{M}{\tau_s}i_{r_q} + v_{s_q} \quad (1)$$

$$\frac{d\psi_{s_d}}{dt} = \omega_{dq}\psi_{s_q} - \frac{1}{\tau_s}\psi_{s_d} + \frac{M}{\tau_s}i_{r_d} + v_{s_d} \quad (2)$$

$$\frac{di_{r_q}}{dt} = \frac{\beta}{\tau_s}\psi_{s_q} + \beta\omega_r\psi_{s_d} - \gamma_2 i_{r_q} - (\omega_{dq} - \omega_r)i_{r_d} - \beta v_{s_q} + \frac{1}{\sigma L_r}v_{r_q} \quad (3)$$

$$\frac{di_{r_d}}{dt} = -\beta\omega_r\psi_{s_q} + \frac{\beta}{\tau_s}\psi_{s_d} + (\omega_{dq} - \omega_r)i_{r_q} - \gamma_2 i_{r_d} - \beta v_{s_d} + \frac{1}{\sigma L_r}v_{r_d} \quad (4)$$

$\psi_{s_q}, \psi_{s_d}, i_{r_q}, i_{r_d}$ are the stator flux and the rotor currents

$v_{s_q}, v_{s_d}, v_{r_q}, v_{r_d}$ are the stator and rotor voltages

L_s and L_r are the stator and rotor inductances

ω_r is the rotor's angular velocity

M is the mutual inductance

R_s and R_r are the stator and rotor resistances

Example 3: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the doubly-fed induction generators

Moreover, the following parameters are defined

$$\sigma = 1 - \frac{M^2}{L_r L_s} \qquad \beta = \frac{1-\sigma}{M\sigma} \qquad \tau_s = \frac{L_s}{R_s}$$

$$\tau_r = \frac{L_r}{R_r} \qquad \gamma_2 = \left(\frac{1-\sigma}{\sigma\tau_s}\right)$$

The angle of the vectors that describe the magnetic flux $\psi_{s\alpha}$ and ψ_{sb} is first defined for the stator

$$\rho = tan^{-1}\left(\frac{\psi_{s_b}}{\psi_{s_a}}\right)$$

Moreover, it holds that

$$cos(\rho) = \frac{\psi_{s_a}}{||\psi||},\ sin(\rho) = \frac{\psi_{s_b}}{||\psi||},\ \text{and}\ ||\psi|| = \sqrt{\psi_{s_\alpha}^2 + \psi_{s_b}^2}$$

Therefore, in the rotating d−q frame of the generator, and under the condition of field orientation, there will be only one non-zero component of the magnetic flux ψ_{sd} while the component of the flux along the q axis equals 0

The dynamic model of the doubly-fed induction generator can be also written in state-space equations form by defining the following state variables:

$$x_1 = \theta,\ x_2 = \omega_r,\ x_3 = \psi_{s_d},\ x_4 = \psi_{s_q},\ x_5 = i_{r_d}\ \text{and}\ x_6 = i_{r_q}.$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the doubly-fed induction generators

Thus, the state-space model of the induction generator becomes

$$\dot{x}_1 = x_2 \tag{5}$$

$$\dot{x}_2 = -\frac{K_m}{J}x_2 - \frac{T_m}{J} + \frac{\eta}{J}(i_{r_q}x_3 - i_{r_d}x_4) \tag{6}$$

$$\dot{x}_3 = -\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d} \tag{7}$$

$$\dot{x}_4 = -\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q} \tag{8}$$

$$\dot{x}_5 = -\beta x_2 x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2 x_5 + \frac{1}{\sigma L_r}v_{r_d} - \beta v_{s_d} \tag{9}$$

$$\dot{x}_6 = \frac{\beta}{\tau_s}x_4 + \beta x_2 x_3 - (\omega_{dq} - x_2)x_5 - \gamma_2 x_6 + \frac{1}{\sigma L_r}v_{r_q} - \beta v_{s_q} \tag{10}$$

The state-space model of the asynchronous generator is also written in the affine-in-the-input form

$$\dot{x} = f(x) + g_a(x)v_{r_d} + g_b(x)v_{r_q} \tag{11}$$

with

$$x = [x_1, x_2, x_3, x_4, x_5, x_6]^T$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the doubly-fed induction generator

and

$$f(x)=\begin{pmatrix} x_2 \\ -\frac{K_m}{J}x_2-\frac{T_m}{J}+\frac{n}{J}(i_{r_q}x_3-i_{r_d}x_4) \\ -\frac{1}{T_s}x_3+\omega_{dq}x_4+\frac{M}{T_s}x_5+v_{s_d} \\ -\omega_{dq}x_3-\frac{1}{T_s}x_4+\frac{M}{T_s}x_6+v_{s_q} \\ -\beta x_2x_4+\frac{\beta}{T_s}x_3+(\omega_{dq}-x_2)x_6-\gamma_2x_5-\beta v_{s_d} \\ \frac{\beta}{T_s}x_4+\beta x_2x_3-(\omega_{dq}-x_2)x_5-\gamma_2x_6-\beta v_{s_q} \end{pmatrix} \quad (12)$$

$$g_a(x)=\begin{pmatrix}0 & 0 & 0 & 0 & \frac{1}{\sigma L_r} & 0\end{pmatrix}^T \quad (13)$$

$$g_b(x)=\begin{pmatrix}0 & 0 & 0 & 0 & 0 & \frac{1}{\sigma L_r}\end{pmatrix}^T \quad (14)$$

while the active and reactive power of the generator are given by

$$P_s=Re\{U_sI_s^*\}=v_{s_d}i_{s_d}+v_{s_q}i_{s_q} \quad (15)$$

$$Q_s=Im\{U_sI_s^*\}=v_{s_d}i_{s_q}-v_{s_q}i_{s_d} \quad (16)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

The flat outputs of the system are defined as

$$\begin{gathered} y_1=\theta \text{ or } y=x_1 \\ y_2=\psi_{s_d}^2+\psi_{s_q}^2 \text{ or } y_2=x_3^2+x_4^2 \end{gathered} \tag{17}$$

It holds that

$$\begin{gathered} \dot{y}_1=\omega \text{ or } \dot{y}_1=x_2 \Rightarrow \\ \ddot{y}_1=\dot{\omega}=-\frac{K_m}{J}x_2-\frac{T_m}{J}+\frac{\eta}{J}(x_6x_3-x_5x_4) \Rightarrow \\ \ddot{y}_1=\dot{\omega}=-\frac{K_m}{J}\dot{y}_1-\frac{T_m}{J}+\frac{\eta}{J}(x_6x_3-x_5x_4) \end{gathered} \tag{18}$$

Deriving the last row of the previous equation with respect to time one obtains

$$\begin{array}{l} y_1^{(3)}=-\frac{K_m}{J}\ddot{y}_1+\frac{\eta}{J}(\dot{x}_6x_3+x_6\dot{x}_3-\dot{x}_5x_4-x_5\dot{x}_4) \Rightarrow \\ y_1^{(3)}=-\frac{K_m}{J}\ddot{y}_1+\frac{\eta}{J}x_3\{[\frac{\beta}{\tau_s}x_4+\beta x_2x_3+(\omega_{dq}-x_2)x_5- \\ -\gamma_2x_6-\beta v_{s_q}]+\frac{1}{\sigma L_r}u_1\}+\frac{\eta}{J}x_6[-\frac{1}{\tau_s}x_3+\omega_{dq}x_4+\frac{M}{\tau_s}x_5+v_{s_d}] \\ -\frac{\eta}{J}x_4\{[-\beta x_2x_4+\frac{\beta}{\tau_s}x_3+(\omega_{dq}-x_2)x_6-\gamma_2x_5- \\ -\beta v_{s_d}]+\frac{1}{\sigma L_r}u_2\}-\frac{\eta}{J}x_5[-\omega_{dq}x_3-\frac{1}{\tau_s}x_4+\frac{M}{\tau_s}x_6+v_{s_q}] \end{array} \tag{19}$$

Moreover, about the second flat output it holds

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

$$\begin{array}{c}\dot{y}_2 = 2x_3\dot{x}_3 + 2x_4\dot{x}_4 \Rightarrow \\ \dot{y}_2 = 2x_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] + \\ +2x_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] \Rightarrow \end{array} \quad (20)$$

Consequently, it holds

$$\begin{array}{c}\ddot{y}_2 = 2\dot{x}_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] + \\ +2x_3[-\frac{1}{\tau_s}\dot{x}_3 + \omega_{dq}\dot{x}_4 + \frac{M}{\tau_s}\dot{x}_5] + \\ 2\dot{x}_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] + \\ +2x_4[-\omega_{dq}\dot{x}_3 - \frac{1}{\tau_s}\dot{x}_4 + \frac{M}{\tau_s}\dot{x}_6] \end{array}$$

or equivalently

$$\begin{array}{c}\ddot{y}_2 = 2[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}]^2 + \\ -\frac{2}{\tau_s}x_3[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] \\ -2\omega_{dq}x_3[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] \\ +\frac{2M}{\tau_s}x_3\{[-\beta x_2x_4 + \frac{\beta}{\tau_s}x_3 + (\omega_{dq} - x_2)x_6 - \\ -\gamma_2x_5 - \beta v_{s_d}] + \frac{1}{\sigma L_r}u_1\} \\ +2[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}]^2 \\ -2\omega_{dq}x_4[-\frac{1}{\tau_s}x_3 + \omega_{dq}x_4 + \frac{M}{\tau_s}x_5 + v_{s_d}] \\ -\frac{2}{\tau_s}x_4[-\omega_{dq}x_3 - \frac{1}{\tau_s}x_4 + \frac{M}{\tau_s}x_6 + v_{s_q}] \\ 2x_4\frac{M}{\tau_s}\{[\frac{\beta}{\tau_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \\ -\gamma_2x_6 - \beta v_{s_q}] + \frac{1}{\sigma L_r}u_2\} \end{array} \quad (21)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

It holds that

$$x_1 = y_1 \,,\ x_2 = \dot{y}_1 . \qquad (22)$$

Considering that the field orientation condition $x_4 = \psi_{s_q} = 0$

one obtains

$$x_3 = \sqrt{y_2} . \qquad (23)$$

Moreover, for the first flat output it holds

$$\ddot{y}_1 = -\frac{K_m}{J}\dot{y}_1 - \frac{T_m}{J} + \frac{\eta}{J}\sqrt{y_2}x_6 \Rightarrow$$
$$x_6 = \frac{\ddot{y}_1 + \frac{K_m}{J}\dot{y}_1 + \frac{T_m}{J}}{\frac{\eta}{J}\sqrt{y_2}}, \ \ y_2 \neq 0 \qquad (24)$$

Additionally, for the second flat output it holds

$$\dot{y}_2 = -\frac{2}{\tau_s}x_3^2 + \frac{2M}{\tau_s}x_3x_5 + 2v_{s_d}x_3 \Rightarrow$$
$$\dot{y}_2 + \left(\frac{2}{\tau_s}x_3 - 2v_{s_d}\right)x_3 = \frac{2M}{\tau_s}x_3x_5 \Rightarrow$$
$$x_5 = \frac{\dot{y}_2 + \left(\frac{2}{\tau_s}\sqrt{y_2} - 2v_{s_d}\right)\sqrt{y_2}}{\frac{2M}{\tau_s}\sqrt{y_2}} \ \ y_2 \neq 0 \qquad (25)$$

Therefore, x_5 is also a function of the flat output and of its derivatives.

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

Moreover, by solving the system of equations of. 19 and. 21 with respect to .u_1, u_2 one obtains that the control inputs are functions of the flat output and its derivatives.

Therefore, the model of the DFIG is a differentially flat one.

Next, to design the flatness-based controller for the DFIG the following transformation of the state variables is introduced:

$$z_1 = y_1,\ z_2 = \dot{y}_1,\ z_3 = \ddot{y}_1,\ z_4 = y_2,\ z_5 = \dot{y}_2$$

Using a notation of variables as in the case of Lie algebra-based linearization it holds

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= f_1(x) + g_{1a}(x)u_1 + g_{1b}(x)u_2 \\ \dot{z}_4 &= z_5 \\ \dot{z}_5 &= f_2(x) + g_{2a}(x)u_1 + g_{2b}(x)u_2 \end{aligned} \quad (26)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

where

$$
\begin{aligned}
f_1(x) = & -\frac{K_m}{J}[-\frac{K_m}{J}x_2 - \frac{T_m}{J} + \frac{\eta}{J}(x_6x_3 - x_5x_4)] \\
& +\frac{\eta}{J}x_6[-\frac{1}{T_s}x_3 + \omega_{dq}x_4 + \frac{M}{T_s}x_5 + v_{s_d}] \\
& -\frac{\eta}{J}x_5[-\omega_{dq}x_3 - \frac{1}{T_s}x_4 + \frac{M}{T_s}x_6 + v_{s_q}] \\
& -\frac{\eta}{J}x_4[-\beta x_2x_4 + \frac{\beta}{T_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2x_5 - \beta v_{s_d}] \\
& +\frac{\eta}{J}x_3[\frac{\beta}{T_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \gamma_2x_6 - \beta v_{s_q}]
\end{aligned}
$$

$$
g_{1a}(x) = -\frac{\eta}{J}\frac{1}{\sigma L_r}x_4 \qquad g_{1b}(x) = \frac{\eta}{J}\frac{1}{\sigma L_r}x_3
$$

$$
\begin{aligned}
f_2(x) = & \\
& (-\frac{4}{T_s}x_3 - \frac{2M}{T_s}x_5 + 2v_{s_d})[-\frac{1}{T_s}x_3 + \omega_{dq}x_4 + \frac{M}{T_s}x_5 + v_{s_d}] + \\
& (-\frac{4}{T_s}x_4 + \frac{2M}{T_s}x_6 + 2v_{s_q})[-\omega_{dq}x_3 - \frac{1}{T_s}x_4 + \frac{M}{T_s}x_6 + v_{s_q}] + \\
& (\frac{2M}{T_s}x_3)[-\beta x_2x_4 + \frac{\beta}{T_s}x_3 + (\omega_{dq} - x_2)x_6 - \gamma_2x_5 - \beta v_{s_d}] + \\
& (\frac{2M}{T_s}x_3)[\frac{\beta}{T_s}x_4 + \beta x_2x_3 + (\omega_{dq} - x_2)x_5 - \gamma_2x_6 - \beta v_{s_q}]
\end{aligned}
$$

$$
g_{2a}(x) = \frac{2M}{T_s}\frac{1}{\sigma L_s}x_3 \qquad g_{2b}(x) = \frac{2M}{T_s}\frac{1}{\sigma L_s}x_4
$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

Therefore, one obtains the decoupled and linearized representation of the system

$$\begin{pmatrix} \dot{z}_1^{(3)} \\ \ddot{z}_4 \end{pmatrix} = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} + \begin{pmatrix} g_{1a}(x) & g_{1b}(x) \\ g_{2a}(x) & g_{2b}(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad (27)$$

or equivalently

$$\begin{pmatrix} z_1^{(3)} \\ \ddot{z}_4 \end{pmatrix} = f_a + \tilde{M} u \quad (28)$$

where

$$f_a = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} \qquad \breve{M} = \begin{pmatrix} g_{1a}(x) & g_{1b}(x) \\ g_{2a}(x) & g_{2b}(x) \end{pmatrix}$$

Moreover, by defining the new control inputs

$$v_1 = f_1(x) + g_{1a}(x)u_1 + g_{1b}(x)u_2 \quad (29)$$

$$v_2 = f_2(x) + g_{2a}(x)u_1 + g_{2b}(x)u_2 \quad (30)$$

one can also have the description in the MIMO canonical form

Example 3: Nonlinear control and state estimation using Lyapunov methods

3. Differential flatness properties of the DFIG system dynamics

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (30)$$

The control input for the linearized and decoupled model of the DFIG is chosen as follows

$$\begin{aligned} v_1 &= z_1^{d(3)} - k_1^{(1)}(\ddot{z}_1 - \ddot{z}_1^d) - k_2^{(1)}(\dot{z}_1 - \dot{z}_1^d) - k_3^{(1)}(z_1 - z_1^d) \\ v_2 &= \ddot{z}_4^d - k_1^{(2)}(\dot{z}_4 - \dot{z}_4^d) - k_2^{(2)}(z_4 - z_4^d) \end{aligned} \quad (31)$$

The control input that is finally applied on the initial nonlinear model of the DFIG is

$$u = \tilde{M}^{-1}(-f_a + v) \quad (32)$$

The turn angle $x_1=\theta$ of the rotor can be directly measured

Moreover, the following relation allows to compute the magnetic flux from stator currents measurements

$$\begin{aligned} \psi_{s_d} &= L_s i_{s_d} + M i_{r_d} \\ \psi_{s_q} &= 0 \end{aligned}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

It is assumed now that **after defining the flat outputs of the initial MIMO nonlinear system**, and after expressing the system state variables and control inputs as functions of the flat output and of the associated derivatives, the system can be **transformed in the Brunovsky canonical form**

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\cdots \\
\dot{x}_{r_1-1} &= x_{r_1} \\
\dot{x}_{r_1} &= f_1(x) + \textstyle\sum_{j=1}^{p} g_{1_j}(x)u_j + d_1 \\
\\
\dot{x}_{r_1+1} &= x_{r_1+2} \\
\dot{x}_{r_1+2} &= x_{r_1+3} \\
&\cdots \\
\dot{x}_{p-1} &= x_p \\
\dot{x}_p &= f_p(x) + \textstyle\sum_{j=1}^{p} g_{p_j}(x)u_j + d_p
\end{aligned}
$$

$$
\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_{r_1-1} \\
&\cdots \\
y_p &= x_{n-r_p+1}
\end{aligned}
$$

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$x = [x_1, \cdots, x_n]^T$: is the state vector

$u = [u_1, \cdots, u_p]^T$: is the inputs vector

$y = [y_1, \cdots, y_p]^T$: is the outputs vector

Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Next **the following vectors and matrices** can be defined

$$f(x)=[f_1(x), \quad ..., \quad f_n(x)]^T$$

$$g(x)=[g_1(x), \quad ..., \quad g_n(x)]^T$$

with $g_i(x)=[g_{1i}(x), \quad ..., \quad g_{pi}(x)]^T$

$$A=diag[A_1,...,A_p], \; B=diag[B_1,...,B_p]$$

$$C^T=diag[C_1,...,C_p], \; d=[d_1,...,d_p]^T$$

where matrix A has the **MIMO canonical form**, i.e. with elements

$$A_i=\begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 \\ 0 & 0 & 0 & ... & 0 \end{bmatrix}_{r_i \times r_i}$$

$$B_i^T=[0 \quad 0 \quad ... \quad 0 \quad 1]_{1\times r_i} \qquad C_i=[1 \quad 0 \quad ... \quad 0 \quad 0]_{1\times r_i}$$

Thus, the initial nonlinear system can be writtenin the **state-space form**

$$\dot{x}=Ax+B[f(x)+g(x)u+\tilde{d}]$$
$$y=Cx \qquad (34)$$

or equivalently in the state space form

$$\dot{x}=Ax+Bv+B\tilde{d}$$
$$y=Cx$$

where $\quad v=f(x)+g(x)u$

For the case of the **MIMO diesel engine model** it is assumed that the functions $f(x)$ and $g(x)$ are unknown and have to be approximated by **neuro-fuzzy networks**

Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Thus, the nonlinear system can be written in **state-space form**

$$\dot{x} = Ax + B[f(x) + g(x)u + \tilde{d}]$$
$$y = C^T x$$

which equivalently can be written as

$$\dot{x} = Ax + Bv + B\tilde{d}$$
$$y = C^T x$$

where $v = f(x) + g(x)u.$

The **reference setpoints** for the system's outputs $y_1, \cdots, y_p$ are denoted as $y_{1m}, \cdots, y_{pm}$ and the associated tracking errors are defined as

$$e_1 = y_1 - y_{1m}$$
$$e_2 = y_2 - y_{2m}$$
$$\cdots$$
$$e_p = y_p - y_{pm}$$

The **error vector of the outputs** of the transformed MIMO system is denoted as

$$E_1 = [e_1, \cdots, e_p]^T$$
$$y_m = [y_{1m}, \cdots, y_{pm}]^T$$
$$\cdots$$
$$y_m^{(r)} = [y_{1m}^{(r)}, \cdots, y_{pm}^{(r)}]^T$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.2. Control law

The **control signal of the MIMO nonlinear system** contains the **unknown nonlinear functions** f(x) and g(x) which can be approximated by

$$\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f, \quad \hat{g}(x|\theta_g) = \Phi_g(x)\theta_g \qquad (35)$$

where $\Phi_f(x) = \left(\xi_f^1(x), \xi_f^2(x), \cdots \xi_f^n(x)\right)^T,$

$$\xi_f^i(x) = \left(\phi_f^{i,1}(x), \phi_f^{i,2}(x), \cdots, \phi_f^{i,N}(x)\right)$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_f{}^T = \left(\theta_f^1, \theta_f^2, \cdots \theta_f^N\right).$

Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.2. Control law

Similarly, it holds $\Phi_g(x) = \left(\xi_g^1(x), \xi_g^2(x), \cdots \xi_g^N(x)\right)^T$

$$\xi_g^i(x) = \left(\phi_g^{i,1}(x), \phi_g^{i,2}(x), \cdots, \phi_g^{i,N}(x)\right)$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_g = \left(\theta_g^1, \theta_g^2, \cdots, \theta_g^p\right)^T$

However, here each row of θ_g is vector thus giving

$$\theta_g = \begin{pmatrix} \theta_{g1}^1 & \theta_{g1}^2 & \cdots & \theta_{g1}^p \\ \theta_{g2}^1 & \theta_{g2}^2 & \cdots & \theta_{g2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{gN}^1 & \theta_{gN}^2 & \cdots & \theta_{gN}^p \end{pmatrix}$$

If the state variables of the system are available for measurement then a **state-feedback control law can be formulated** as

$$u = \hat{g}^{-1}(x|\theta_g)\left[-\hat{f}(x|\theta_f) + y_m^{(r)} + K_c^T e + u_c\right]$$

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Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.3. Estimation of the state vector

The control of the system described by becomes more complicated **when the state vector x is not directly measurable** and has to be reconstructed through a state observer. The following definitions are used

$e = x - x_m$: is the error of the state vector

$\hat{e} = \hat{x} - x_m$ is the error of the estimated state vector

$\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$ is the observation error

When an **observer is used to reconstruct the state vector**, the control law

$$u = \hat{g}^{-1}(\hat{x}|\theta_g)[-\hat{f}(\hat{x}|\theta_f) + y_m^{(r)} - K^T\hat{e} + u_c]$$

By applying the previous feedback control law one obtains the closed-loop dynamics

$$y^{(n)} = f(x) + g(x)\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$

$$y^{(n)} = f(x) + [g(x) - \hat{g}(\hat{x}) + \hat{g}(\hat{x})]\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$

$$y^{(n)} = [f(x) - \hat{f}(\hat{x})] + [g(x) - \hat{g}(\hat{x})]u + y_m^{(n)} - K^T\hat{e} + u_c + d$$

It holds $e = x - x_m \Rightarrow y^{(n)} = e^{(n)} + y_m^{(n)}$

and by substituting $y^{(n)}$ n the **previous tracking error dynamics** gives

Example 3: Nonlinear control and state estimation using Lyapunov methods

4. Design of an adaptive neurofuzzy controller for the DFIG system

4.3. Estimation of the state vector

the **new tracking error dynamics**

$$e^{(n)} + y_m^{(n)} = y_m^{(n)} - K^T\hat{e} + u_c + [f(x) - \hat{f}(\hat{x})] + \\ +[g(x) - \hat{g}(\hat{x})]u + d$$

or equivalently

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\{[f(x) - \hat{f}(\hat{x})] + \\ +[g(x) - \hat{g}(\hat{x})]u + d\} \quad (37)$$

$$e_1 = C^Te$$

where $e = [e^1, e^2, \cdots, e^p]^T$ with $e^i = [e_i, \dot{e}_i, \ddot{e}_i, \cdots, e_i^{r_i-1}]^T,\ i = 1, 2, \cdots, p$

and equivalently $\hat{e} = [\hat{e}^1, \hat{e}^2, \cdots, \hat{e}^p]^T$ with $\hat{e}^i = [\hat{e}_i, \dot{\hat{e}}_i, \ddot{\hat{e}}_i, \cdots, \hat{e}_i^{r_i-1}]^T,\ i = 1, 2, \cdots, p.$

A state observer is designed as:

$$\dot{\hat{e}} = A\hat{e} - BK^T\hat{e} + K_o[e_1 - C^T\hat{e}] \quad (38)$$

$$\hat{e}_1 = C^T\hat{e}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

5. Application of adaptive neurofuzzy control to the DFIG system

5.1. Tracking error dynamics under feedback control

By **applying differential flatness theory, and in the presence of disturbances**, the dvnamic model of the DFRM comes to the form

$$\begin{pmatrix} \ddot{x}_1 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} u + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad (39)$$

The following **control input** is defined:

$$u = \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \dot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} \quad (40)$$

where: $[u_{c_1}\ u_{c_2}]^T$ is a **robust control term** that is used for the compensation of the model's uncertainties as well as of the external disturbances

and: $K_i^T = [k_1^i, k_2^i, \cdots, k_{n-1}^i, k_n^i]$ is the feedback gain

Substituting the control input (40) into the system (39) one obtains

$$\begin{pmatrix} \ddot{x}_1 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot$$

$$\cdot \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \dot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

5. Application of adaptive neurofuzzy control to the DFIG system

5.1. Tracking error dynamics under feedback control

Moreover, using again Eq. 40 one obtains the **tracking error dynamics**

$$\begin{pmatrix}\ddot{e}_1\\ \dot{e}_3\end{pmatrix}=\begin{pmatrix}f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u-\begin{pmatrix}K_1^T\\ K_2^T\end{pmatrix}e+\begin{pmatrix}u_{c_1}\\ u_{c_2}\end{pmatrix}+\begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

The **approximation error** is defined $w=\begin{pmatrix}f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u$

Using matrices A,B,K, $A=\begin{pmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{pmatrix},\ B=\begin{pmatrix}0&0\\1&0\\0&0\\0&1\end{pmatrix}$ $K^T=\begin{pmatrix}K_1^1&K_2^1&K_3^1&K_4^1\\ K_1^2&K_2^2&K_3^2&K_4^2\end{pmatrix}$

and considering that **the estimated state vector is used in the control loop** the following description of the tracking error dynamics is obtained:

$$\dot{e}=Ae-BK^T\hat{e}+Bu_c+B\left\{\begin{pmatrix}f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u+\tilde{d}\right\}$$

When the **estimated state vector** is used in the loop the **approximation error is written as**

$$w=\begin{pmatrix}f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u$$

while the **tracking error dynamics becomes** $\dot{e}=Ae-BK^T\hat{e}+Bu_c+Bw+B\tilde{d}$

Example 3: Nonlinear control and state estimation using Lyapunov methods

5. Application of adaptive neurofuzzy control to the DFIG system

5.2. Dynamics of the observation error

The **observation error** is defined as: $\tilde{e} = e - \hat{e} = x - \hat{x}$.

By subtracting Eq. 38 from Eq. 37 one obtains:

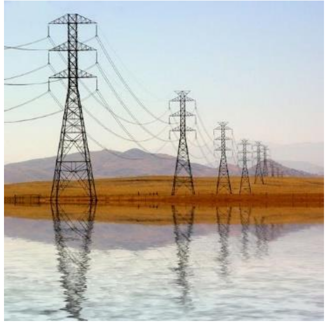
$$\dot{e} - \dot{\hat{e}} = A(e - \hat{e}) + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + \\ +[g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T(e - \hat{e})$$

$$e_1 - \hat{e}_1 = C^T(e - \hat{e})$$

or equivalently:

$$\dot{\tilde{e}} = A\tilde{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T\tilde{e}$$

$$\tilde{e}_1 = C^T\tilde{e}$$



which can be also written as:

$$\dot{\tilde{e}} = (A - K_o C^T)\tilde{e} + Bu_c + Bw + \tilde{d}\}$$

$$\tilde{e}_1 = C^T\tilde{e}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

5. Application of adaptive neurofuzzy control to the DFIG system

5.3. Approximation of functions f(x,t) and g(x,t)

Next, the **first of the approximators** of the unknown system dynamics is defined

$$\hat{f}(\hat{x}) = \begin{pmatrix} \hat{f}_1(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_1(\hat{x}|\theta_f) & \in & R^{1\times 1} \\ \hat{f}_2(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_2(\hat{x}|\theta_f) & \in & R^{1\times 1} \end{pmatrix}$$

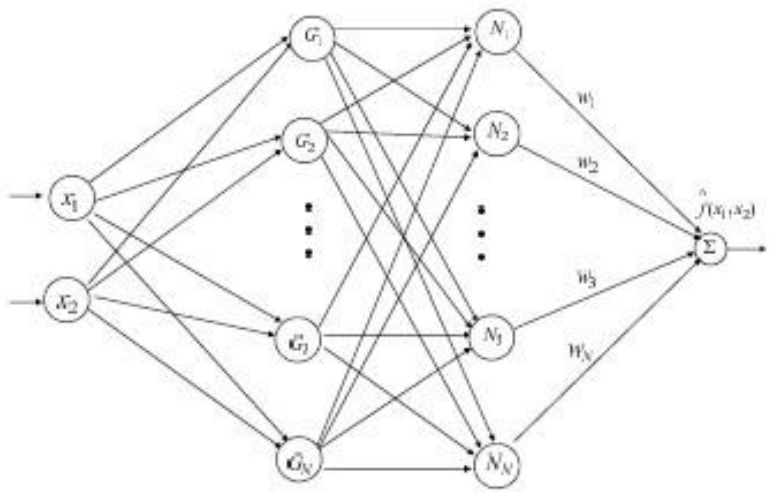


Fig 2: Neurofuzzy approximator

containing kernel functions $\phi_f^{l,j}(\hat{x}) = \frac{\prod_{j=1}^{n}\mu_{A_j^l}(\hat{x}_j)}{\sum_{l=1}^{N}\prod_{j=1}^{n}\mu_{A_j^l}(\hat{x}_j)}$

where $\mu_{A_j^l}(\hat{x})$ are fuzzy membership functions

appearing in the antecedent part of the *l-th* fuzzy rule

Example 3: Nonlinear control and state estimation using Lyapunov methods

5. Application of adaptive neurofuzzy control to the DFIG system

5.3. Approximation of functions f(x,t) and g(x,t)

Similarly, **the second of the approximators** of the unknown system dynamics is defined

$$\hat{g}(\hat{x}) = \begin{pmatrix} \hat{g}_1(\hat{x}|\theta_g) \ \hat{x}\in R^{4\times 1} \ \hat{g}_1(\hat{x}|\theta_g) & \in & R^{1\times 2} \\ \hat{g}_2(\hat{x}|\theta_g) \ \hat{x}\in R^{4\times 1} \ \hat{g}_2(\hat{x}|\theta_g) & \in & R^{1\times 2} \end{pmatrix}$$

The **values of the weights that result in optimal approximation** are

$$\theta_f^* = arg\ min_{\theta_f \in M_{\theta_f}} [sup_{\hat{x}\in U_{\hat{x}}}(f(x) - \hat{f}(\hat{x}|\theta_f))]$$
$$\theta_g^* = arg\ min_{\theta_g \in M_{\theta_g}} [sup_{\hat{x}\in U_{\hat{x}}}(g(x) - \hat{g}(\hat{x}|\theta_g))]$$

The variation ranges for the weights are given by

$$M_{\theta_f} = \{\theta_f \in R^h : \ ||\theta_f|| \leq m_{\theta_f}\}$$
$$M_{\theta_g} = \{\theta_g \in R^h : \ ||\theta_g|| \leq m_{\theta_g}\}$$

The **value of the approximation error** that corresponds to the optimal values of the weights vectors is

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f^*)\right) + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

5. Application of adaptive neurofuzzy control to the DFIG system

5.3. Approximation of functions f(x,t) and g(x,t)

which is next written as

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f) + \hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)\right) + \\ + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g) + \hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)\right) u \qquad (41)$$

which can be also written in the following form

$$w = \ (w_a + w_b)$$

with

$$w_a = \{[f(x,t) - \hat{f}(\hat{x}|\theta_f)] + [g(x,t) - \hat{g}(\hat{x}|\theta_g)]\} u$$

and

$$w_b = \{[\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x}, \theta_g) - \hat{g}(\hat{x}|\theta_g^*)]\} u$$

Moreover, the following **weights error vectors are defined**

$$\tilde{\theta}_f = \theta_f - \theta_f^*$$
$$\tilde{\theta}_g = \theta_g - \theta_g^*$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

The **following Lyapunov function** is considered:

$$V = \frac{1}{2}\hat{e}^T P_1 \hat{e} + \frac{1}{2}\bar{e}^T P_2 \bar{e} + \frac{1}{2\gamma_1}\bar{\theta}_f^T \bar{\theta}_f + \frac{1}{2\gamma_2} tr[\bar{\theta}_g^T \bar{\theta}_g] \quad (42)$$

The selection of the **Lyapunov function** is based on the following principle of indirect adaptive control

$$\hat{e} : \lim_{t\to\infty} \hat{x}(t) = x_d(t)$$
$$\bar{e} : \lim_{t\to\infty} \hat{x}(t) = x(t).$$

this results into

$$\lim_{t\to\infty} x(t) = x_d(t)$$

By deriving the **Lyapunov function** with respect to time one obtains:

$$\dot{V} = \frac{1}{2}\dot{\hat{e}}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \dot{\hat{e}} + \frac{1}{2}\dot{\bar{e}}^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 \dot{\bar{e}} + + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g] \Rightarrow$$

$$\dot{V} = \frac{1}{2}\left\{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\right\}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\}$$
$$+ \frac{1}{2}\left\{(A - K_o C^T)\tilde{e} + Bu_c + B\tilde{d} + Bw\right\}^T P_2 \tilde{e}$$
$$+ \frac{1}{2}\tilde{e}^T P_2 \{(A - K_o C^T)\tilde{e} + Bu_c + B\tilde{d} + Bw\} + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr\left[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g\right] \Rightarrow .$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

The equation is rewritten as:

$$\dot V = \frac{1}{2}\{\hat e^T(A-BK^T)^T + \tilde e^T C K_o^T\}P_1\hat e + \frac{1}{2}\hat e^T P_1\{(A-BK^T)\hat e + K_o C^T \tilde e\} + \\ +\frac{1}{2}\{\tilde e^T(A-K_oC^T)^T + u_c^T B^T + w^T B^T + \tilde d^T B^T\}P_2\tilde e + \\ \frac{1}{2}\tilde e^T P_2\{(A-K_oC^T)\tilde e + Bu_c + Bw + B\tilde d\} + \frac{1}{\gamma_1}\dot{\tilde\theta}_f^T\tilde\theta_f + \frac{1}{\gamma_2}tr[\dot{\tilde\theta}_g^T\tilde\theta_g] \Rightarrow$$

which finally takes the form:

$$\dot V = \frac{1}{2}\hat e^T(A-BK^T)^T P_1\hat e + \frac{1}{2}\tilde e^T C K_o^T P_1\hat e + \\ +\frac{1}{2}\hat e^T P_1(A-BK^T)\hat e + \frac{1}{2}\hat e^T P_1 K_o C^T\tilde e + \\ +\frac{1}{2}\tilde e^T(A-K_oC^T)^T P_2\tilde e + \frac{1}{2}(u_c^T + w^T + \tilde d^T)B^T P_2\tilde e + \\ +\frac{1}{2}\tilde e^T P_2(A-K_oC^T)\tilde e + \frac{1}{2}\tilde e^T P_2 B(u_c + w + \tilde d) + \\ +\frac{1}{\gamma_1}\dot{\tilde\theta}_f^T\tilde\theta_f + \frac{1}{\gamma_2}tr[\dot{\tilde\theta}_g^T\tilde\theta_g]$$

Assumption 1: For given positive definite matrices Q1 and Q2 there exist positive definite matrices P1 and P2, which are the solution of the following **Riccati equations**

$$(A-BK^T)^T P_1 + P_1(A-BK^T) + Q_1 = 0 \qquad (43)$$

$$(A-K_oC^T)^T P_2 + P_2(A-K_oC^T) - \\ -P_2B(\frac{2}{r} - \frac{1}{\rho^2})B^T P_2 + Q_2 = 0 \qquad (44)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

By substituting the conditions from the previous Riccati equations into the derivative of the Lyapunov function one gets:

$$\begin{aligned}\dot{V} = & \tfrac{1}{2}\hat{e}^T\{(A-BK^T)^T P_1 + P_1(A-BK^T)\}\hat{e} + \tilde{e}^T C K_o^T P_1 \hat{e} + \\ & + \tfrac{1}{2}\tilde{e}^T\{(A-K_oC^T)^T P_2 + P_2(A-K_oC^T)\}\tilde{e} + \\ & + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \tfrac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]\end{aligned}$$

or:

$$\begin{aligned}\dot{V} = & -\tfrac{1}{2}\hat{e}^T Q_1 \hat{e} + \tilde{e}^T C K_o^T P_1 \hat{e} - \tfrac{1}{2}\tilde{e}^T\{Q_2 - P_2 B(\tfrac{2}{r} - \tfrac{1}{\rho^2})B^T P_2\}\tilde{e} + \\ & + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \tfrac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]\end{aligned}$$

- The **supervisory control term** u_c consists of two terms: u_a and u_b.

$$u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a \qquad (45)$$

where assuming that the measurable elements of vector $\tilde{e}$ are $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$,

the term Δu_a Is given by

$$-\tfrac{1}{r}\tilde{e}^T P_2 B + \Delta u_a = -\tfrac{1}{r}\begin{pmatrix} p_{11}\tilde{e}_1 + p_{13}\tilde{e}_3 + \cdots + p_{1k}\tilde{e}_k \\ p_{13}\tilde{e}_1 + p_{33}\tilde{e}_3 + \cdots + p_{3k}\tilde{e}_k \\ \cdots \;\; \cdots\cdots \\ p_{1k}\tilde{e}_1 + p_{3k}\tilde{e}_3 + \cdots + p_{kk}\tilde{e}_k \end{pmatrix}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

- The control ter u_b. Is given by

$$u_b = -[(P_2B)^T(P_2B)]^{-1}(P_2B)^T C K_o^T P_1 \hat{e}$$ (46)

u_a is an H-infinity control used for the compensation of the approximation error w and the additive disturbance $\tilde{d}$.

Its first component $-\frac{1}{r}\tilde{e}^T P_2 B$ has been chosen so as to compensate for the term $\frac{1}{r}\tilde{e}^T P_2 B B^T P_2 \tilde{e}$, which appears in the previously computed function about $\dot{V}$.

By including also the second component Δu_a one has that u_a is computed based on the feedback only the measurable variables $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$ out of the complete vector $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$

Eq. $u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$ finally rewritten as $u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$.

- u_b is a control used for the compensation of the observation error (the control term has been chosen so as to satisfy the condition

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

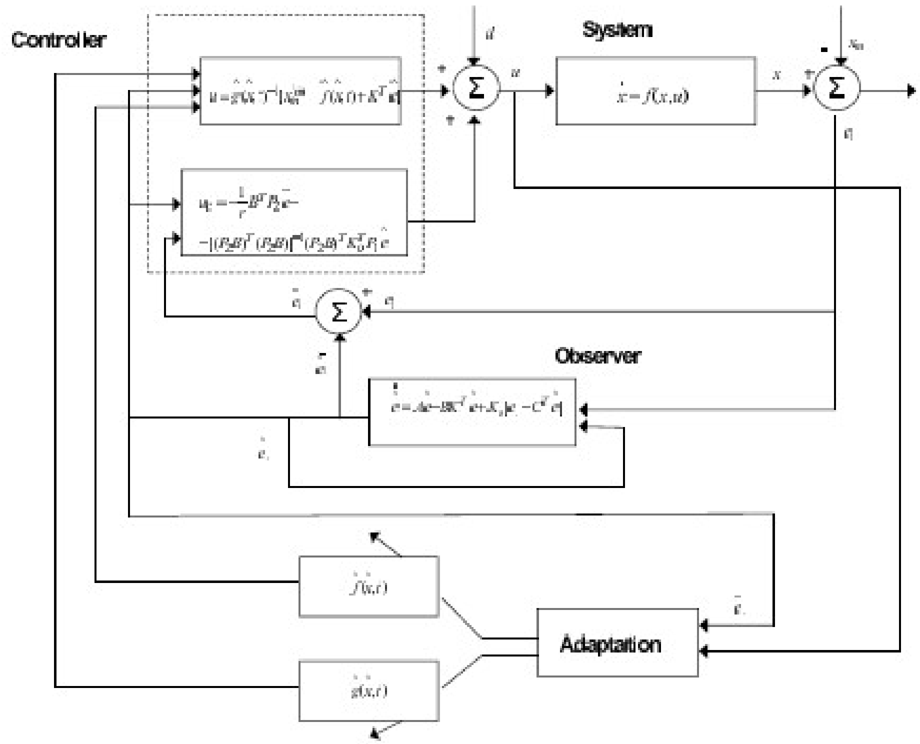


Fig. 3 Adaptive neurofuzzy control scheme for the DFIG

By substituting the supervisory control term in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} + \tilde{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} + \frac{1}{r}\tilde{e}^T P_2 B B^T P_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ + \tilde{e}^T P_2 B u_a + \tilde{e}^T P_2 B u_b + \tilde{e}^T P_2 B (w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

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or equivalently

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ +\bar{e}^T P_2 B(w+\tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Besides, about the adaptation of the weights of the neurofuzzy network it holds

$$\dot{\tilde{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f \qquad \dot{\tilde{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g.$$

and also

$$\dot{\theta}_f = -\gamma_1 \Phi(\hat{x})^T B^T P_2 \bar{e}$$
$$\dot{\theta}_g = -\gamma_2 \Phi(\hat{x})^T B^T P_2 \bar{e} u^T$$

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By substituting the above relations in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w+\tilde{d}) + \\ +\frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ +\frac{1}{\gamma_2}(-\gamma_2) tr[u\bar{e}^T P_2 B \Phi(\hat{x})(\theta_g - \theta_g^*)]$$

or

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w+\tilde{d}) + \\ +\frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ +\frac{1}{\gamma_2}(-\gamma_2) tr[u\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

Taking into account that $u \in R^{2\times 1}$ and $\bar{e}^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1\times 2}$

one gets

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \tilde{d}) +$$
$$+\frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) +$$
$$+\frac{1}{\gamma_2}(-\gamma_2) tr[\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u]$$

Since $\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \in R^{1\times 1}$

it holds

$$tr(\bar{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)u) =$$
$$= \bar{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u$$

Therefore, one finally obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \tilde{d}) +$$
$$+\frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) +$$
$$+\frac{1}{\gamma_2}(-\gamma_2)\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u$$

Next, the following **approximation error** is defined

$$w_a = [\hat{f}(\hat{x}|\theta_f^*) - \hat{f}(\hat{x}|\theta_f)] + [\hat{g}(\hat{x}|\theta_g^*) - \hat{g}(\hat{x}|\theta_g)]u$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

Thus, one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + B^T P_2 \bar{e}(w + \bar{d}) + \bar{e}^T P_2 B w_a$$

Denoting the **aggregate approximation e**rror and disturbances vector as

$$w_1 = w + \bar{d} + w_a$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \bar{e}^T P_2 B w_1$$

which in turn is written as

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e}$$

Lemma: The following inequality holds

$$\frac{1}{2}\bar{e}^T P_2 B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \\ \leq \frac{1}{2}\rho^2 w_1^T w_1$$

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Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

Proof:

The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow$$
$$\frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow$$
$$\frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$

By substituting $a = w_1$ and $b = \bar{e}^T P_2 B$ one gets

$$\frac{1}{2}w_1^T B^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 B w_1 - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Moreover, by substituting the above inequality into the derivative of the Lyapunov function one gets

$$\dot{V} \leq -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{2}\rho^2 w_1^T w_1 \quad (49)$$

which is also written as $\dot{V} \leq -\frac{1}{2}E^T Q E + \frac{1}{2}\rho^2 w_1^T w_1$

with $E = \begin{pmatrix} \hat{e} \\ \bar{e} \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} = diag[Q_1, Q_2]$

Example 3: Nonlinear control and state estimation using Lyapunov methods

6. Lyapunov stability analysis

Hence, the H_∞ performance criterion is derived. For sufficiently small ρ the inequality will be true and the H_∞ tracking criterion will be satisfied. In that case, the integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)\,dt \leq -\frac{1}{2}\int_0^T ||E||^2 dt + \frac{1}{2}\rho^2 \int_0^T ||w_1||^2 dt \Rightarrow$$
$$2V(T) - 2V(0) \leq -\int_0^T ||E||_Q^2 dt + \rho^2 \int_0^T ||w_1||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||E||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||w_1||^2 dt$$

It is assumed that there exists a positive constant $M_w > 0$ such that

$$\int_0^\infty ||w_1||^2 dt \leq M_w$$

Therefore for the integral $\int_0^T ||E||_Q^2 dt$ one gets

$$\int_0^\infty ||E||_Q^2 dt \leq 2V(0) + \rho^2 M_w$$

Thus, the integral $\int_0^\infty ||E||_Q^2 dt$ is bounded and **according to Barbalat's Lemma**

$$\lim_{t \to \infty} e(t) = 0$$

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Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

● The efficiency of the proposed flatness-based control method for doubly-fed induction generators has been confirmed with the use of simulation experiments.

● The dynamic model of the DFIG was taken to be completely unknown. The system's dynamics were identified with the used of the previously analyzed neurofuzzy approximators

● There was no need to measure the entire state vector of the generator. Measurements were obtained in real-time only about the rotation angle of the rotor and the stator currents

● There was no need to measure or estimate the mechanical excitation provided by the wind. Efficient control can be achieved by adjusting only the rotor currents

● Under the proposed control scheme the machine can function at variable operating conditions and under variable mechanical excitation. This makes the DFIG more efficient In energy harvesting

● Fast and accurate tracking of the setpoints was achieved. The transients of the state variables did not exhibit abrupt changes and the variations of the control input were smooth

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 1

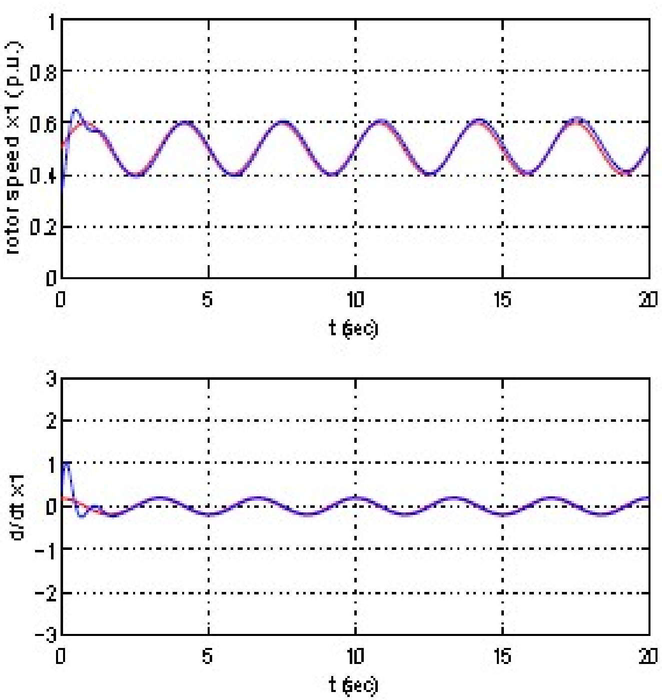


Fig 4a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

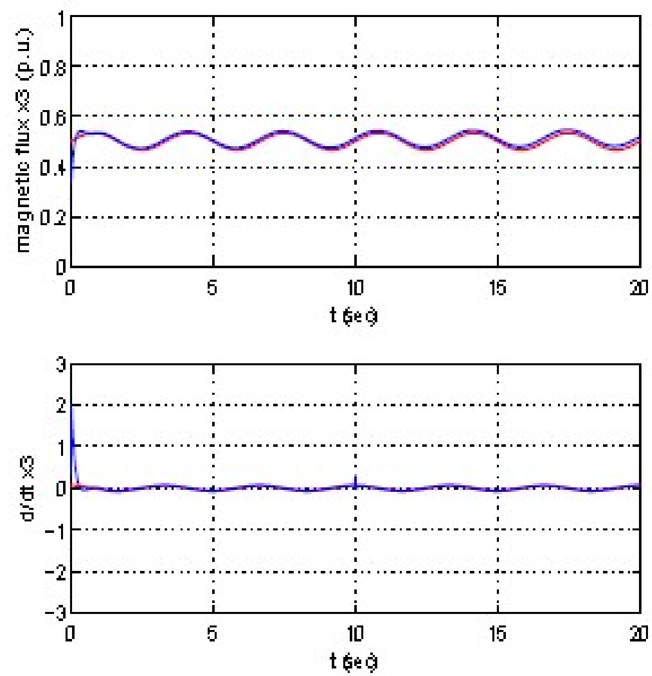
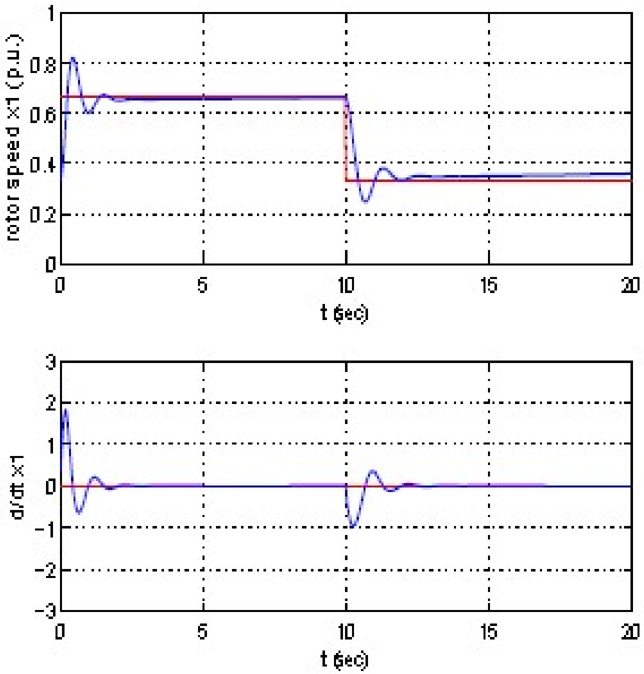


Fig 4b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 2



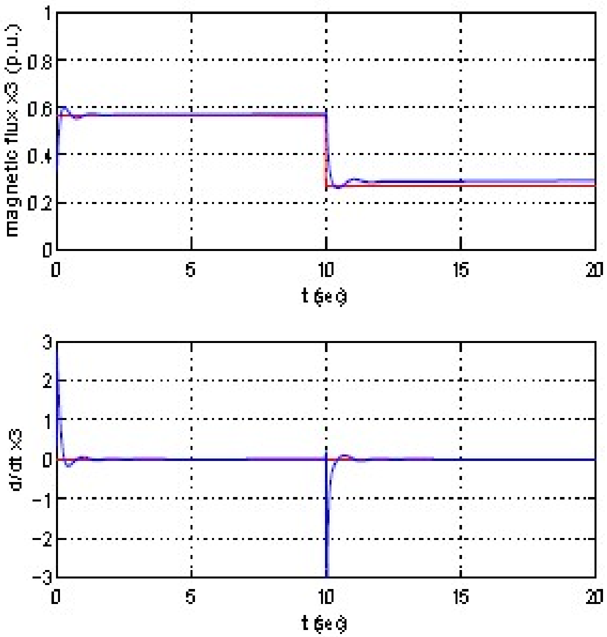


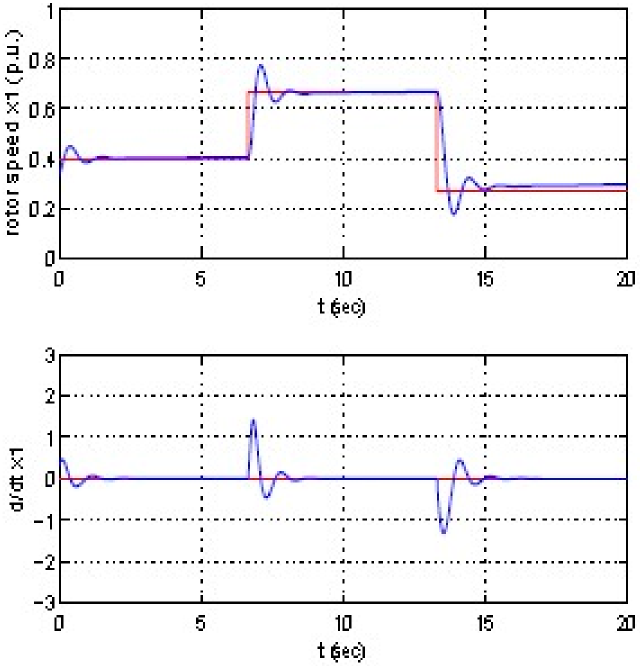
Fig 5a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

Fig 5b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 3



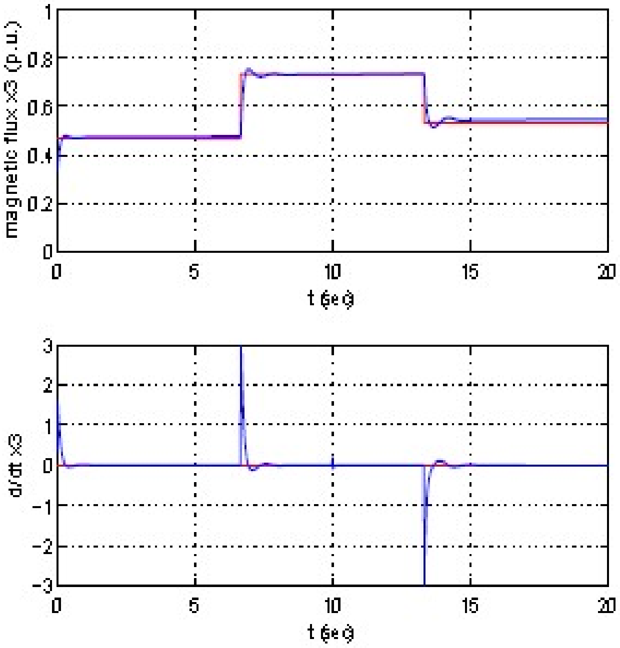


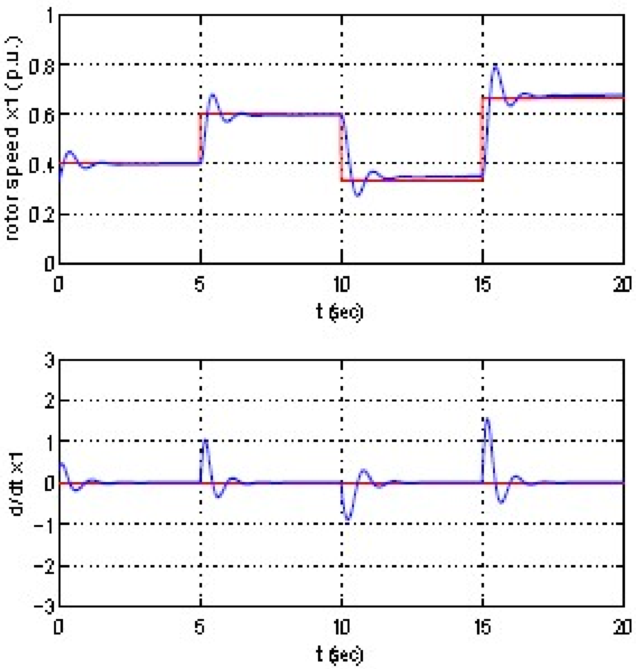
Fig 6a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

Fig 6b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 4



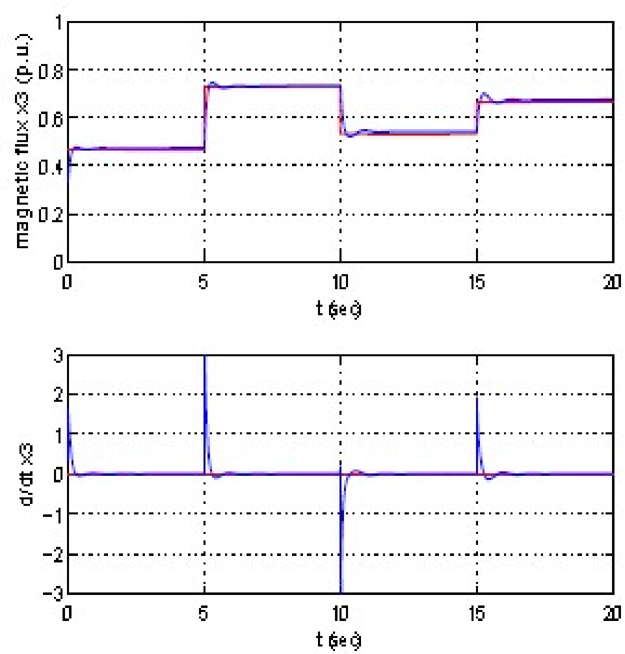


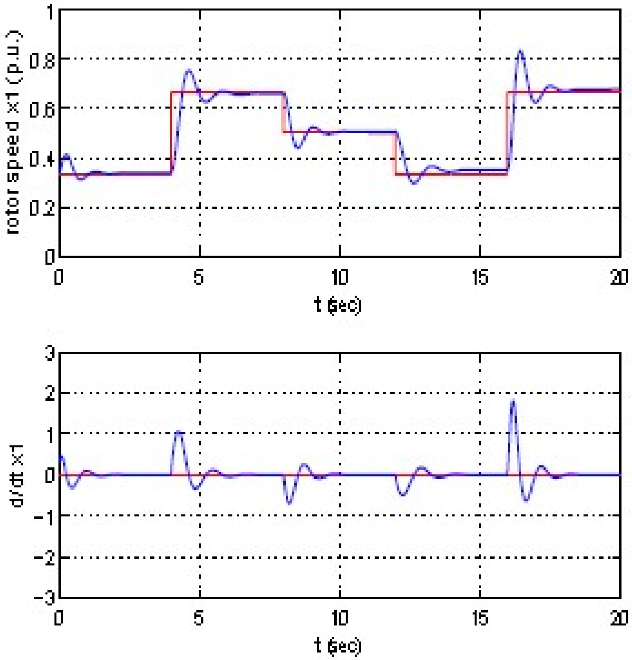
Fig 7a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

Fig 7b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 5



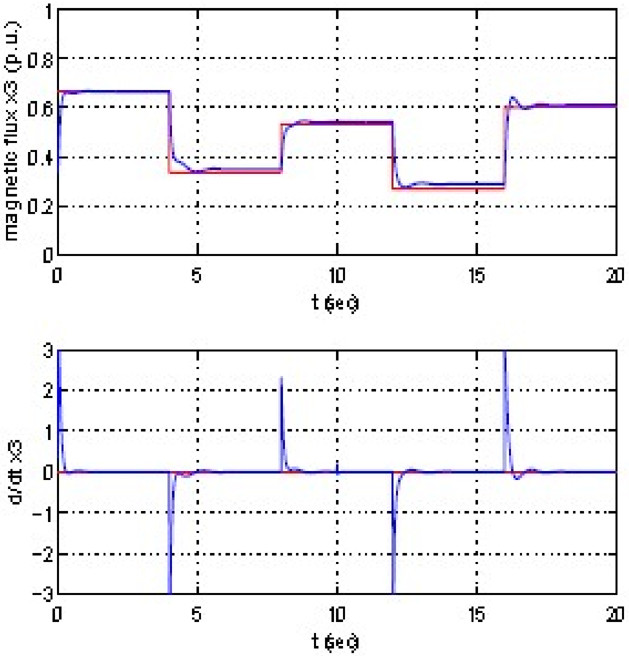


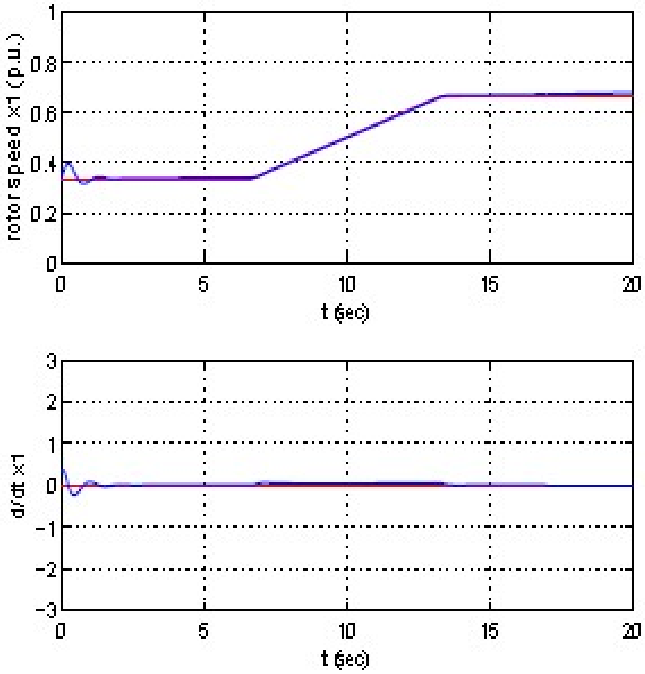
Fig 8a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

Fig 8b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 6



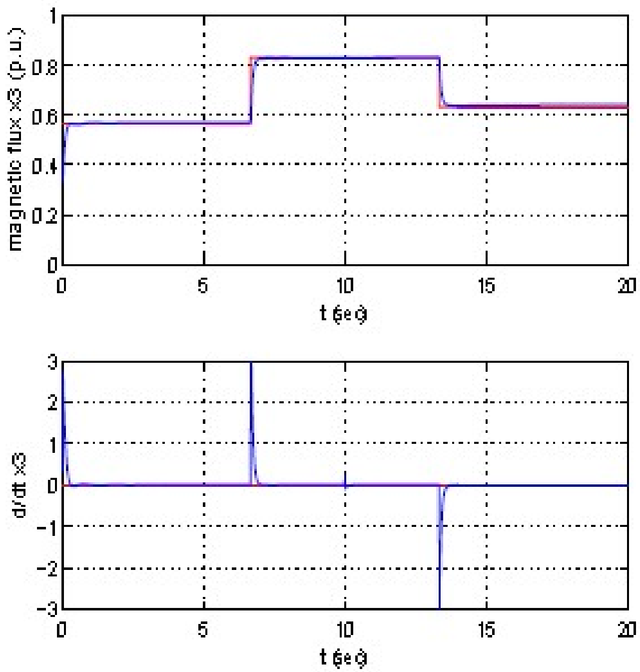


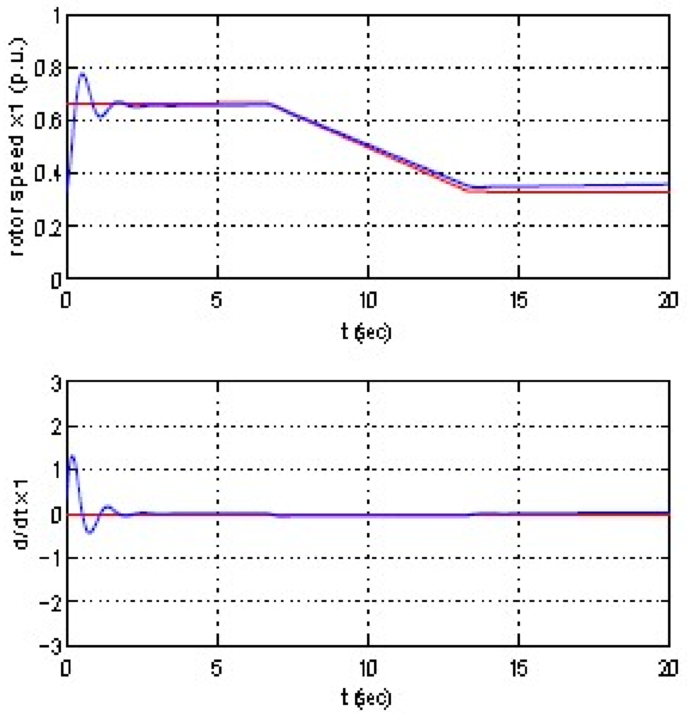
Fig 9a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

Fig 9b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

7. Simulation tests

Setpoint 7



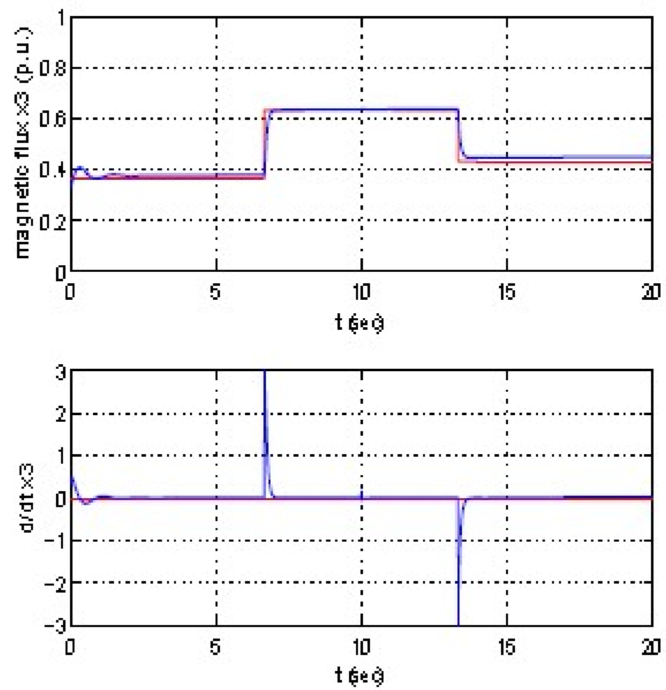


Fig 10a, Convergence of the rotor's speed x_1 and of its derivative x_2 to their reference setpoints

Fig 10b, Convergence of the stator's magnetic flux x_3 and of its derivative x_4 to their reference setpoints

Example 3: Nonlinear control and state estimation using Lyapunov methods

8. Conclusions

● A solution to the problem of model-free adaptive control for doubly-fed Induction generators has been proposed

● It was proven that the dynamic model of the DFIG is a differentially flat one. The flat outputs of the model were taken to be the rotor's turn speed and the magnetic flux of the stator.

● By proving differential flatness properties for the machine, the transformation of its model to the linear canonical form was achieved.

● In this new linearized description the control inputs comprised nonlinear terms which were related to the system's unknown dynamics.

● These terms were dynamically identified with the use of neurofuzzy approximators. These estimates of the unknown dynamics were used in turn in the computation of a feedback control input, thus establishing an indirect adaptive control scheme.

● It was also assumed that only the output of the DFIG could be directly measured and that the rest of the state vector elements of the machine had to be computed with the use of a state-observer.

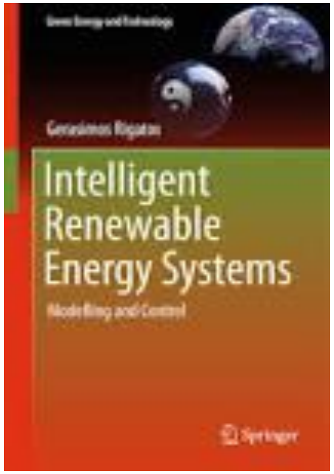
● The stability of the control loop was proven with the use of Lyapunov analysis.

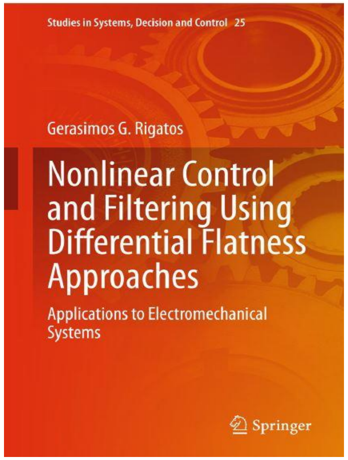
8. Conclusions

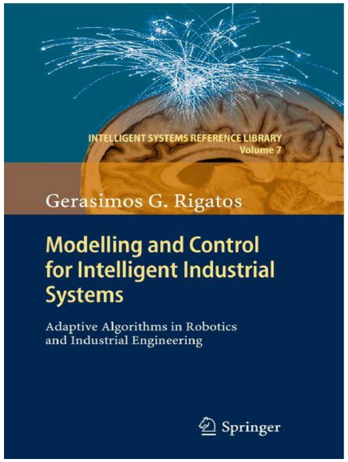
● New approaches to nonlinear and optimal control of electric power systems have been analyzed, and their applications to electric power generators and power electronics have been shown

● The main approaches for nonlinear control have been: (i) **control with global linearization** method (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods (adaptive control)** in case that the model of the electric power system is unknown

● Flatness-based control and its adaptive fuzzy implementation have been shown as very efficient for controlling a wide class of electric power systems. Besides a novel method of Nonlinear optimal control for electric power systems has been analyzed







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