Nonlinear control and state estimation for electric power systems

Gerasimos Rigatos

Electrical and Comp. Eng., PH.D.
Research Director
SMIEEE, MIET, CEng

email: grigat@ieee.org

1 . Outline

● The reliable functioning of electric power systems relies on the solution of the associated nonlinear control and state estimation problems

● The main approaches followed towards the solution of nonlinear control problem are as follows: (i) **control with global linearization** methods (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods** (adaptive control methods) when the dynamic model of the electric power systems is unknown

● The main approaches followed towards the solution of the nonlinear state estimation problems are as follows: (i) state estimation with methods global linearization (ii) state estimation with methods of approximate (asymptotic) linearization

● Factors of major importance for the control loop of electric power systems are as follows (i) global stability conditions for the related nonlinear control scheme (ii) global stability conditions for the related nonlinear state estimation scheme (iii) global asymptotic stability for the joint control and state estimation scheme

2 . Nonlinear control and state estimation with global linearization

● To this end the differential flatness control theory is used

● The method can be applied to all nonlinear systems which are subject to an input-output linearization and actually such systems posses the property of differential flatness

● The state-space description for the dynamic model of the electric power systems is transformed into a more compact form that is input-output linearized. This is achieved after defining the system's flat outputs

● A system is differentially flat if the following two conditions hold: (i) all state variables and control inputs of the system can be expressed as differential functions of its flat outputs (ii) the flat outputs of the system and their time-derivatives are differentially independent, which means that they are not connected through a relying having the form of an ordinary differential equation

● With the applications of change of variables (diffeomorphisms) that rely on the differential flatness property (i), the state-space description of the electric power system is written into the linear canonical form. For the latter state-space description it is possible to solve both the control and the state estimation problem for the electric power system.

3 . Nonlinear control and state estimation with approximate linearization

● To this end the theory of optimal H-infinity control and the theory of optimal H-infinity state estimation are used

● The nonlinear state-space description of the electric power system undergoes approximate linearization around a temporary operating point which is updated at each iteration of the control and state estimation algorithm

● The linearization relies on first order Taylor series expansion around the temporary operating point and makes use of the computation of the associated Jacobian matrices

● The linearization error which is due to the truncation error of higher-order terms in the Taylor series expansion is considered to be a perturbation that is finally compensated by the robustness of the control algorithm

● For the linearized description of the state-space model an optimal H-infinity controller is designed. For the selection of the controller's feedback gains an algebraic Riccati equation has to be solved at each time step of the control algorithm

● Through Lyapunov stability analysis, the global stability properties of the control method are proven

● For the implementation of the optimal control method through the processing of measurements from a small number of sensors in the electric power system, the H-infinity Kalman Filter is used as a robust state estimator

4 . Nonlinear control and state estimation with Lyapunov methods

● By initially proving the differential flatness properties for the electric power system and by defining its flat outputs a transformation of Its state-space description into an equivalent input-output linearized form is achieved.

● The unknown dynamics of the electric power systems is incorporated into the transformed control inputs of the system, which now appear in its equivalent input-output linearized state-space description

● The control problem for the electric power systems of unknown dynamics in now turned into a problem of indirect adaptive control. The computation of the control inputs of the system is performed simultaneously with the identification of the nonlinear functions which constitute its unknown dynamics.

● The estimation of the unknown dynamics of the electric power system is performed through the adaptation of neurofuzzy approximators. The definition of the learning parameters takes place through gradient algorithms of proven convergence, as demonstrated by Lyapunov stability analysis

● The Lyapunov stability method is the tool for selecting both the gains of the stabilizing feedback controller and the learning rate of the estimator of the unknown system's dynamics

● Equivalently through Lyapunov stability analysis the feedback gains of the state estimators of the electric power system are chosen. Such observers are included in the control loop so as to enable feedback control through the processing of a small number of sensor measurements

Example 1: Nonlinear control and state estimation using global linearization

5.1. Outline

• **Decentralized control for parallel inverters** connected to the power grid is developed using **differential flatness theory** and **the Derivative-free nonlinear Kalman Filter**.

• The problem is of elevated difficulty comparing to the control of stand-alone inverters because in this case in the dynamics of each inverter one has also to **compensate for interaction terms** which are due to the coupling with other inverters.

• T**he model of inverters, is differentially flat** and thus the multiple inverters model can be transformed into a set of local inverter models which are **decoupled and linearized**.

• For each local inverter the design of a **state feedback controller** becomes possible, e.g. using pole placement methods. Such a controller processes measurements not only coming from the individual inverter but also coming from other inverters connected to the grid.

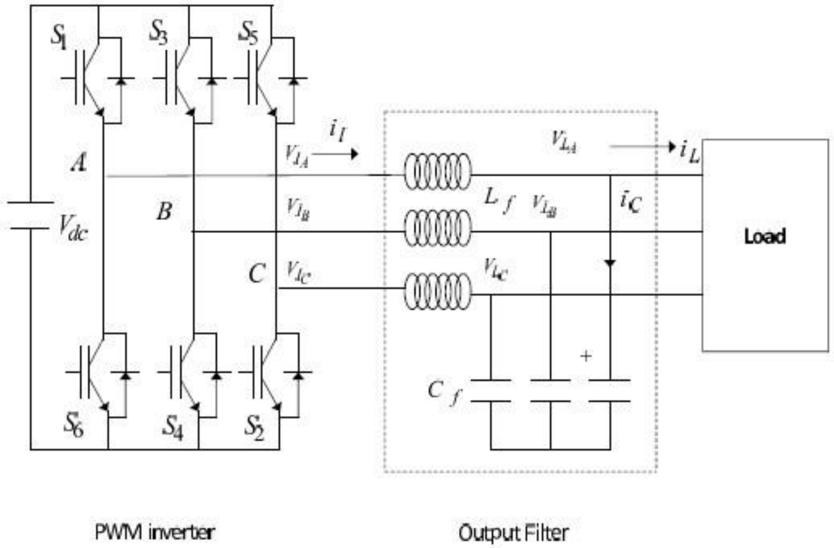
• Moreover, to **estimate the non-measurable state variables** of each local inverter, the **Derivative-free nonlinear Kalman Filter** is used. This consists of the Kalman Filter recursion applied to the local linearized model of the inverter and of an inverse transformation that is based on differential flatness theory, which enables to compute estimates of the state variables of the initial nonlinear model of the inverter.

• Furthermore, by **redesigning the aforementioned filter as a disturbance observer** it becomes also possible to estimate and compensate for disturbance terms that affect each local inverter.

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

Voltage inverters (DC to AC converters) are usually connected to their output to **a LC or a LCL filter**



By applying **Kirchhoff's voltage and current laws** one obtains

$$\frac{d}{dt}i_I = \frac{1}{L_f}V_I - \frac{1}{L_f}V_L$$
$$\frac{d}{dt}V_L = \frac{1}{C_f}i_I - \frac{1}{C_f}i_L$$

A

For the representation of the voltage and current variables, denoted as $X = \{I, V\}$ in the ab static reference frame one has

$$X_{ab} = X_a e^{j0} + X_b e^{\frac{j2\pi}{3}} + X_c e^{\frac{j4\pi}{3}}$$

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

Using the **Park transformation** this is also written as a complex variable in the form

$$X_{ab} = X_a + jX_b$$

Next, the voltage and current variables are represented in the **rotating *dq* reference** frame

$$X_{dq} = X_{ab}e^{-j\theta} \Rightarrow X_{ab} = X_{dq}e^{j\theta}$$
$$\text{where } \theta(t) = \int_0^t \omega(t)dt + \theta_0$$

By differentiating with respect to time one obtains the following description

$$\dot{X}ab = \frac{d}{dt}X_{dq} + j\omega X_{dq}$$

Thus, one has for the current and voltage variables respectively,

$$\begin{aligned} \dot{i}_{I,ab} &= \frac{d}{dt}i_{I,dq} + (j\omega)i_{I,dq} \\ \dot{V}_{L,ab} &= \frac{d}{dt}V_{L,dq} + (j\omega)V_{L,dq} \end{aligned} \quad \text{B}$$

By substituting Eq. B into Eq. A one obtains

$$\begin{aligned} \frac{d}{dt}i_{I,dq} + j\omega i_{I,dq} &= \frac{1}{L_f}V_{I,dq} - \frac{1}{L_f}V_{L,dq} \\ \frac{d}{dt}V_{L,dq} + j\omega V_{L,dq} &= \frac{1}{C_f}i_{I,dq} - \frac{1}{C_f}i_{L,dq} \end{aligned}$$

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

Thus one arrives at a **description of the inverter's dynamics in the *dq* reference frame**

$$
\begin{aligned}
\frac{d}{dt}V_{L,d} &= \omega V_{L,q} + \frac{1}{C_f}i_{I,d} - \frac{1}{C_f}i_{L,d}\\
\frac{d}{dt}V_{L,q} &= -\omega V_{L,d} + \frac{1}{C_f}i_{I,q} - \frac{1}{C_f}i_{L,q}\\
\frac{d}{dt}i_{I,d} &= \omega i_{I,q} + \frac{1}{L_f}V_{I,d} - \frac{1}{L_f}V_{L,d}\\
\frac{d}{dt}i_{I,q} &= -\omega i_{I,d} + \frac{1}{L_f}V_{I,q} - \frac{1}{L_f}V_{L,q}
\end{aligned}
$$

The **state vector of the system** is taken to be

$$\tilde{X} = [V_{L_d}, V_{L_q}, i_{I,d}, i_{I,q}]^T$$

The **active and the reactive power of the inverter** are used next

$$p_f = V_{L_d} i_{L_d} + V_{L_q} i_{L_q} \tag{C}$$

$$q_f = V_{L_q} i_{L_d} - V_{L_d} i_{L_q} - \omega C_f (V_{L_d}^2 + V_{L_q}^2) + \omega L_f (i_{I,d}^2 + i_{I,q}^2) \tag{D}$$

By solving Eq. C and Eq. D with respect to the **load currents** one obtains

$$i_{L_d} = \frac{p_f V_{L_d} + q_f V_{L_q}}{V_{L_d}^2 + V_{L_q}^2} + \omega C_f V_{L_q} - \frac{\omega L_f V_{L_q}(i_{I_d}^2 + i_{I_q}^2)}{(V_{L_d}^2 + V_{L_q}^2)}$$

$$i_{L_q} = \frac{p_f V_{L_q} - q_f V_{L_d}}{V_{L_d}^2 + V_{L_q}^2} - \omega C_f V_{L_d} + \frac{\omega L_f V_{L_d}(i_{I_d}^2 + i_{I_q}^2)}{(V_{L_d}^2 + V_{L_q}^2)}$$

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

and by using the state variables notation $x_1 = V_{L_d},\ x_2 = V_{L_q},\ x_3 = i_{L_d}$ and $x_4 = i_{L_q}$

one finally obtains the **state-space description of the inverter's dynamics**

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \omega x_2 + \frac{1}{C_f}x_3 - \frac{1}{C_f}\frac{p_f x_1 + q_f x_2}{x_1^2+x_2^2} + \omega C_f x_2 - \frac{\omega L_f x_2(x_3^2+x_4^2)}{(x_1^2+x_2^2)} \\ -\omega x_1 + \frac{1}{C_f}x_4 - \frac{1}{C_f}\frac{p_f x_2 - q_f x_1}{x_1^2+x_2^2} - \omega C_f x_1 + \frac{\omega L_f x_1(x_3^2+x_4^2)}{(x_1^2+x_2^2)} \\ \omega x_4 - \frac{1}{L_f}x_1 \\ -\omega x_3 - \frac{1}{L_f}x_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} V_{L_d} \\ V_{L_q} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

thus, the inverter's model is written in the **nonlinear state-space form**

$$\begin{aligned} \dot{x} &= f(x) + G(x)u \\ y &= h(x) \end{aligned}$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

- **Differential flatness theory** has been developed as a **global linearization control method** by M. Fliess (Ecole Polytechnique, France) and co-researchers (Lévine, Rouchon, Mounier, Rudolph, Petit, Martin, Zhu, Sira-Ramirez et. al)

- A dynamical system can be written in the ODE form $S_i(w,\dot{w},\ddot{w},\ldots,w^{(i)}),\ \ i=1,2,\ldots,q$

  where $w^{(i)}$ stands for the i-th derivative of either a state vector element or of a control input

- The system is said to be **differentially flat** with respect to the **flat output**

$$y_i=\phi(w,\dot{w},\ddot{w},\ldots,w^{(a)}),\ i=1,\ldots,m \quad \text{where} \quad y=(y_1,y_2,\ldots,y_m)$$

if the following two conditions are satisfied

(i) There does not exist any differential relation of the form

$$R(y,\dot{y},\ddot{y},\ldots,y^{(\beta)})=0$$

which means that **the flat output and its derivatives are linearly independent**

(ii) All system variables are **functions of the flat output and its derivatives**

$$w^{(i)}=\psi(y,\dot{y},\ddot{y},\ldots,y^{(\gamma_i)})$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

The **flat output of the inverter** is taken to be the vector

$$y = [y_1, y_2] = [V_{L_d}, V_{L_q}]$$

The first row of the state-space equations is

$$\dot{x}_1 = \omega x_2 + \frac{1}{C_f}x_3 - \frac{1}{C_f}\frac{p_f x_1 + q_f x_2}{x_1^2+x_2^2} - \omega x_2 + \frac{1}{C_f}\frac{\omega L_f x_2(x_3^2+x_4^2)}{x_1^2+x_2^2} \tag{E}$$

The second row of the state-space equations is

$$\dot{x}_2 = \omega x_1 + \frac{1}{C_f}x_4 - \frac{1}{C_f}\frac{p_f x_2 + q_f x_1}{x_1^2+x_2^2} + \omega x_1 + \frac{1}{C_f}\frac{\omega L_f x_1(x_3^2+x_4^2)}{x_1^2+x_2^2} \tag{F}$$

These equations are rewritten as follows

$$\frac{1}{C_f}\frac{\omega L_f x_2(x_3^2+x_4^2)}{x_1^2+x_2^2} = \dot{x}_1 - \omega x_2 - \frac{1}{C_f}x_3 + \frac{1}{C_f}\frac{p_f x_1 + q_f x_2}{(x_1^2+x_2^2)} + \omega x_2$$

$$\frac{1}{C_f}\frac{\omega L_f x_1(x_3^2+x_4^2)}{x_1^2+x_2^2} = \dot{x}_2 + \omega x_1 - \frac{1}{C_f}x_4 + \frac{1}{C_f}\frac{p_f x_2 - q_f x_1}{(x_1^2+x_2^2)} - \omega x_1$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

By dividing the above two equations one gets

$$-\frac{x_2}{x_1}=\frac{\dot{x}_1-\omega x_2-\frac{1}{C_f}x_3+\frac{1}{C_f}\frac{p_f x_1+q_f x_2}{(x_1^2+x_2^2)}+\omega x_2}{\dot{x}_2+\omega x_1-\frac{1}{C_f}x_4+\frac{1}{C_f}\frac{p_f x_2-q_f x_1}{(x_1^2+x_2^2)}-\omega x_1}$$

while using in the notation the elements of the flat output vector this give

$$\begin{aligned}&-\frac{y_2}{y_1}\dot{y}_2-\omega y_2+\frac{1}{C_f}\left(\frac{y_2}{y_1}\right)\frac{p_f y_2-q_f y_1}{(y_1^2+y_2^2)}+\omega y_2= \\ &=\dot{y}_1-\omega y_2-\frac{1}{C_f}x_3+\frac{1}{C_f}\frac{p_f y_1+q_f y_2}{(y_1^2+y_2^2)}+\omega y_2\end{aligned}$$

By solving the above equation with respect to x_3 gives

$$\begin{aligned}x_3=-\frac{y_2}{y_1}x_4+C_f\{&\frac{y_2}{y_1}\dot{y}_2+\omega y_2+\frac{1}{C_f}\frac{(y_2}{y_1)}\frac{p_f y_2-q_f y_1}{(y_1^2+y_2^2)}-\omega y_2 \\ &+\dot{y}_1-\omega y_2+\frac{1}{C_f}\frac{p_f y_1+q_f y_2}{(y_1^2+y_2^2)}+\omega y_2\}\end{aligned}$$

which is also written as $x_3=-(\frac{y_2}{y_1})x_4+f_a(y_1,\dot{y}_1,y_2,\dot{y}_2)$ (G)

Next Eq. (G) is substituted into Eq. (E) which gives.

$$\begin{aligned}\dot{x}_2=&-\omega x_1+\frac{1}{C_f}x_4-\frac{1}{C_f}\frac{p_f x_2-q_f x_1}{(x_1^2+x_2^2)}+\omega x_1- \\ &-\frac{1}{C_f}\frac{\omega L_f x_1\{[-\frac{(y_2}{y_1})x_4+f_a(y_1,\dot{y}_1,y_2,doty_2)]^2+x_4^2\}}{(x_1^2+x_2^2)}\end{aligned}$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

or equivalently.

$$\dot{y}_2 = -\omega y_1 + \frac{1}{C_f}x_4 - \frac{1}{C_f}\frac{p_f y_2 - q_f y_1}{(y_1^2+y_2^2)} + \omega y_1 + \\ -\frac{1}{C_f}\frac{\omega L_f y_1\{[-(\frac{y_2}{y_1})x_4 + f_a(y_1,\dot{y}_1,y_2,\dot{y}_2)]^2 + x_4^2\}}{(y_1^2+y_2^2)}$$

which finally gives.

$$x_4 = f_b(y_1,\dot{y}_1,y_2,\dot{y}_2) \qquad \text{H}$$

Moreover, by substituting Eq. H into Eq. G one gets

$$x_3 = -(\frac{y_2}{y_1})f_b(y_1,\dot{y}_1,y_2,\dot{y}_2) + f_4(y_1,\dot{y}_1,y_2,\dot{y}_2)$$

From the last two rows of the state-space equations one has that

$$\dot{x}_3 = \omega x_4 - \frac{1}{L_f}x_1 + \frac{1}{L_f}u_1 \qquad \text{I}$$

$$\dot{x}_4 = -\omega x_3 - \frac{1}{L_f}x_2 + \frac{1}{L_f}u_2 \qquad \text{J}$$

Thus, one obtains

$$u_1 = L_f\{\dot{x}_3 - \omega x_4 + \frac{1}{L_f}x_1\} \Rightarrow u_1 = f_c(y_1,\dot{y}_1,y_2,\dot{y}_2)\}$$

$$u_2 = L_f\{\dot{x}_4 - \omega x_3 + \frac{1}{L_f}x_2\} \Rightarrow u_2 = f_d(y_1,\dot{y}_1,y_2,\dot{y}_2)\}$$

This **confirms the differential flatness of the model**

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

By considering the **active and reactive power** of the inverter as **piecewise constant** and by deriving and by deriving the first row of the state-space equations in time, one has

$$\ddot{x}_1=\omega\dot{x}_2+\frac{1}{C_f}\dot{x}_3-\frac{1}{C_f}\frac{\{(p_f\dot{x}_1+q_f\dot{x}_2)(x_1^2+x_2^2)-(p_fx_1+q_fx_2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2\}}-$$

$$-\omega\dot{x}_2+\frac{1}{C_f}\{\frac{\omega L_f\dot{x}_2(x_3^2+x_4^2)(x_1^2+x_2^2)+\omega L_f x_2(2x_3\dot{x}_3+2x_4\dot{x}_4)(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2}\}$$

$$-\frac{\omega L_f x_2(x_3^2+x_4^2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}$$

The time derivatives are substituted from the associated rows of the state-space equations.

$$\ddot{x}_1=\omega\dot{x}_2+\frac{1}{C_f}(\omega x_4-\frac{1}{L_f}x_1+\frac{1}{L_f}u_1)-\frac{1}{C_f}\frac{\{(p_f\dot{x}_1+q_f\dot{x}_2)(x_1^2+x_2^2)-(p_fx_1+q_fx_2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2\}}-$$

$$-\omega\dot{x}_2+\frac{1}{C_f}\{\frac{\omega L_f\dot{x}_2(x_3^2+x_4^2)(x_1^2+x_2^2)+\omega L_f x_2(x_3^2+x_4^2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$+\frac{\omega L_f x_2}{C_f}\frac{2x_3(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2}(\omega x_4-\frac{1}{L_f}x_1+\frac{1}{L_f}u_1)+\frac{\omega L_f x_2}{C_f}\frac{2x_4(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2}(-\omega x_3-\frac{1}{L_f}x_2+\frac{1}{L_f}u_2)$$

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

The previous relation can be also written using the **notation of the Lie algebra-based linearization**

$$\ddot{x}_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$

where

$$L_f^2 h_1(x) = \omega \dot{x}_2 + \frac{1}{C_f}(\omega x_4 - \frac{1}{L_f} x_1) - \frac{1}{C_f}\{\frac{(p_f \dot{x}_1 + q_f \dot{x}_2)(x_1^2 + x_2^2) - (p_f x_1 + q_f x_2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$-\omega \dot{x}_2 + \frac{1}{C_f}\{\frac{\omega L_f \dot{x}_2 (x_3^2 + x_4^2)(x_1^2 + x_2^2) - \omega L_f x_2 (x_3^2 + x_4^2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$+\frac{\omega L_f x_2}{C_f} \cdot \frac{2x_3(\omega x_4 - \frac{1}{L_f} x_1)}{(x_1^2+x_2^2)} + \frac{\omega L_f x_2}{C_f} \cdot \frac{2x_1(-\omega x_3 - \frac{1}{L_f} x_2)}{(x_1^2+x_2^2)}$$

$$L_{g_a} L_f h_1(x) = \frac{1}{C_f}\{\frac{\omega L_f x_2 (2x_3)}{(x_1^2+x_2^2)L_f} + \frac{1}{L_f}\}$$

$$L_{g_b} L_f h_1(x) = \frac{\omega L_f x_2}{C_f} \frac{2x_4}{(x_1^2+x_2^2)} \frac{1}{L_f}$$

In a similar manner, by differentiating the second row of the state-space equations with respect to time one has

$$\ddot{x}_2 = -\omega \dot{x}_1 + \frac{1}{C_f}\dot{x}_4 - \frac{1}{C_f}\frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2 + x_2^2) - (p_f x_2 - g_f x_1(2x_1\dot{x}_1 + 2x_2\dot{x}_2))}{(x_1^2+x_2^2)^2} +$$

$$+\omega \dot{x}_1 - \frac{1}{C_f}\{\frac{\omega L_f \dot{x}_1 (x_3^2 + x_4^2)(x_1^2 + x_2^2) + \omega L_f x_1 (2x_3\dot{x}_3 + 2x_4\dot{x}_4)(x_1^2 + x_2^2)}{(x_1^2+x_2^2)^2} -$$

$$\frac{-\omega L_f x_1 (x_3^2 + x_4^2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

The previous relation can be also written using the **notation of the Lie algebra-based linearization**

$$\ddot{x}_2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

where

$$L_f^2 h_2(x) = -\omega \dot{x}_1 + \frac{1}{C_f}(-\omega x_3) - \frac{1}{C_f}\{\frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2+x_2^2) - (p_f x_2 - g_f x_1)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\} +$$

$$\omega \dot{x}_1 - \frac{1}{C_f}\{\frac{\omega L_f \dot{x}_1 (x_3^2+x_4^2)(x_1^2+x_2^2) - \omega L_f x_1 (x_3^2+x_4^2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$-\frac{1}{C_f}\frac{\omega L_f x_1 2x_3(\omega x_4 - \frac{1}{L_f}x_1)}{(x_1^2+x_2^2)} - \frac{1}{C_f}\frac{\omega L_f x_1 2x_4(-\omega x_3 - \frac{1}{L_f}x_2)}{(x_1^2+x_2^2)}$$

$$L_{g_a} L_f h_2(x) = -\frac{1}{C_f}\frac{\omega L_f x_1 2x_3}{(x_1^2+x_2^2)}\frac{1}{L_f}$$

$$L_{g_b} L_f h_2(x) = -\frac{1}{C_f}\{\frac{\omega L_f x_1 2x_4}{(x_1^2+x_2^2)}\frac{1}{L_f} + \frac{1}{L_f}\}$$

Thus, one obtains **an input-output linearized description of the inverter**

$$\ddot{x}_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$
$$\ddot{x}_2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

or equivalently

$$\ddot{x}_1 = v_1 \qquad \ddot{x}_2 = v_2$$

with

$$v_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$
$$v_2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

For this form of the system's dynamics the design of a **state feedback controller** is easy. This takes the form

$$v_1 = \ddot{x}_1^d - k_d^1(\dot{x}_1 - \dot{x}_1^d) - k_p^1(x_1 - x_1^d)$$
$$v_2 = \ddot{x}_2^d - k_d^2(\dot{x}_2 - \dot{x}_2^d) - k_p^2(x_2 - x_2^d)$$

The **control input** that is actually applied to the inerter is given form $\tilde{v} = \tilde{f} + \tilde{M}u$

or equivalently $$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} + \begin{pmatrix} L_{g_a} L_f h_1(x) & L_{g_b} L_f h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

which means that the control input that is finally applied to the system is

$$\tilde{u} = \tilde{M}^{-1}(\tilde{v} - \tilde{f})$$

Moreover, by defining the **new state variables** $z_1 = x_1, \; z_2 = \dot{x}_1, \; z_3 = x_2 \text{ and } z_4 = \dot{x}_2,$ the following state-space description is obtained

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} z_1^m \\ z_2^m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

Example 1: Nonlinear control and state estimation using global linearization

5.5. Equivalence between inverters and synchronous generators

Synchronization between parallel inverters is considered next. The functioning of the i-th inverter is shown to be equivalent to a synchronous generator with turn speed denoted as ω_i

The **deviation from the synchronous speed** is shown to be proportional to the deviation of the produced active power from a reference value

$$\Delta\delta_i = \omega_i - \omega_d = -k_{p_i}(P_i^m - P_i^d) \qquad \text{K}$$

P_i^m **measured active power** of the i-th power generation unit

P_i^d **desirable active power**

k_{p_i} ”**droop” gain** which is practically computed by dividing the range of variation of the inverter's frequency $(\omega_{max}\text{-}\omega_{min})$ by the maximum active power $P_{i_{max}}$

Since the measured active power is obtained from the inverter's real active power with a **time delay in measurement**, it holds that

$$P_i^m(s) = e^{-s\tau_{p_i}}P(s) \quad \text{or equivalently} \quad \tau_{p_i}\dot{P}_i^m = -P_i^m + P_i \qquad \text{L}$$

Thus the i-th **inverter's dynamics** is expressed as

$$\Delta\dot{\delta}_i = \Delta\omega_i$$
$$\tau_{p_i}\dot{P}_i^m = -P_i^m + P_i$$

Example 1: Nonlinear control and state estimation using global linearization

5.5. Equivalence between inverters and synchronous generators

By differentiating Eq. K one obtains

$$J_i \Delta\dot{\omega}_i = -\frac{1}{k_{p_i}}\Delta\omega_i + P_i^d - P_i \quad \text{M}$$

Moreover, from Eq. L one obtains

$$\dot{P}_i^m = -\frac{1}{\tau_{p_i}}P_i^m + \frac{1}{\tau_{p_i}}P_i \quad \text{N}$$

By substituting Eq. N Into Eq. M one obtains

$$\Delta\dot{\omega}_i = -k_{p_i}\left(-\frac{1}{\tau_{p_i}}P_i^m + \frac{1}{\tau_{p_i}}P_i\right) + k_{p_i}\dot{P}_i^d$$

and using that $\dot{P}_i^d = 0$ one has

$$\Delta\dot{\omega}_i = \frac{k_{p_i}}{\tau_{p_i}}P_i^m - \frac{k_{p_i}}{\tau_{p_i}}P_i$$ or equivalently $$J_i\Delta\dot{\omega}_i = P_i^m - P_i$$ with $J_i = \tau_{p_i}/k_{p_i}$

Additionally, from Eq. K one has

$$\omega_i - \omega_d - k_{p_i}P_i^d = -k_{p_i}P_i^m \Rightarrow P_i^m = -\frac{1}{k_{p_i}}\omega_i + \frac{1}{k_{p_i}}\omega_d + P_i^d \Rightarrow$$
$$P_i^m = -\frac{1}{k_{p_i}}\Delta\omega_i + P_i^d$$

From the previous two equations one gets

$$\Delta\dot{\omega}_i = -k_{p_i}\dot{P}_i^m + k_{p_i}\dot{P}_i^d$$ or $$J_i\Delta\dot{\omega}_i = -D_{p_i}\Delta\omega_i + P_i^d - P_i \quad \text{O}$$

Example 1: Nonlinear control and state estimation using global linearization

5.5. Equivalence between inverters and synchronous generators

In ideal conditions there is no **interaction (power exchange) between distributed power units** connected to the same electricity grid.

However, frequently such interaction exists and in the latter case Eq. O should be enhanced by including an interaction term

$$J_i\Delta\dot{\omega}_i = -D_{p_i}\Delta\omega_i + (P_i^d - P_i) + \sum_{j=1,j\neq i}^{n} G_{ij} sin(\delta_i - \delta_j)$$

where δ_i is the virtual turn angle that is associated with the i-th power generation unit (inverter).

About the **coupling coefficients** G_{ij} these are functions of the conductance of the grid line which connects the *i-th* to the *j-th* power generation unit, as well as of the grid voltage that is measured at points *i* and *j* respectively

Thus, finally the **dynamics of the i-th power generation** unit (inverter) is **described as a synchronous generator,** which interacts with other generators In the grid

$$\Delta\dot{\delta}_i(t) = \Delta\omega_i(t)$$
$$J_i\Delta\dot{\omega}_i(t) = -D_{p_i}\Delta\omega_i(t) + (P_i^d(t) - P_i(t)) + \sum_{j=1}^{N} G_{ij} sin(\delta_i - \delta_j) \quad \text{P}$$

In this approach, it is considered that the **i-th local controller** not only processes measurements coming from the associated power generation unit, but also **uses measurements coming from the other power units** which are connected to the grid

Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

By **representing the inverter as a virtual synchronous generator** then one has that its dynamics is composed of two parts (i) the rotation part and (ii) the electrical part.

(i) **Rotation part**

$$\begin{gathered}\Delta\dot{\delta}_i(t)=\Delta\omega_i(t)\\ J_i\Delta\dot{\omega}_i(t)=-D_{p_i}\Delta\omega_i(t)+(P_i^d(t)-P_i(t))+\sum_{j=1,j\neq i}^{N}G_{ij}sin(\delta_i-\delta_j)\end{gathered}\tag{Q}$$

(i) **Electrical part**

$$\frac{d}{dt}\begin{pmatrix}V_{L_d}\\ V_{L_q}\\ i_{I_d}\\ i_{I_q}\end{pmatrix}=\begin{pmatrix}\omega V_{L_q}+\frac{1}{C_f}i_{I_d}-\frac{1}{C_f}\frac{p_fV_{L_d}+q_fV_{L_q}}{V_{L_d}^2+V_{L_q}^2}+\omega C_fV_{L_q}-\frac{\omega L_fV_{L_q}(i_{I_d}^2+i_{I_q}^2)}{(V_{L_d}^2+V_{L_q}^2)}\\ -\omega V_{L_d}+\frac{1}{C_f}i_{I_q}-\frac{1}{C_f}\frac{p_fV_{L_q}-q_fV_{L_d}}{V_{L_d}^2+V_{L_q}^2}-\omega C_fV_{L_d}+\frac{\omega L_fV_{L_d}(i_{I_d}^2+i_{I_q}^2)}{(V_{L_d}^2+V_{L_q}^2)}\\ \omega i_{I_q}-\frac{1}{L_f}V_{L_d}\\ -\omega i_{I_d}-\frac{1}{L_f}V_{L_q}\end{pmatrix}+\begin{pmatrix}0&0\\0&0\\ \frac{1}{L_f}&0\\0&\frac{1}{L_f}\end{pmatrix}\begin{pmatrix}V_{I_d}\\ V_{I_q}\end{pmatrix}\tag{R}$$

The **synchronizing control approach** for the i-th inverter makes use of Eq. Q and of the linearized inverter model given in Eq. R

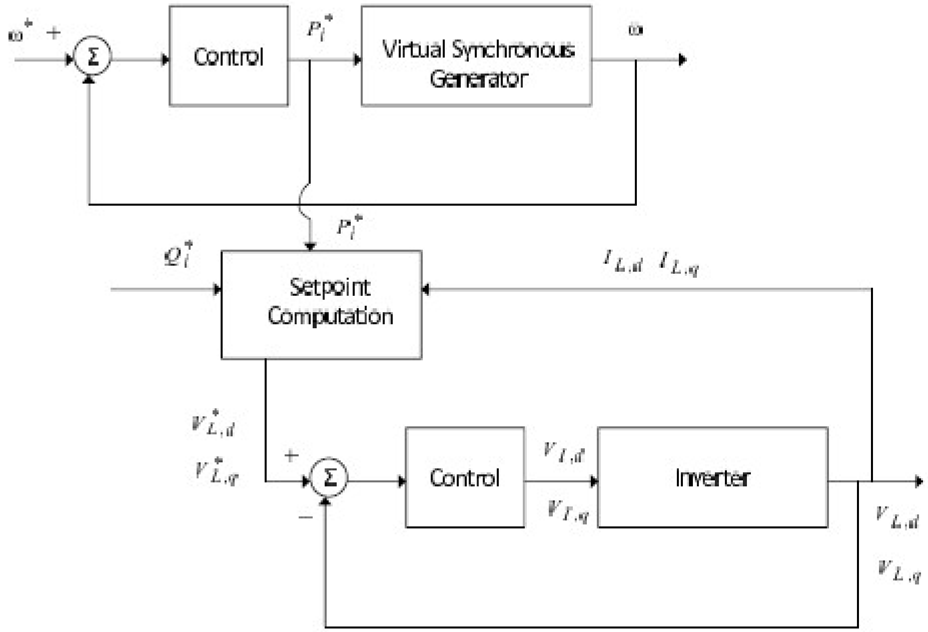
Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

First, the value of P_i , that is the **active power** that the i-th inverter should inject to the grid, is found from the solution of the control problem of Eq. (Q)

Subsequently P_i is used in the computation of the solution of the control problem of Eq. (R)

The **computation of setpoints** for the control of the **electric part of the inverter** is shown in the following diagram



Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

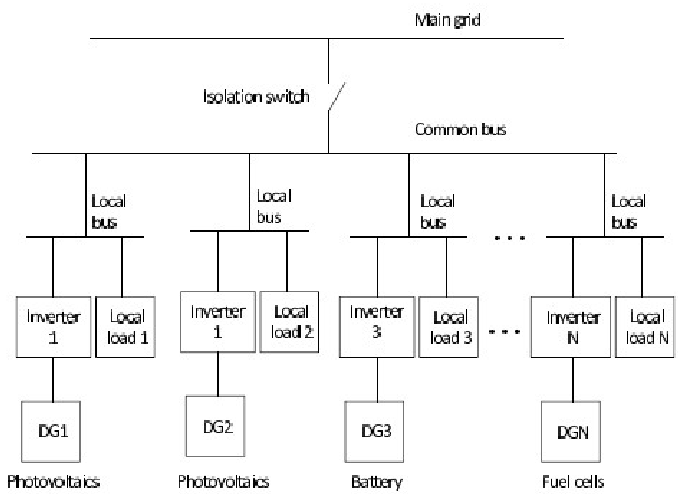
It can be proven that the model of **N-parallel inverters connected to the electricity grid** is a **differentially flat** one

By defining as **flat output** a **generalization of the state vector** of the stand-alone inverter, that is

$$Y = [y_1^1, y_2^1, y_3^1, y_1^2, y_2^2, y_3^2 \cdots, y_1^N, y_2^N, y_3^N]$$

or equivalently

$$Y = [\delta^1, V_{L_d}^1, V_{L_q}^1, \ \delta^2, V_{L_d}^2, V_{L_q}^2, \cdots, \delta^N, V_{L_d}^N, V_{L_q}^N]$$



It can be confirmed that **all state variables and control inputs** for **the model of the N coupled inverters** can be expressed as **functions of the aforementioned flat output Y** and of its derivatives.

Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

Using the previous flat output definition, and the state variables

$$z_1^i = y_1,\ z_2^i = \dot{y}_1,\ z_3^i = y_2,\ z_4^i = \dot{y}_2,\quad z_5^i = y_3,\ z_6^i = \dot{y}_3$$

one has the state-space description

$$\begin{pmatrix} \dot{z}_1^i \\ \dot{z}_2^i \\ \dot{z}_3^i \\ \dot{z}_4^i \\ \dot{z}_5^i \\ \dot{z}_6^i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^i \\ z_2^i \\ z_3^i \\ z_4^i \\ z_5^i \\ z_6^i \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1^i \\ v_2^i \\ v_3^i \end{pmatrix}$$

where the control inputs of this model are defined as

$$v_1^i = \frac{1}{J_i}[-D_{p_i}\Delta\omega_i(t) + (P_i^d(t) - P_i(t)) + \sum_{j=1}^N G_{ij} sin(\delta_i - \delta_j)] \qquad S$$

$$\begin{aligned} v_2^i &= L_f^2 h_1{}^i(x) + L_{g_a} L_f h_1^i(x) u_1^i + L_{g_b} L_f h_1^i(x) u_2^i \\ v_3^i &= L_f^2 h_2{}^i(x) + L_{g_a} L_f h_2^i(x) u_1^i + L_{g_b} L_f h_2^i(x) u_2^i \end{aligned} \qquad T$$

The above mean that for the **synchronization of the i-th virtual generator (inverter)** the **control input** (in the form of active power) is finally given by

$$\begin{aligned} P_i = &-J_i \ddot{x}_{1,i}^d - D_{p_1} x_2 + P_i^d + \sum_{j=1}^N G_{ij} sin(x_{1,i} - x_{1,j}) + \\ &+ J_i K_{d_i}(\dot{x}_{1,i} - \dot{x}_{1,i}^d) + J_i K_{p_i}(x_{1,i} - x_{1,i}^d) \end{aligned} \qquad U$$

Example 1: Nonlinear control and state estimation using global linearization

5.7. Disturbances estimation with Kalman Filtering

A **state estimator for each local power generation unit** can be also designed in the form of a disturbance observer.

It is considered that the **linearized model of the i-th inverter** is affected by **additive input disturbances**

$$\ddot{z}_1^i = v_1^i + \tilde{d}_1^i$$
$$\ddot{z}_3^i = v_2^i + \tilde{d}_2^i$$
$$\ddot{z}_5^i = v_3^i + \tilde{d}_3^i$$

The **disturbances' dynamics can be represented by the n-th order derivative** of the disturbances variables together with the associated initial conditions.

Thus the **additive disturbances** are equivalently described in the form

$$\tilde{d}_1^{(n)} = f_{d_1},\ \tilde{d}_2^{(n)} = f_{d_2} \text{ and } \tilde{d}_3^{(n)} = f_{d_3}$$

The state vector is extended by **including as additional state variables** the disturbances and their derivatives. Thus, one has

$$\dot{z}_1^i = z_2^i,\ \dot{z}_2^i = z_7 + v_1^i,\ \dot{z}_3^i = z_4^i,\ \dot{z}_4^i = z_9 + v_2^i,\ \dot{z}_5^i = z_6^i,\ \dot{z}_6^i = z_{11}^i + v_3^i,$$

$$\dot{z}_7^i = z_8^i,\ \dot{z}_8^i = f_{d_1}^i,\ \dot{z}_9^i = z_{10}^i, \quad z_{11}^i = \tilde{d}_3 \text{ and } z_{12}^i = \dot{\tilde{d}}_3.$$

Example 1: Nonlinear control and state estimation using global linearization

5.7. Disturbances estimation with Kalman Filtering

Therefore, one has the system's dynamics in the **extended spate-space form**

$$\dot{z}_e = A_e\hat{z}_e + B_e v_e$$

where the **extended inputs vector** is $v_e = [v_1^i, v_2^i, v_3^i, f_{d_1}, f_{d_2}, f_{d_3}]^T$

while

$$A_e = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad B_e = \begin{pmatrix} 0&0&0&0&0&0\\ 1&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&1 \end{pmatrix} \quad C_e^T = \begin{pmatrix} 1&0&0\\ 0&0&0\\ 0&1&0\\ 0&0&0\\ 0&0&1\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0\\ 0&0&0 \end{pmatrix}$$

For the extended state-space description of the system the **state observer** becomes

$$\dot{\hat{z}}_e = A_o\hat{z}_e + B_o v_e + K_f(C_o z_e - C_o)\hat{z}_e$$

where $A_o = A_e$ and $C_o = C_e$

while B_o differs from B_e In the elements of the 10th and 12th rows which are all set to 0

Example 1: Nonlinear control and state estimation using global linearization

5.7. Disturbances estimation with Kalman Filtering

For the **linearized model** of the parallel inverters, **state estimation** is performed with the use of the **Kalman Filter (Derivative-free nonlinear Kalman Filter)**

In the filter's algorithm, the previously defined matrices A_e, B_e and C_e are substituted by their discrete-time equivalents A_{e_d}, B_{e_d} and C_{e_d} This is done through common discretization methods

The **filter's recursion** is:

measurement update:

$$\begin{gathered}K_f(k) = P^-(k)C_d^T[C_{e_d}P^-(k)C_{e_d}^T + R(k)]^{-1}\\ \hat{x}(k) = \hat{x}^-(k) + K_f(k)[C_{e_d}z(k) - C_{e_d}\hat{z}(k)]\\ P(k) = P^-(k) - K(k)C_{e_d}P^-(k)\end{gathered}$$

time update:

$$\begin{gathered}P^-(k+1) = A_{e_d}P(k)A_{e_d}^T + Q(k)\\ \hat{x}^-(k+1) = A_{e_d}\hat{x}(k) + B_{e_d}v(k)\end{gathered}$$

After identifying the disturbance terms, the control input of the inverter is modified as follows:

$$\begin{aligned}v_1^i &= \ddot{z}_1 - k_d^1(\dot{z}_1 - \dot{z}_1^d) - k_p^1(z_1 - z_1^d) - \hat{z}_7\\ v_2^i &= \ddot{z}_3 - k_d^2(\dot{z}_3 - \dot{z}_3^d) - k_p^2(z_3 - z_3^d) - \hat{z}_9\\ v_3^i &= \ddot{z}_5 - k_d^3(\dot{z}_5 - \dot{z}_5^d) - k_p^3(z_5 - z_5^d) - \hat{z}_{11}\end{aligned}$$

The inclusion of the **disturbance estimation term** $\hat{z}_7$, $\hat{z}_9$ and $\hat{z}_{11}$ in the feedback control inputs enables to compensate for effects of the perturb $\tilde{d}_1$, $\tilde{d}_2$ and $\tilde{d}_3$

Example 1: Nonlinear control and state estimation using global linearization

5.8. Simulation tests

The performance of the proposed **distributed control scheme for the synchronization of parallel inverters** was tested through simulation experiments. A **model of *N* = *3* distributed power generation units** was considered, while each one of these units was connected to the grid through an inverter

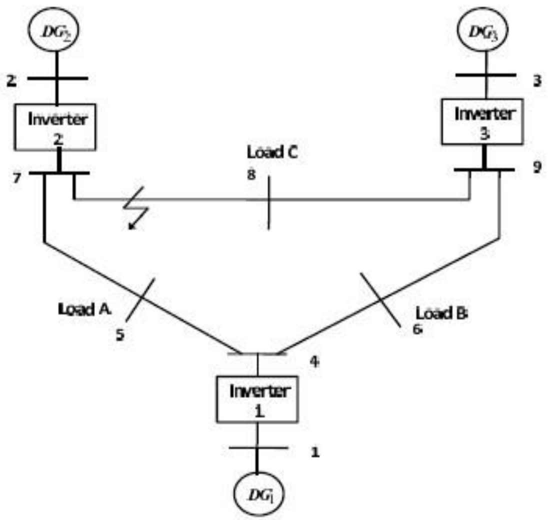


Table I
Parameters of the Inverters

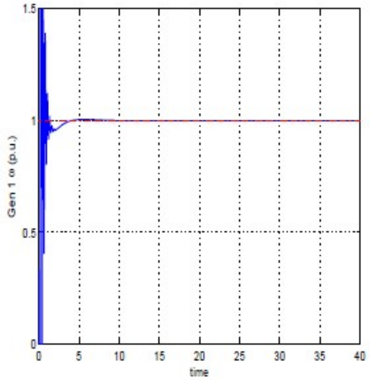
	Inv_1	Inv_2	Inv_3
$L_f\ (mH)$	10.5	10.3	10.1
$C_f\ (mF)$	0.04	0.03	0.02
$p_f\ (Kw)$	21.1	22.3	23.6

The three interconnected inverters, shown in Fig. 4, are assumed to have different model parameters which are described in **Table I.**

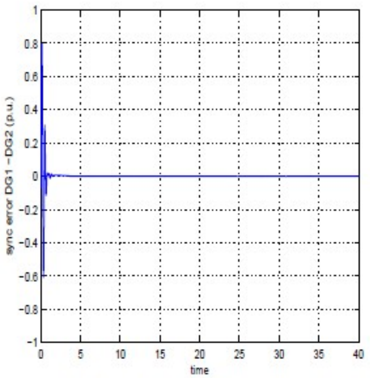
The objective is that all inverters (virtual synchronous generators) finally attain the same frequency ω_i

Example 1: Nonlinear control and state estimation using global linearization

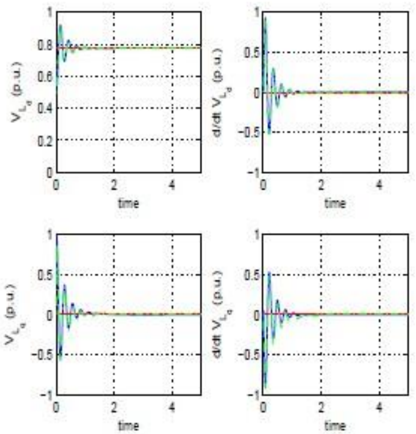
5.8. Simulation tests



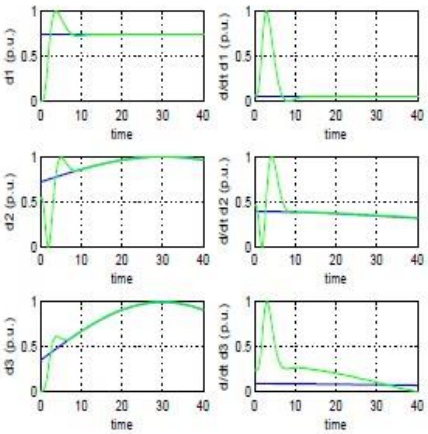
Test 1: (a) Angular speed of power generation unit 1



Test 1: synchronization error between power generation units 1 and 2



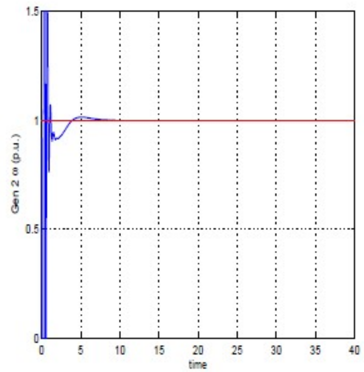
Test 1: Voltage components (in dq frame) and their derivatives



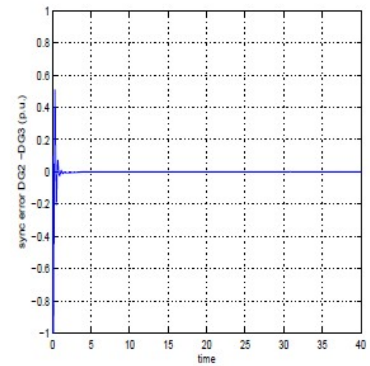
Test 1: Estimation of disturbance inputs

Example 1: Nonlinear control and state estimation using global linearization

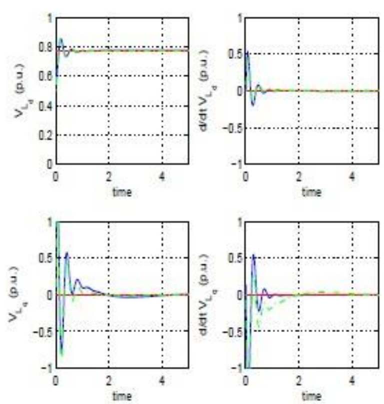
5.8. Simulation tests



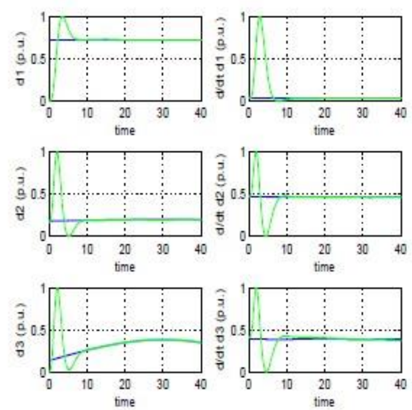
Test 2: (a) Angular speed of power generation unit 2



Test 2: synchronization error between power generation units 2 and 3



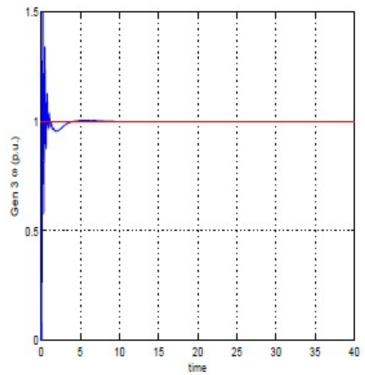
Test 2: Voltage components (in dq frame) and their derivatives



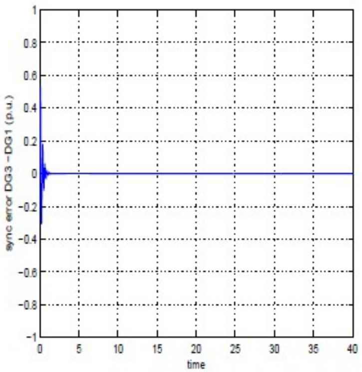
Test 2: Estimation of disturbance inputs

Example 1: Nonlinear control and state estimation using global linearization

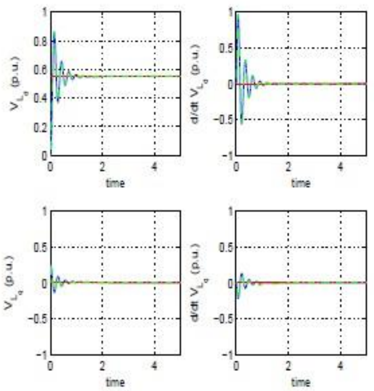
5.8. Simulation tests



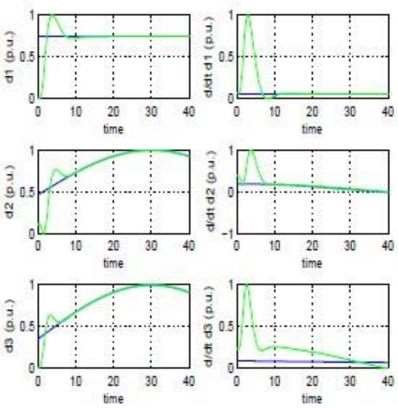
Test 3: (a) Angular speed of power generation unit 3



Test 3: synchronization error between power generation units 3 and 1



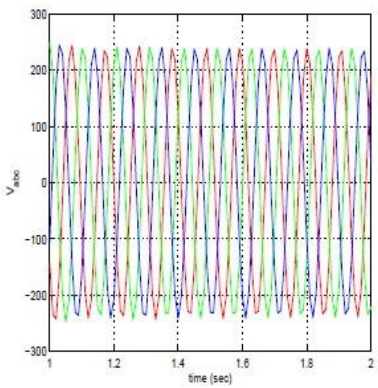
Test 3: Voltage components (in dq frame) and their derivatives



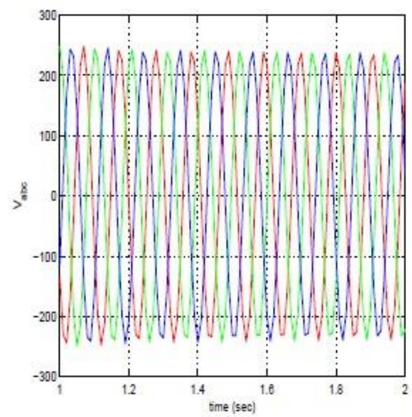
Test 3: Estimation of disturbance inputs

Example 1: Nonlinear control and state estimation using global linearization

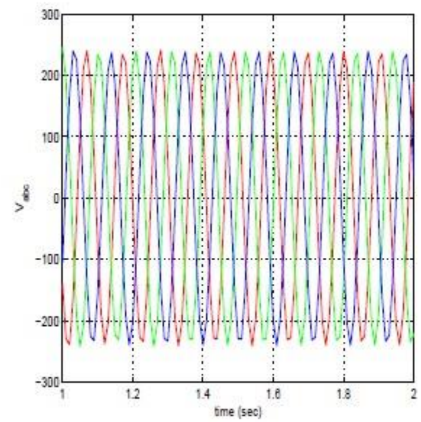
5.8. Simulation tests



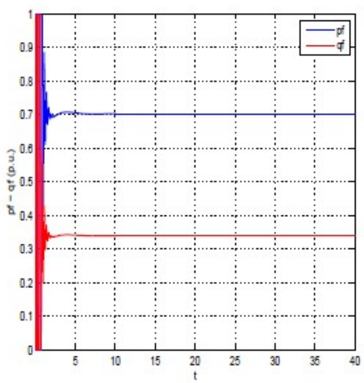
Test 1: Three-phase voltage variables



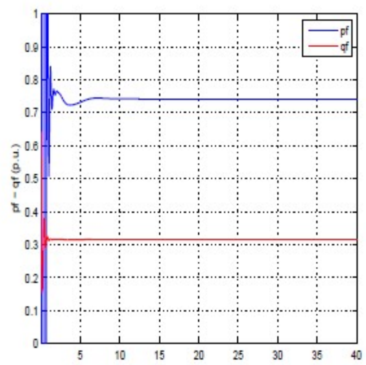
Test 2: Three-phase voltage variables



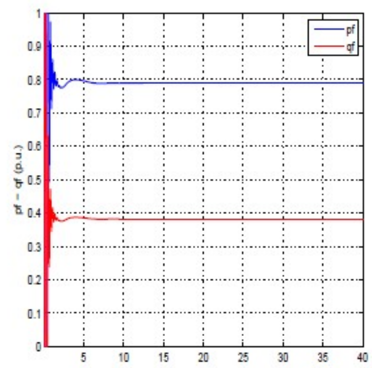
Test 3: Three-phase voltage variables



Test 1: Active and reactive power of the inverter



Test 2: Active and reactive power of the inverter



Test 3: Active and reactive power of the inverter

Example 1: Nonlinear control and state estimation using global linearization

5.8. Simulation tests

The presented simulation experiments demonstrated the **efficiency of the control method** in tracking rapidly changing reference setpoints while also achieving good transients. The associated results are outlined in Table II

Table II
RMSE for the distributed inverters

	$RMSE_1$	$RMSE_2$	$RMSE_3$
ω	0.0225	0.0427	0.0199
V_{L_d}	0.0180	0.0008	0.0003
V_{L_q}	0.0246	0.0020	0.0010

The disturbances appearing in the simulation experiments could be met in **adverse operating conditions** of the distributed power generation system.

Even for the latter case the good performance of the control loop is confirmed.

Such **disturbances can be due to modelling errors** (e.g. parametric changes in the inverters' model) or **due to external perturbations** (e.g. grid faults or disturbances due to the connection or disconnection from the grid of power generation units).

Example 1: Nonlinear control and state estimation using global linearization

5.9. Conclusions

• The **inverter's model satisfies differential flatness properties**, which allows to transform the inverter's model to the **linear canonical form**.

• Next, the problem of **control and synchronization of parallel inverters** connected to the grid was analysed. It has been shown that, the **dynamics of each inverter** can be written in a form that is equivalent to the **model of the synchronous power generator.**

• Using the latter description one can compute **the active power** that each inverter should be contributing so as **to remain synchronized** with the reference frequency of the grid.

• The **active power** and the **frequency associated with the inverter** were used next to **compute the control input** that is applied to the inverter's electrical model.

• Thus, finally the **synchronization problem of each local inverter** was turned into a problem of **nonlinear feedback control** for the associated inverter's electrical model.

• To compensate for **additive disturbance terms** that affect the local inverters' models, the **Derivative-free nonlinear Kalman Filter** was redesigned as a **disturbance observer**.

• The performance of the proposed distributed feedback control scheme for parallel inverters was tested through simulation experiments

Example 2: Nonlinear control and state estimation using approximate linearization

6.1. Outline

● A new nonlinear H-infinity control approach is applied to PEM fuel cells. First, the dynamic model of the PEM fuel cells undergoes approximate linearisation, through Taylor series expansion, round local operating points which are defined at each time instant by the present value of the system's state vector and the last value of the control input exerted on it.

● The linearization procedure requires the computation of Jacobian matrices. The modelling error, which is due to the truncation of higher order terms in the Taylor series expansion is perceived as a perturbation that should be compensated by the robustness of the control loop. Next, for the linearized PEM fuel cells model, an H-infinity feedback control loop is designed.

● This approach, is based on the concept of a differential game that takes place between the control input (which tries to minimize the deviation of the state vector from the reference setpoints) and the disturbance input (that tries to maximize it).

● In such a case, the computation of the optimal control input requires the solution of an algebraic Riccati equation at each iteration of the control algorithm. The known robustness properties of H-infinity control enable compensation of model uncertainty and perturbations

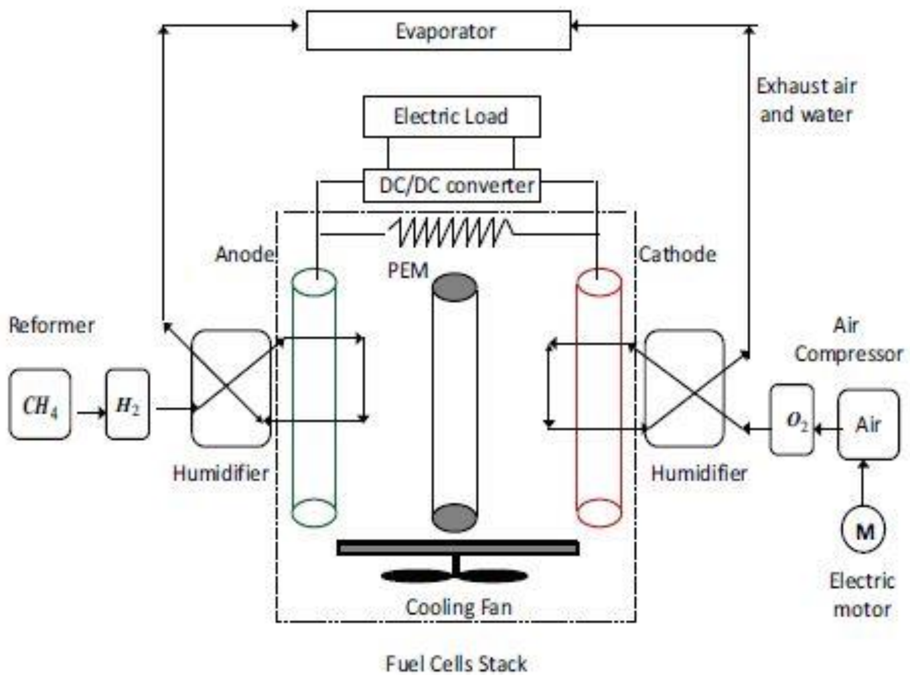
● The stability of the control loop is proven through Lyapunov analysis. Actually, it is shown that H-infinity tracking performance is succeeded, while conditionally the asymptotic stability of the control loop is also demonstrated.

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Nonlinear dynamics of the PEM Fuel Cells

6.2.1. Nonlinear state equations model of the PEM fuel cells

The PEM fuel cells system is depicted in the following diagram



Focusing on the cathode, the state vector of the model is defined as

$$x = [p_{O_2}, p_{N_2}, \omega_{cp}, p_{sm}]^T$$

pO_2 oxygen concentration at the cathode

pN_2 nitrogen concentration at the cathode

ω_{cp} compressor's rotation speed

p_{sm} Isupply manifold pressure

By applying the ideal gas law and by considering that the volume of the cathode is known one has

$$\frac{dp_{O_2}}{dt} = \frac{RT}{M_{O_2}V_{ca}}(W_{O_{2,in}} - W_{O_2,out} - W_{O_{2,react}})$$

$$\frac{dp_{N_2}}{dt} = \frac{RT}{M_{N_2}V_{ca}}(W_{N_{2,in}} - W_{N_2,out})$$

1

Example 2: Nonlinear control and state estimation using approximate linearization

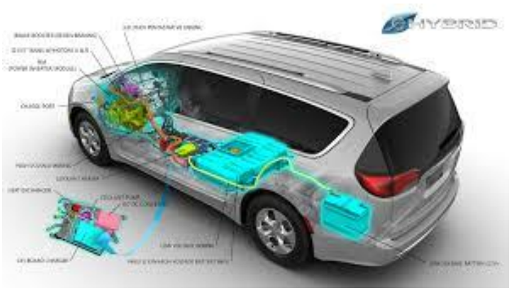
6.2.Nonlinear dynamics of the PEM Fuel Cells

6.2.1. Nonlinear state equations model of the PEM fuel cells

M_v is the mass of the vapor in mole,

$M_{a,ca,in}$ is the mass of the air in mole,

ϕ_{ca} is the relative humidity in ambient conditions,



$p_{sat}(T_{atm})$ is the saturation pressure in ambient temperature,

p_{atm} is the atmospheric pressure

$k_{ca,in}$ is the cathode inlet orifice constant.

The outlet flow rates of oxygen and nitrogen $W_{o_2,out}$ and $W_{N_2,out}$ are calculated from the mass fraction of oxygen and nitrogen in the stack after reaction

$$W_{O_2,out} = \frac{M_{O_2} p_{O_2}}{M_{O_2} p_{O_2} + M_{N_2} p_{N_2} + M_v p_{sat}} W_{ca,out}$$

$$W_{N_2,out} = \frac{M_{N_2} p_{N_2}}{M_{O_2} p_{O_2} + M_{N_2} p_{N_2} + M_v p_{sat}} W_{ca,out} \quad 5$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Nonlinear dynamics of the PEM Fuel Cells

6.2.1. Nonlinear state equations model of the PEM fuel cells

The flow rate at the cathode's exit $W_{ca,out}$ is calculated by the nozzle flow equation

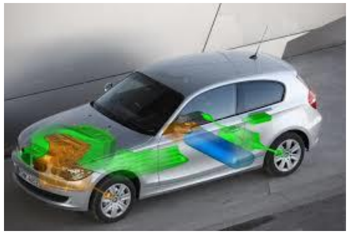
$$W_{ca,out} = \frac{C_D A_T p_{ca}}{\sqrt{RT}} \left(\frac{p_{atm}}{p_{ca}}\right)^{\frac{1}{T}}$$
$$\text{if } \frac{p_{atm}}{p_{ca}} > \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \left\{\frac{2\gamma}{\gamma-1}\left[1-\left(\frac{p_{atm}}{p_{ca}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\} \quad (6)$$

where γ is the ratio of the specific heat capacities of the air

and the pressure of the cathode is given by $p_{ca} = p_{O_2} + p_{N_2} + P_{sat}$

The mass flow rate of oxygen is expressed as

$$W_{O_2,react} = \frac{n I_{st}}{4F} M_{O_2} \quad (7)$$



where n is the number of cells in the stack, F is the Faraday number and Ist is the stack current. The compressor's turn speed is related to the associated mechanical torque

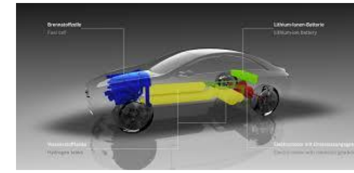
$$\frac{d\omega_{cp}}{dt} = \frac{1}{J_{cp}}(\tau_{cm} - \tau_{cp}) \quad (8)$$

where τ_{cm} is the mechanical input torque and τ_{cp} is the load torque

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Nonlinear dynamics of the PEM Fuel Cells

6.2.1. Nonlinear state equations model of the PEM fuel cells



In (1) V is the volume of the cathode, R is the universal gas constant,

and M_{0_2} M_{N_2} are the mass concentrations (in mole) of oxygen and nitrogen.

The incoming flow rates of oxygen and nitrogen are given by

$$W_{O_2,in} = x_{O_2} W_{ca,in}$$

$$W_{N_2,in} = (1 - x_{O_2}) W_{ca,in} \qquad (2)$$

where x_{0_2} is the oxygen mass fraction of the inlet air, $1 - x_{0_2}$ is the nitrogen mass fraction of the inlet air, and $W_{ca,in}$ is the mass flow rate entering the cathode which is given by

$$W_{ca,in} = \frac{1}{1+\omega_{atm}} k_{ca,in}(p_{sm} - p_{in}) \qquad (3)$$

where w_{atm} is the humidity ratio

$$\omega_{atm} = \frac{M_v}{M_{a,ca,in}} \frac{\phi_{ca} p_{sat}(T_{atm})}{p_{atm} - \phi_{ca} p_{sat}(T_{atm})} \qquad (4)$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Nonlinear dynamics of the PEM Fuel Cells

6.2.1. Nonlinear state equations model of the PEM fuel cells

$$\tau_{cm} = \eta_{cm}\frac{K_v}{R_{cm}}(v_{cm})k_v\omega_{cp} \quad (9)$$

$$\tau_{cp} = \frac{C_p}{\omega_{cp}}\frac{T_{atm}}{\eta_{cp}}[(\frac{p_{atm}}{p_{ca}})^{\frac{\gamma-1}{\gamma}}-1]W_{cp} \quad (10)$$

where k_v, R_{cm} are motor constants,

C_p is the specific heat capacity of air

W_{cp} is the compressor mass flow rate.

The dynamics of the air pressure in the supply manifold depend on the compressor flow into the supply manifold $W_{cp} = A_{wcp}$, on the flow out of the supply manifold into the cathode Wco,in and on the compressor flow temperature T_{cp}

$$\frac{dp_{sm}}{dt} = \frac{RT_{cp}}{M_a V_{sm}}[W_{cp} - k_{ca,in}(p_{sm} - p_{ca})] \quad (11)$$

where V_{sm} is the supply manifold volume and T_{cp} is the temperature of the air leaving the compressor

$$T_{cp} = T_{atm} + \frac{T_{atm}}{\eta_{cp}}[(\frac{p_{sm}}{p_{atm}})^{\frac{\gamma-1}{\gamma}}-1] \quad (12)$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Nonlinear dynamics of the PEM Fuel Cells

6.2.1. Nonlinear state equations model of the PEM fuel cells

The nonlinear state-space model of the PEM fuel-cells model is based on

Eq. 1 8 11

$$\dot{x}_1 = c_1(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_1 W_{ca,out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7\zeta \quad (13)$$

$$\dot{x}_2 = c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_2 W_{ca,out}}{c_4 x_1 + c_5 x_2 + c_6} \quad (14)$$

$$\dot{x}_3 = -c_9 x_3 - c_{10}[(\frac{x_4}{c_{11}})^{c_{12}} - 1] + c_{13} u \quad (15)$$

$$\begin{aligned} \dot{x}_4 = c_{14}\{1 + c_{15}[(\tfrac{x_4}{c_{11}})^{c_{12}} - 1]\}\cdot \\ \cdot[W_{cp} - c_{16}(x_4 - x_1 - x_2 - c_2)] \end{aligned} \quad (16)$$

The control input u depends the motor’s current. The control input ζ is the stack current (which can be considered as an external perturbation to the model

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Nonlinear dynamics of the PEM Fuel Cells

6.2.2. State-space description of the PEM fuel cells

The previous set of state-space equations is also written in the form

$$\dot{x} = f(x) + g(x)u \Rightarrow$$

where

$$f(x) = \begin{pmatrix} c_1(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_1 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} - c_7 \zeta \\ c_8(x_4 - x_1 - x_2 - c_2) - \frac{c_3 x_2 W_{co,out}}{c_4 x_1 + c_5 x_2 + c_6} \\ -c_9 x_3 - c_{10}[(\frac{x_4}{c_{11}})^{c_{12}} - 1] \\ c_{14}\{1 + c_{15}[(\frac{x_4}{c_{11}})^{c_{12}} - 1]\} \cdot [W_{cp} - c_{16}(x_4 - x_1 - x_2 - c_2)] \end{pmatrix}$$

and

$$g(x) = \begin{pmatrix} 0 & 0 & c_{13} & 0 \end{pmatrix}^T$$

Although global linearization of this nonlinear model is possible through elaborated state variables transformations (diffeomorphisms), the approach to be followed next is approximate linearization and H-infinity (optimal) control.

Example 2: Nonlinear control and state estimation using approximate linearization

6.3. Linearization of the PEMs Fuel Cells model

The system's dynamic model undergoes **linearization** round its present operating point (x^*,u^*), where x^* is the present value of the finance system's state vector and u^* is the last value of the control input vector that was applied on it.

Thus one arrives at the **approximately linearized description** of the system:

$$\dot{x} = Ax + Bu + \tilde{d}$$

where d_1 is the linearization error due to truncation of higher-order terms in the **Taylor series expansion** and

$$A = \nabla_x [f(x) + g(x)u] \mid_{x^*,u^*}$$

In a similar manner, one has that

$$B = \nabla_u [f(x) + g(x)u] \mid_{x^*,u^*} \Rightarrow B = g(x)$$

After **linearization** round its current operating point the system's **model** is written as

$$\dot{x} = Ax + Bu + d_1 \qquad \text{A}$$

Parameter d_1 stands for the **linearization error** in the system's model

At every time instant the control input u^* is assumed to differ from the control input u appearing in A by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.3. Linearization of the PEMs Fuel Cells model

The dynamics of the system of Eq. (A) can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad \text{(C)}$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad \text{(D)}$$

By subtracting Eq. (D) from Eq. (A) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad \text{(E)}$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$ the **tracking error dynamics** becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad \text{(F)}$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.4. Design of the H-infinity feedback controller

The initial PEM fuel cells system is assumed to be in the form

$$\dot{x} = f(x,u) \quad x \in R^n, \ u \in R^m$$

where the **linearization point (temporary equilibrium)** is defined by the present value of the system's state vector and the last value of the control inputs vector exerted on it

$$(x^*, u^*) = (x(t), u(t - T_s)).$$

The **linearized equivalent of the system** is described by

$$\dot{x} = Ax + Bu + Ld \quad x \in R^n, \ u \in R^m, \ d \in R^q$$

where matrices A and B are obtained from the **computation of the Jacobians**

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} |_{(x^*, u^*)} \qquad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} |_{(x^*, u^*)}$$

and vector d denotes disturbance terms due to linearization errors.

The problem of **disturbance rejection** for the linearized model that is described by

$$\begin{aligned} \dot{x} &= Ax + Bu + Ld \\ y &= Cx \end{aligned}$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

The **tracking error dynamics** for the PEM fuel cells system is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \qquad (1)$$

where in the **case of the considered DC power system** $L = I \in R^5$ with I being the identity matrix. The following **Lyapunov function** is considered

$$V = \frac{1}{2}e^T P e \qquad (2)$$

where $e = x - x_d$ Is the state vector's tracking error

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow$$

$$\dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ +\frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}e^T A^T Pe + \frac{1}{2}u^T B^T Pe + \frac{1}{2}\tilde{d}^T L^T Pe + \\ \frac{1}{2}e^T PAe + \frac{1}{2}e^T PBu + \frac{1}{2}e^T PL\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(A^TP + PA)e + (\frac{1}{2}u^TB^TPe + \frac{1}{2}e^TPBu) + \\ +(\frac{1}{2}\tilde{d}^TL^TPe + \frac{1}{2}e^TPL\tilde{d})$$

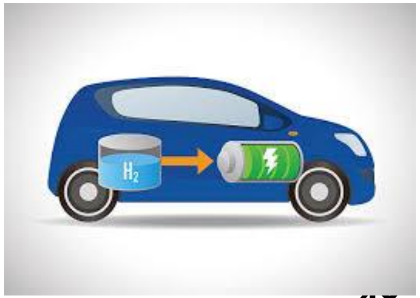
Assumption: For given positive definite matrix Q *and coefficients* r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \qquad 3$$

Moreover, the following **feedback control law** is applied to the PEM fuel cells model

$$u = -\frac{1}{r}B^TPe \qquad 4$$

By substituting Eq. 3 and Eq. 4 one obtains



$$\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P]e + \\ +e^TPB(-\frac{1}{r}B^TPe + e^TPL\tilde{d} \Rightarrow$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

Continuing with computations one obtains

$$\dot{V} = -\frac{1}{2}e^TQe + (\frac{1}{r}PBB^TPe - \frac{1}{2\rho^2}e^TPLL^T)Pe$$
$$-\frac{1}{r}e^TPBB^TPe + e^TPL\tilde{d}$$

which next gives

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe + e^TPL\tilde{d}$$

or equivalently

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe +$$
$$+\frac{1}{2}e^TPL\tilde{d} + \frac{1}{2}\tilde{d}^TL^TPe \qquad 5$$

Lemma: The following inequality holds

$$\frac{1}{2}e^TL\tilde{d} + \frac{1}{2}\tilde{d}L^TPe - \frac{1}{2\rho^2}e^TPLL^TPe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

Proof : *The binomial* $(\rho a - \frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d} - \frac{1}{2\rho^2}e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (6)$$

Eq. (6) is substituted in Eq. (5) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (7)$$

Eq. (7) shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of $\dot{V}$ from 0 to T gives

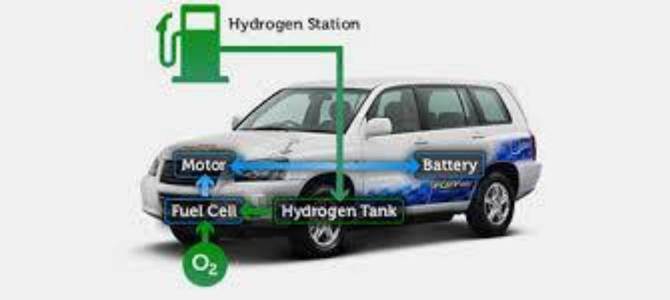
$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T ||\tilde{d}||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||\tilde{d}||^2 dt$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d$$



then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded.

Moreover, V(T) is bounded and from the definition of the Lyapunov function V it becomes clear that **e(t) will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$lim_{t \to \infty} e(t) = 0.$$

8

This completes the stability proof.

Example 2: Nonlinear control and state estimation using approximate linearization

6.6. Robust state estimation with the use of the H-infinity Kalman Filter

- The control loop has to be implemented with the use of information provided by a **small number of measurements** of the state variables of the PEM fuel cells system

- To reconstruct the missing information about the state vector of the PEM fuel cells system it is proposed to **use a filter** and based on it to apply state **estimation-based control** .

- The **recursion of the H-infinity Kalman Filter**, for the **PEM fuel cells model**, can be formulated in terms of a measurement update and a time update part

Measurement update

$$D(k) = [I - \theta W(k)P^{-}(k) + C^{T}(k)R(k)^{-1}C(k)P^{-}(k)]^{-1}$$
$$K(k) = P^{-}(k)D(k)C^{T}(k)R(k)^{-1}$$
$$\hat{x}(k) = \hat{x}^{-}(k) + K(k)[y(k) - C\hat{x}^{-}(k)]$$

Time update

$$\hat{x}^{-}(k+1) = A(k)x(k) + B(k)u(k)$$
$$P^{-}(k+1) = A(k)P^{-}(k)D(k)A^{T}(k) + Q(k)$$

where it is assumed that parameter θ is sufficiently small to assure that the **covariance matrix**

$$P^{-}(k) - \theta W(k) + C^{T}(k)R(k)^{-1}C(k)$$

Is **positive definite**

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

• The performance of the proposed nonlinear **H-nfinity control scheme** for the **PEM fuel cells system** is tested through simulation:

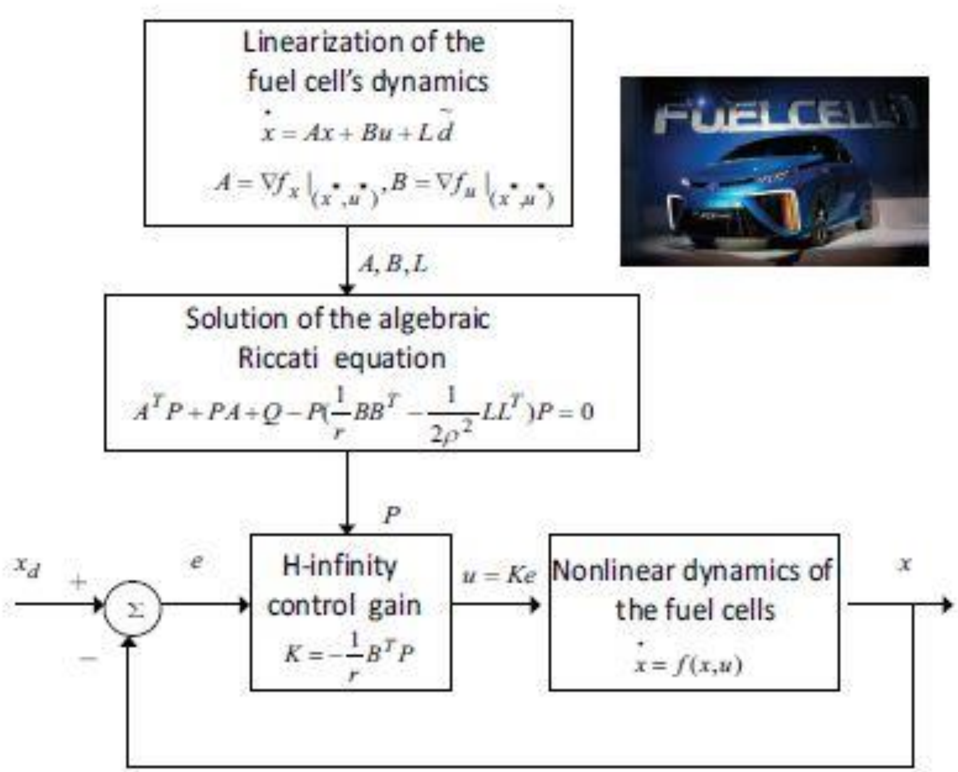
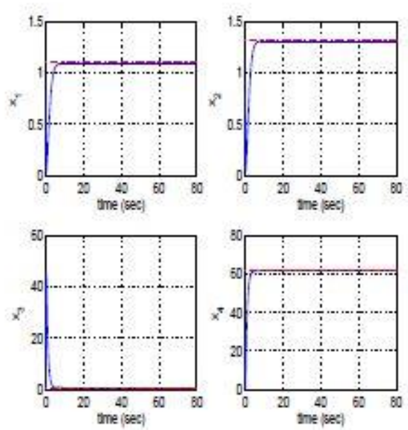


Fig.2 Diagram of the nonlinear optimal control

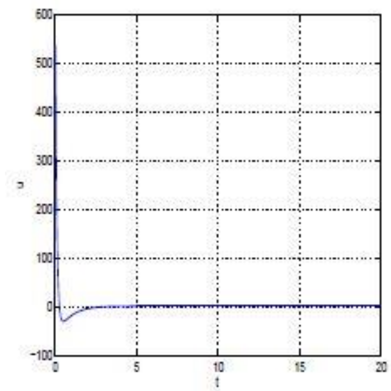
With the use of the proposed H-infinity control method, fast and accurate tracking of the reference setpoints of the **PEM fuel cells system's state variables** was achieved

Example 2: Nonlinear control and state estimation using approximate linearization

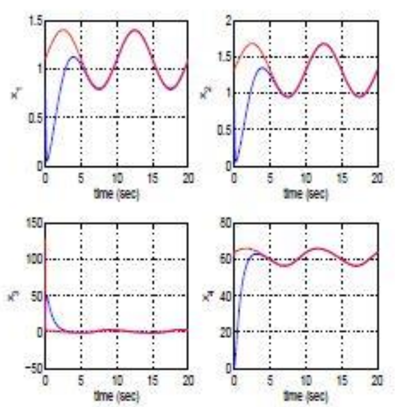
6.7. Simulation tests



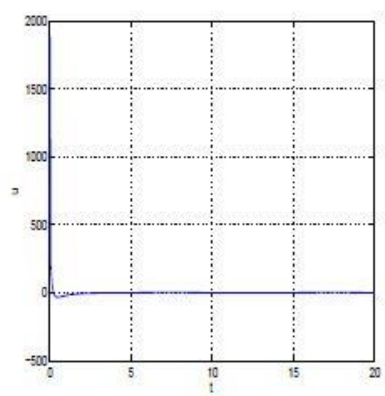
(a) **Test 1:** Convergence of state variables $x_1 - x_4$ (green line) to setpoint 1 (red line)



(b) **Test 1:** Control input u applied to the PEM fuel cells model



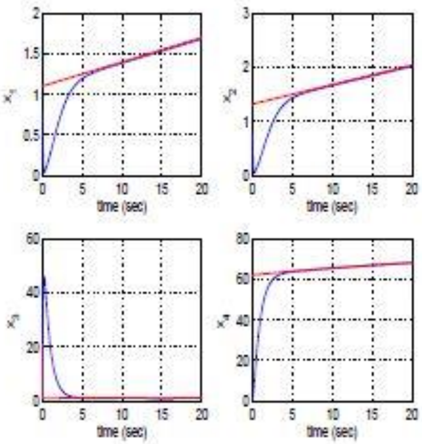
(a) **Test 2:** Convergence of state variables $x_1 - x_4$ (green line) to setpoint 1 (red line)



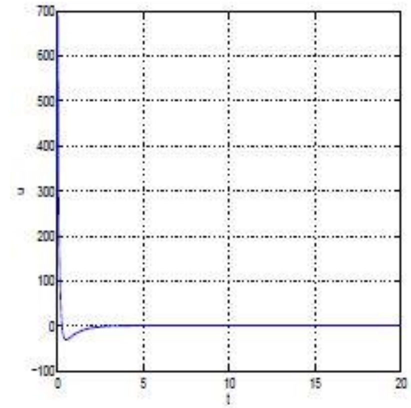
(b) **Test 2:** Control input u applied to the PEM fuel cells model

Example 2: Nonlinear control and state estimation using approximate linearization

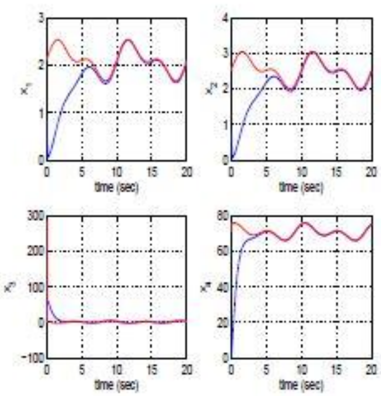
6.7. Simulation tests



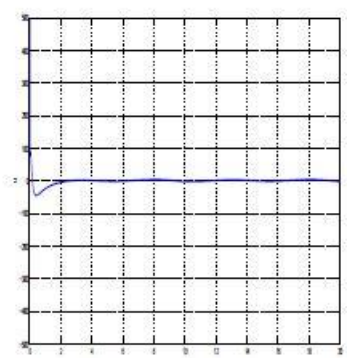
(a) **Test 3:** Convergence of state variables $x_1 - x_4$ (green line) to setpoint 1 (red line)



(b) **Test 3:** Control input u applied to the PEM fuel cells model



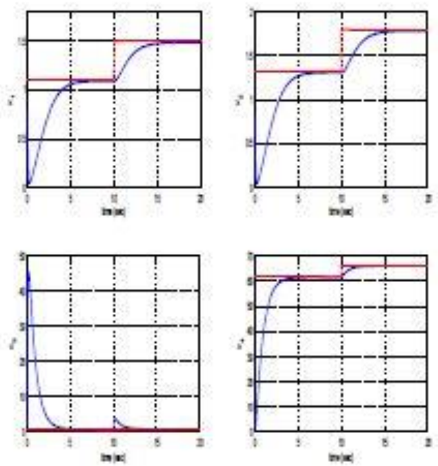
(a) **Test 4:** Convergence of state variables $x_1 - x_4$ (green line) to setpoint 1 (red line)



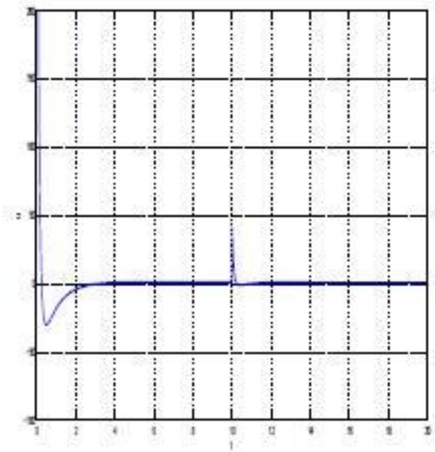
(b) **Test 4:** Control input u applied to the PEM fuel cells model

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests



(a) **Test 5:** Convergence of state variables $x_1 - x_4$ (green line) to setpoint 1 (red line)



(b) **Test 5:** Control input u applied to the PEM fuel cells model

Table I: RMSE of the fuel cell's state variables

parameter	p_{O_2}	p_{N_2}	ω_{c_p}	p_{s_m}
$RMSE_1$	$7.40{\cdot}10^{-5}$	$8.88{\cdot}10^{-5}$	$1.08{\cdot}10^{-5}$	$7.91{\cdot}10^{-4}$
$RMSE_2$	$9.82{\cdot}10^{-5}$	$1.17{\cdot}10^{-4}$	$4.35{\cdot}10^{-4}$	$14.0{\cdot}10^{-4}$
$RMSE_3$	$7.94{\cdot}10^{-5}$	$9.53{\cdot}10^{-5}$	$7.52{\cdot}10^{-6}$	$6.76{\cdot}10^{-4}$
$RMSE_4$	$2.34{\cdot}10^{-4}$	$2.81{\cdot}10^{-4}$	$9.16{\cdot}10^{-4}$	$18.0{\cdot}10^{-4}$
$RMSE_5$	$1.86{\cdot}10^{-4}$	$2.24{\cdot}10^{-4}$	$1.48{\cdot}10^{-5}$	$6.64{\cdot}10^{-4}$

Example 2: Nonlinear control and state estimation using approximate linearization

6.8. Conclusions

● A new nonlinear H-infinity control method has been developed for the dynamic model of PEM fuel cells. The first stage for the method's implementation has been the linearization of the fuel cells' dynamic model round local operating points.

● At every time instant, these equilibria consisted of the present value of the system's state vector and of the last value of the control input that was exerted on it.

● For this linearization, Taylor series expansion has been applied to the fuel cells' dynamic model and the associated Jacobian matrices have been computed.

● For the linearized equivalent model of the system H-infinity nonlinear optimal control has been applied.

● The modelling errors which were due to the approximate linearization of the system were perceived as disturbances affecting the fuel cells' dynamics and were compensated by the robustness of the H-infinity controller.

● Moreover, conditions which assure the asymptotic stability of the control loop have been formulated. The efficiency of the nonlinear H-infinity control method has been further confirmed through simulation experiments.

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.1. Outline

● The article proposes an adaptive control approach that is capable of compensating for model uncertainty and parametric changes of the doubly-fed reluctance machines (DFRMs), as well as for the lack of measurements about the DFRM's state vector elements.

● First it is proven that the DFRM's model is a differentially flat one. By exploiting differential flatness properties it is shown that the DFRM model can be transformed into the linear canonical form.

● For the latter description, the new control inputs comprise unknown nonlinear functions which can be identified with the use of neurofuzzy approximators. The estimated dynamics of the machine is used by a feedback controller thus establishing an indirect adaptive control scheme.

● Moreover, to enforce the robustness of the control loop, a supplementary control term is computed using H-infinity control theory.

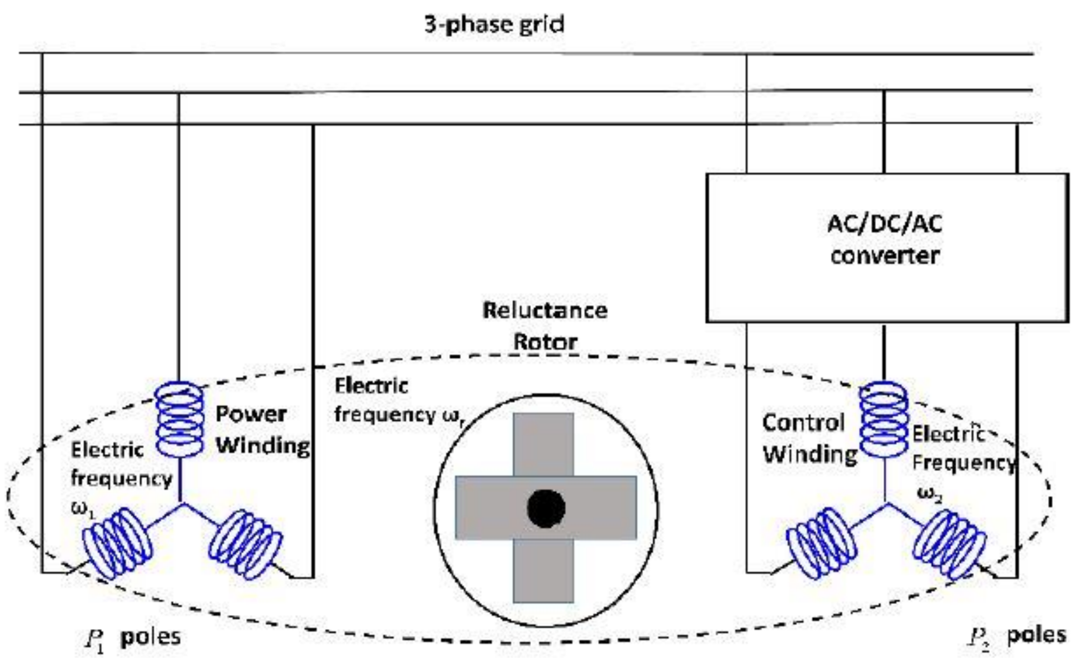
● Another problem that has to be dealt with comes from partial measurements of the state vector of the generator. Thus, a state observer is implemented in the control loop.

● The stability of the considered observer-based adaptive control approach is proven using Lyapunov analysis.Moreover, the performance of the control scheme is evaluated through simulation experiments.

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the doubly-fed reluctance machine

The brushless doubly-fed reluctance machine has two separate stator windings where the first one is noted as power winding while the second one is noted as control winding. The power winding is directly connected to the 3-phase grid and using the dq reference notation one has two voltage components v_d and v_q



The power winding has p_1 poles while the secondary (control) winding has p_2 poles. The rotor of the machine has no windings and the number of poles in it is usually chosen to be $p=(p_1+p_2)/2$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the doubly-fed reluctance machine

The frequency of current at the primary winding is the one of the grid and is denoted as .ω_1
The secondary winding is connected to the grid through an AC/DC/AC converter thus it can have current at a frequency different from the one of the grid, which is denoted as ω_2

By applying Kirchhoff's laws at the primary and the secondary winding of the reluctance machine one obtains the dynamic model of its electrical part. Thus, at the primary winding one has

$$\begin{aligned}\dot{\lambda}_{1d} &= -R_1 i_{1d} + \omega_1 \lambda_{1q} + v_{1d} \\ \dot{\lambda}_{1q} &= -R_1 i_{1q} - \omega_1 \lambda_{1d} + v_{1q}\end{aligned} \quad (1)$$

while at the secondary winding it holds

$$\begin{aligned}\dot{\lambda}_{2d} &= -R_2 i_{2d} + \omega_2 \lambda_{2q} + v_{2d} \\ \dot{\lambda}_{2q} &= -R_2 i_{2q} - \omega_2 \lambda_{2d} + v_{2q}\end{aligned} \quad (2)$$

The magnetic flux at the primary winding is the result of the inductance of this winding and of the mutual inductance (coupling) with the secondary winding

$$\begin{aligned}\lambda_{1d} &= L_1 i_{1d} + L_{12} i_{2d} \\ \lambda_{1q} &= L_1 i_{1q} - L_{12} i_{2q}\end{aligned} \quad (3)$$

Equivalently, the magnetic flux at the secondary winding is the result of the inductance of this winding and of the mutual inductance (coupling) with the primary winding

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the doubly-fed reluctance machine

$$\lambda_{2d} = L_2 i_{2d} + L_{12} i_{1d}$$
$$\lambda_{2q} = L_2 i_{2q} - L_{12} i_{1q} \quad (4)$$

v_{1d}, v_{1q} grid voltage components through connection to primary winding

v_{2d}, v_{2q} voltage components at the AC/DC/AC converter of the secondary winding

i_{1d}, i_{1q} grid current components through connection to primary winding

i_{2d}, i_{2q} current components at the AC/DC/AC converter of the secondary winding

$\lambda_{1d}, \lambda_{1q}$ components of the magnetic flux at the primary winding

$\lambda_{2d}, \lambda_{2q}$ components of the magnetic flux at the secondary winding

R_1, L_1 resistance and inductance of the primary winding

R_2, L_2 resistance and inductance of the secondary winding

L_{12} mutual inductance (coupling) between the primary and secondary winding

The electromagnetic torque of the machine is $T_e = \frac{3}{2}\frac{L_{12}}{L_1} p_r(\lambda_{1d} i_{2q} + \lambda_{1q} i_{2d}).$ (5)

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the doubly-fed reluctance machine

The dynamics of the rotational motion of the machine is given by

$$\dot{\omega} = \frac{1}{J_r}(T_e + T_m) = \frac{1}{J_r}[\frac{3}{2}\frac{L_{12}}{L_1}p_r(\lambda_{1d}i_{2q} + \lambda_{1q}i_{2d}) + T_m] \quad (5)$$

The active and the reactive power of the reluctance machine are given by

$$P^* = \frac{3}{2}(v_{1d}i_{1d} + v_{1q}i_{1d}), \qquad Q^* = \frac{3}{2}(v_{1d}i_{1q} - v_{1q}i_{1d}) \quad (6)$$

By combining the previous equations 1 to 3 of the machine's dynamics one has

$$\dot{\omega} = \frac{1}{J_r}(\frac{3}{2}\frac{L_{12}}{L_1}p_r(L_1 i_{1d}i_{2q} + L_{12}i_{2d}i_{2q} + L_1 i_{1q}i_{2d} + L_{12}i_{2q}i_{2d})) \quad (7)$$

$$\begin{aligned}
\frac{d}{dt}i_{1d} &= \sigma\{-R_1L_2i_{1d} + \omega_1L_1L_2i_{1q} - \omega_1L_2L_{12}i_{2q} + L_2V_s + \\
&\quad + R_2L_{12}i_{2d} - \omega_2L_2L_{12}i_{2q} + \omega_2L_{12}^2i_{1q} - L_{12}v_{2d}\} \\
\frac{d}{dt}i_{1q} &= \sigma\{-R_1L_2i_{1q} + \omega_1L_1L_2i_{1d} + \omega_1L_2L_{12}i_{2d} - \\
&\quad - R_2L_{12}i_{2q} - \omega_2L_2L_{12}i_{2d} + \omega_2L_{12}^2i_{1d} - L_{12}v_{2q}\} \\
\frac{d}{dt}i_{2d} &= \sigma\{-R_1L_{12}i_{2d} - \omega_1L_1L_{12}i_{1q} + \omega_1L_{12}^2i_{2q} - L_{12}V_s - \\
&\quad - R_2L_1i_{2d} + \omega_2L_1L_2i_{2q} - \omega_2L_1L_{12}i_{1q} + L_1v_{2d}\} \\
\frac{d}{dt}i_{2q} &= \sigma\{-R_1L_{12}i_{1q} - \omega_1L_1L_{12}i_{1d} - \omega_1L_{12}^2i_{2d} - \\
&\quad - R_2L_1i_{2q} - \omega_2L_1L_2i_{2d} - \omega_2L_1L_{12}i_{1d} + L_1v_{2q}\}
\end{aligned} \quad (8)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the doubly-fed reluctance machine

Next, by defining the state vector

$$x_1 = \omega,\ x_2 = i_{1d},\ x_3 = i_{1q},\ x_4 = i_{2d},\ x_5 = i_{2q}$$

the state-space description of the system becomes

$$\dot{x} = f(x) + g(x)u \qquad (9)$$

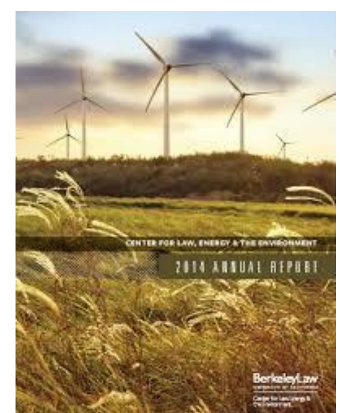
with $x \in R^{5\times 1}$, $u \in R^{2\times 1}$, $f(x) \in R^{5\times 1}$ and $g(x) \in R^{5\times 2}$,

where

$$f(x) = \begin{pmatrix} -\frac{b}{J}x_1 + \frac{3}{2J}\frac{L_{12}}{L_1}p_r(L_1x_2x_5 + L_{12}x_3x_5 + L_1x_4x_5 + L_{12}x_3x_5) + \frac{T_m}{J} \\ -\sigma R_1 L_2 x_2 - \sigma\omega_1 L_1 L_2 x_3 - \sigma\omega_1 L_2 L_{12} x_5 + \sigma L_2 V_s - \\ -\sigma R_2 L_{12} x_4 - \sigma\omega_2 L_2 L_{12} x_5 + \sigma\omega_2 L_{12}^2 x_3 \\ -\sigma R_1 L_2 x_3 - \sigma\omega_1 L_1 L_2 x_2 + \sigma\omega_1 L_2 L_{12} x_4 - \\ -\sigma R_2 L_{12} x_5 - \sigma\omega_2 L_2 L_{12} x_4 + \sigma\omega_2 L_{12}^2 x_2 \\ -\sigma R_1 L_{12} x_2 - \sigma\omega_1 L_1 L_{12} x_3 + \sigma\omega_1 L_{12}^2 x_5 - \sigma L_{12} V_s - \\ -\sigma R_2 L_1 x_4 + \sigma\omega_2 L_1 L_2 x_5 - \sigma\omega_2 L_1 L_{12} x_3 \\ -\sigma R_1 L_{12} x_3 - \sigma\omega_1 L_1 L_{12} x_2 - \sigma\omega_1 L_{12}^2 x_4 - \\ -\sigma R_2 L_1 x_5 - \sigma\omega_2 L_1 L_2 x_4 - \sigma\omega_2 L_1 L_{12} x_2 \end{pmatrix}$$

and

$$g(x) = \begin{pmatrix} 0 & 0 \\ -\sigma L_{12} & 0 \\ 0 & \sigma L_{12} \\ \sigma L_1 & 0 \\ 0 & \sigma L_2 \end{pmatrix} \qquad (10)$$



Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Differential flatness properties of the DFRM system dynamics

7.3.1. Proof of differential flatness of the DFRM model

The flat output of the model is chosen to be $Y = [y_1, y_2]$

where $y_1 = x_1 = \omega$ and $y_2 = x_4 = i_{2d}.$

From the first row of the states-space model of Eq. (10) one has

$$
\begin{gathered}
x_2 = \frac{1}{\frac{3}{2J}\frac{L_{12}}{L_1}p_r(L_1x_5)}\cdot[\dot{x}_1 + bx_1 - \frac{3}{2J}\frac{L_{12}}{L_1}p_r(L_{12}x_3x_5 + L_1x_4x_5 + L_{12}x_3x_5 - \frac{T_m}{J})] \Rightarrow \\
x_2 = \frac{1}{\frac{3}{2J}\frac{L_{12}}{L_1}p_r(L_1x_5)}\cdot[\dot{y}_1 + by_1 - \frac{3}{2J}\frac{L_{12}}{L_1}p_r(L_{12}x_3x_5 + L_1y_2x_5 + L_{12}x_3x_5 - \frac{T_m}{J})] \Rightarrow \\
x_2 = \tilde{f}_2(y_1, \dot{y}_1, x_3, x_5)
\end{gathered} \quad (11)
$$

Moreover, from (11) and by differentiating its last row with respect to time one gets

$$
\dot{x}_2 = \tilde{f}_2'(\dot{Y}, \ddot{Y}, \dot{x}_5, \dot{x}_3, x_5, \dot{x}_5) \quad (12)
$$

From the second and fourth row of the state-space model of Eq. (10) one gets

$$
\begin{aligned}
& L_1\dot{x}_2 + L_{12}\dot{x}_4 = \\
& L_1[\sigma R_1L_2x_2 - \sigma\omega_1L_1L_2x_3 - \sigma\omega_1L_2L_{12}x_5 + \sigma L_2V_s - \sigma R_2L_{12}x_4 - \sigma\omega_2L_2L_{12}x_5 + \sigma\omega_2L_{12}^2x_3] + \\
& L_{12}[\sigma R_1L_{12}x_2 - \sigma\omega_1L_1L_{12}x_3 + \sigma\omega_1L_{12}^2x_5 - \sigma L_{12}V_s - \sigma R_2L_1x_4 + \sigma\omega_2L_1L_2x_5 - \sigma\omega_2L_1L_{12}x_3]
\end{aligned} \quad (13)
$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Differential flatness properties of the DFRM system dynamics

7.3.1. Proof of differential flatness of the DFRM model

From the third and fifth row of the state-space model of Eq. (10) one gets

$$\begin{aligned}&L_2\dot{x}_3 - L_{12}\dot{x}_5 = \\ &L_2[-\sigma R_1 L_2 x_3 - \sigma\omega_1 L_1 L_2 x_2 + \sigma\omega_1 L_2 L_{12} x_4 - \sigma R_2 L_{12} x_5 - \sigma\omega_2 L_2 L_{12} x_4 + \sigma\omega_2 L_{12}^2 x_2] - \\ &L_{12}[-\sigma R_1 L_{12} x_3 - \sigma\omega_1 L_1 L_{12} x_2 - \sigma\omega_1 L_{12}^2 x_4 - \sigma R_2 L_1 x_5 - \sigma\omega_2 L_1 L_2 x_4 - \sigma\omega_2 L_1 L_{12} x_2]\end{aligned} \quad (14)$$

By substituting Eq. (11) and Eq. (12) into Eq. (13) one gets

$$a_1 x_3 + a_2\dot{x}_3 + a_3 x_5 + a_4\dot{x}_5 = h_1(Y,\dot{Y},\ddot{Y}) \quad (15)$$

By substituting Eq. (11) and Eq. (12) into Eq. (14) one gets

$$b_1 x_3 + b_2\dot{x}_3 + b_3 x_5 + b_4\dot{x}_5 = h_2(Y,\dot{Y},\ddot{Y}) \quad (16)$$

By solving Eq. (15) and Eq. (16) with respect to x_3 and $\dot{x}_3$ one gets

$$x_3 = m_1(Y,\dot{Y},\ddot{Y},x_5,\dot{x}_5) \quad (17)$$

By substituting Eq. (17) into Eq. (11) one gets

$$c_1 x_5 + c_2\dot{x}_5 = n_1(Y,\dot{Y},\ddot{Y})$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Differential flatness properties of the DFRM system dynamics

7.3.1. Proof of differential flatness of the DFRM model

By substituting Eq. (17) into Eq. (14) one gets

$$d_1 x_5 + d_2 \dot{x}_5 = n_2(Y, \dot{Y}, \ddot{Y}) \quad (19)$$

From Eq. (18) and Eq. (19) one gets $x_5 = p_1(Y, \dot{Y}, \ddot{Y})$ (20)

By substituting Eq. (20) into Eq. (17) one gets $x_3 = m_1(Y, \dot{Y}, \ddot{Y}, \cdots)$ (21)

By substituting Eq. (20) and (21) into Eq. (11) one gets $x_2 = m_1(Y, \dot{Y}, \ddot{Y}, \cdots)$ (22)

Thus all state variables $x_i,\ i = 1, \cdots, 5$ are differential functions of its flat output

Moreover, by solving the fourth and fifth row of Eq. (10) with respect to the control

Inputs one has that these are also differential functions of the flat output

$$u_1 = \tilde{f}_{u_1}(Y, \dot{Y}, \ddot{Y}, \cdots) \qquad u_2 = \tilde{f}_{u_2}(Y, \dot{Y}, \ddot{Y}, \cdots)$$

Consequently, the state-space model of the DFRM is a differentially flat one

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Differential flatness properties of the DFRM system dynamics

7.3.2. Transformation of the DFRM model into a canonical form

Next, the model is transformed into the canonical form. By differentiating the first row of Eq, 10 with respect to time one obtains

$$\ddot{x}_1 = -\frac{b}{J}x_1 + \frac{3}{2J_r}\frac{L_{12}}{L_1}p_r[L_1\dot{x}_2x_5 + L_1x_2\dot{x}_5 + 2L_{12}\dot{x}_3x_5 + 2L_{12}x_3\dot{x}_5 + L_1\dot{x}_4x_5 + L_1x_4\dot{x}_5]$$

and by substituting in the above equation $\dot{x}_4, \dot{x}_5$ from Eq 11 one gets

$$\ddot{x}_1 = f_1(x) + g_{11}(x)u_1 + g_{12}(x)u_2 \quad (23)$$

$$\begin{aligned} f_1(x) = & -\frac{b}{J}x_1 + \frac{3}{2J_r}\frac{L_{12}}{L_1}p_r(L_1x_5)[-\sigma R_1L_2x_2 - \sigma\omega_1L_1L_2x_3 - \sigma\omega_1L_2L_{12}x_5 + \sigma L_2V_s - \\ & -\sigma R_2L_{12}x_4 - \sigma\omega_2L_2L_{12}x_5 + \sigma\omega_2L_{12}^2x_3] + \\ & +\frac{3}{2J_r}\frac{L_{12}}{L_1}p_r(2L_{12}x_5)[-\sigma R_1L_2x_3 - \sigma\omega_1L_1L_2x_2 + \sigma\omega_1L_2L_{12}x_4 - \\ & -\sigma R_2L_{12}x_5 - \sigma\omega_2L_2L_{12}x_4 + \sigma\omega_2L_{12}^2x_2] \\ & +\frac{3}{2J_r}\frac{L_{12}}{L_1}p_r(L_1x_5)[\sigma R_1L_{12}x_2 - \sigma\omega_1L_1L_{12}x_3 + \sigma\omega_1L_{12}^2x_5 - \sigma L_{12}V_s - \\ & -\sigma R_2L_1x_4 + \sigma\omega_2L_1L_2x_5 - \sigma\omega_2L_1L_{12}x_3] + \\ & +\frac{3}{2J_r}\frac{L_{12}}{L_1}p_r(2L_{12}x_3 + L_1x_4)[-\sigma R_1L_{12}x_3 - \sigma\omega_1L_1L_{12}x_2 - \sigma\omega_1L_{12}^2x_4 - \\ & -\sigma R_2L_1x_5 - \sigma\omega_2L_1L_2x_4 - \sigma\omega_2L_1L_{12}x_2] \end{aligned} \quad (24)$$

$$g_{11}(x) = \frac{3}{2J_r}\frac{L_{12}}{L_1}p_r(L_1x_5)(\sigma L_1 - \sigma L_{12}) \quad (25)$$

$$g_{12}(x) = \frac{3}{2J_r}\frac{L_{12}}{L_1}p_r[(2L_{12}x_5)(\sigma L_{12}) + (2L_{12}x_3 + L_1x_4)(\sigma L_1)] \quad (26)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Differential flatness properties of the DFRM system dynamics

7.3.2. Transformation of the DFRM model into a canonical form

Moreover, from the fifth row of the state-space model one gets

$$\dot{x}_4 = f_2(x) + g_{21}(x)u_1 + g_{22}(x)u_2 \quad (27)$$

where

$$\begin{aligned} f_2(x) = \sigma R_1 L_{12} x_2 - \sigma\omega_1 L_1 L_{12} x_3 + \sigma\omega_1 L_{12}^2 x_5 - \sigma L_{12} V_s - \\ -\sigma R_2 L_1 x_4 + \sigma\omega_2 L_1 L_2 x_5 - \sigma\omega_2 L_1 L_{12} x_3 \end{aligned} \quad (28)$$

$$g_{21}(x) = \sigma L_1 \quad \text{and} \quad g_{22}(x) = 0 \quad (29)$$

Thus, by defining the state variables $z_1 = x_1$ and $z_2 = x_4$ one arrives at the input-output linearized form of the system

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (30)$$

with

$$\begin{aligned} v_1 = \ddot{z}_1^d - K_1^1(\dot{z}_1 - \dot{z}_1^d) - K_2^1(z_1 - z_1^d) \\ v_2 = \dot{z}_2^d - K_2^2(z_2 - z_2^d) \end{aligned} \quad (31)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

It is assumed now that **after defining the flat outputs of the initial MIMO nonlinear system**, and after expressing the system state variables and control inputs as functions of the flat output and of the associated derivatives, the system can be **transformed in the Brunovsky canonical form**

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\cdots \\
\dot{x}_{r_1-1} &= x_{r_1} \\
\dot{x}_{r_1} &= f_1(x) + \sum_{j=1}^{p} g_{1_j}(x) u_j + d_1 \\
\\
\dot{x}_{r_1+1} &= x_{r_1+2} \\
\dot{x}_{r_1+2} &= x_{r_1+3} \\
&\cdots \\
\dot{x}_{p-1} &= x_p \\
\dot{x}_p &= f_p(x) + \sum_{j=1}^{p} g_{p_j}(x) u_j + d_p
\end{aligned}
$$

$$
\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_{r_1-1} \\
&\cdots \\
y_p &= x_{n-r_p+1}
\end{aligned}
$$

$x = [x_1, \cdots, x_n]^T$: is the state vector

$u = [u_1, \cdots, u_p]^T$: is the inputs vector

$y = [y_1, \cdots, y_p]^T$: is the outputs vector

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Next **the following vectors and matrices** can be defined

$$f(x)=[f_1(x),\quad ...,\quad f_n(x)]^T$$

$$g(x)=[g_1(x),\quad ...,\quad g_n(x)]^T$$

with $g_i(x)=[g_{1i}(x),\quad ...,\quad g_{pi}(x)]^T$

$$A=diag[A_1,...,A_p],\ B=diag[B_1,...,B_p]$$

$$C^T=diag[C_1,...,C_p],\ d=[d_1,...,d_p]^T$$

where matrix A has the **MIMO canonical form**, i.e. with elements

$$A_i=\begin{bmatrix}0 & 1 & 0 & ... & 0\\ 0 & 0 & 1 & ... & 0\\ ... & ... & ... & ... & ...\\ 0 & 0 & 0 & ... & 1\\ 0 & 0 & 0 & ... & 0\end{bmatrix}_{r_i\times r_i}$$

$$B_i^T=[0\quad 0\quad ...\quad 0\quad 1]_{1\times r_i}\qquad C_i=[1\quad 0\quad ...\quad 0\quad 0]_{1\times r_i}$$

Thus, the initial nonlinear system can be writtenin the **state-space form**

$$\dot{x}=Ax+B[f(x)+g(x)u+\tilde{d}]$$
$$y=Cx$$

or equivalently in the state space form

$$\dot{x}=Ax+Bv+B\tilde{d}$$
$$y=Cx$$

where $v=f(x)+g(x)u$

For the case of the **MIMO diesel engine model** it is assumed that the functions $f(x)$ and $g(x)$ are unknown and have to be approximated by **neuro-fuzzy networks**

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Thus, the nonlinear system can be written in **state-space form**

$$\dot{x} = Ax + B[f(x) + g(x)u + \tilde{d}]$$
$$y = C^T x$$

which equivalently can be written as

$$\dot{x} = Ax + Bv + B\tilde{d}$$
$$y = C^T x$$

where $v = f(x) + g(x)u.$

The **reference setpoints** for the system's outputs $y_1, \cdots, y_p$ are denoted as $y_{1m}, \cdots, y_{pm}$ and the associated tracking errors are defined as

$$e_1 = y_1 - y_{1m}$$
$$e_2 = y_2 - y_{2m}$$
$$\cdots$$
$$e_p = y_p - y_{pm}$$

The **error vector of the outputs** of the transformed MIMO system is denoted as

$$E_1 = [e_1, \cdots, e_p]^T$$
$$y_m = [y_{1m}, \cdots, y_{pm}]^T$$
$$\cdots$$
$$y_m^{(r)} = [y_{1m}^{(r)}, \cdots, y_{pm}^{(r)}]^T$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.2. Control law

The **control signal of the MIMO nonlinear system** contains the **unknown nonlinear functions** f(x) and g(x) which can be approximated by

$$\boxed{\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f, \quad \hat{g}(x|\theta_g) = \Phi_g(x)\theta_g}$$

where $\Phi_f(x) = \left(\xi_f^1(x), \xi_f^2(x), \cdots \xi_f^n(x)\right)^T,$

$$\xi_f^i(x) = \left(\phi_f^{i,1}(x), \phi_f^{i,2}(x), \cdots, \phi_f^{i,N}(x)\right)$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_f{}^T = \left(\theta_f^1, \theta_f^2, \cdots \theta_f^N\right).$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.2. Control law

Similarly, it holds $\Phi_g(x) = \left(\xi_g^1(x), \xi_g^2(x), \cdots \xi_g^N(x)\right)^T$,

$$\xi_g^i(x) = \left(\phi_g^{i,1}(x), \phi_g^{i,2}(x), \cdots, \phi_g^{i,N}(x)\right).$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_g = \left(\theta_g^1, \theta_g^2, \cdots, \theta_g^p\right)^T$.

However, here each row of θ_g is vector thus giving

$$\theta_g = \begin{pmatrix} \theta_{g1}^1 & \theta_{g1}^2 & \cdots & \theta_{g1}^p \\ \theta_{g2}^1 & \theta_{g2}^2 & \cdots & \theta_{g2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{gN}^1 & \theta_{gN}^2 & \cdots & \theta_{gN}^p \end{pmatrix}$$

If the state variables of the system are available for measurement then a **state-feedback control law can be formulated** as

$$u = \hat{g}^{-1}(x|\theta_g)\left[-\hat{f}(x|\theta_f) + y_m^{(r)} + K_c^T e + u_c\right]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.2. Estimation of the state vector

The control of the system described by becomes more complicated **when the state vector x is not directly measurable** and has to be reconstructed through a state observer. The following definitions are used

$e = x - x_m$: is the error of the state vector

$\hat{e} = \hat{x} - x_m$ is the error of the estimated state vector

$\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$ is the observation error

When an **observer is used to reconstruct the state vector**, the control law

$$u = \hat{g}^{-1}(\hat{x}|\theta_g)[-\hat{f}(\hat{x}|\theta_f) + y_m^{(r)} - K^T\hat{e} + u_c]$$

By applying the previous feedback control law one obtains the closed-loop dynamics

$$y^{(n)} = f(x) + g(x)\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(n)} = f(x) + [g(x) - \hat{g}(\hat{x}) + \hat{g}(\hat{x})]\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(n)} = [f(x) - \hat{f}(\hat{x})] + [g(x) - \hat{g}(\hat{x})]u + y_m^{(n)} - K^T\hat{e} + u_c + d$$

It holds $e = x - x_m \Rightarrow y^{(n)} = e^{(n)} + y_m^{(n)}$

and by substituting $y^{(n)}$ in the **previous tracking error dynamics** gives

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the DFRM system

7.4.2. Estimation of the state vector

the **new tracking error dynamics**

$$e^{(n)} + y_m^{(n)} = y_m^{(n)} - K^T\hat{e} + u_c + [f(x) - \hat{f}(\hat{x})] + \\ + [g(x) - \hat{g}(\hat{x})]u + \tilde{d}$$

or equivalently

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\{[f(x) - \hat{f}(\hat{x})] + \\ + [g(x) - \hat{g}(\hat{x})]u + \tilde{d}\}$$

A

$$e_1 = C^T e$$

where $e = [e^1, e^2, \cdots, e^p]^T$ with $e^i = [e_i, \dot{e}_i, \ddot{e}_i, \cdots, e_i^{(r_i - 1)}]^T,\ i = 1, 2, \cdots, p$

and equivalently $\hat{e} = [\hat{e}^1, \hat{e}^2, \cdots, \hat{e}^p]^T$ with $\hat{e}^i = [\hat{e}_i, \dot{\hat{e}}_i, \ddot{\hat{e}}_i, \cdots, \hat{e}_i^{(r_i - 1)}]^T,\ i = 1, 2, \cdots, p.$

A state observer is designed as:

$$\dot{\hat{e}} = A\hat{e} - BK^T\hat{e} + K_o[e_1 - C^T\hat{e}]$$

B

$$\hat{e}_1 = C^T\hat{e}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the DFRM system

7.5.1. Tracking error dynamics under feedback control

By **applying differential flatness theory, and in the presence of disturbances**, the dynamic model of the DFRM comes to the form

$$\begin{pmatrix} \ddot{x}_1 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} u + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad \text{C}$$

The following **control input** is defined:

$$u = \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \dot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} \quad \text{D}$$

where: $[u_{c_1} \; u_{c_2}]^T$ is a **robust control term** that is used for the compensation of the model's uncertainties as well as of the external disturbances

and: $K_i^T = [k_1^i, k_2^i, \cdots, k_{n-1}^i, k_n^i]$ is the feedback gain

Substituting the control input D into the system C one obtains

$$\begin{pmatrix} \ddot{x}_1 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot$$

$$\cdot \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \dot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the DFRM system

7.5.1. Tracking error dynamics under feedback control

Moreover, using again Eq. D one obtains the **tracking error dynamics**

$$\begin{pmatrix}\ddot{e}_1\\ \dot{e}_3\end{pmatrix}=\begin{pmatrix}f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u-\begin{pmatrix}K_1^T\\ K_2^T\end{pmatrix}e+\begin{pmatrix}u_{c_1}\\ u_{c_2}\end{pmatrix}+\begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

The **approximation error** is defined

$$w=\begin{pmatrix}f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u$$

Using matrices A,B,K,

$$A=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\quad B=\begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix}\quad K^T=\begin{pmatrix}K_1^1&K_2^1&K_3^1\\ K_1^2&K_2^2&K_3^2\end{pmatrix}$$

and considering that **the estimated state vector is used in the control loop** the following description of the tracking error dynamics is obtained:

$$\dot{e}=Ae-BK^T\hat{e}+Bu_c+B\left\{\begin{pmatrix}f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u+\tilde{d}\right\}$$

When the **estimated state vector** is used in the loop the **approximation error is written as**

$$w=\begin{pmatrix}f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u$$

while the **tracking error dynamics becomes** $\dot{e}=Ae-BK^T\hat{e}+Bu_c+Bw+B\tilde{d}$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5.2. Dynamics of the observation error

The **observation error** is defined as: $\bar{e} = e - \hat{e} = x - \hat{x}$.

By subtracting Eq. B from Eq. A one obtains:

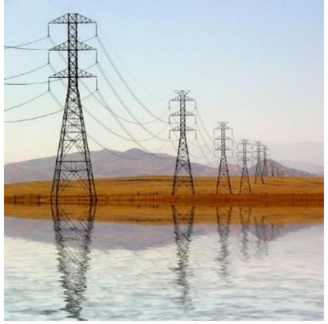
$$\dot{e} - \dot{\hat{e}} = A(e - \hat{e}) + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + \\ + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T(e - \hat{e})$$

$$e_1 - \hat{e}_1 = C^T(e - \hat{e})$$

or equivalently:

$$\dot{\bar{e}} = A\bar{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T \bar{e}$$

$$\bar{e}_1 = C^T \bar{e}$$



which can be also written as:

$$\dot{\bar{e}} = (A - K_o C^T)\bar{e} + Bu_c + Bw + \tilde{d}\}$$

$$\bar{e}_1 = C^T \bar{e}$$

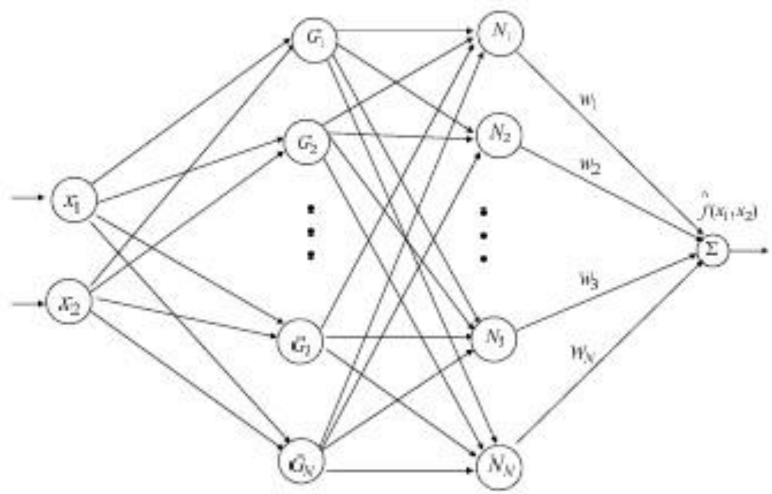
Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the DFRM system

7.5.3. Approximation of functions f(x,t) and g(x,t)

Next, the **first of the approximators** of the unknown system dynamics is defined

$$\hat{f}(\hat{x}) = \begin{pmatrix} \hat{f}_1(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_1(\hat{x}|\theta_f) & \in & R^{1\times 1} \\ \hat{f}_2(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_2(\hat{x}|\theta_f) & \in & R^{1\times 1} \end{pmatrix}$$



containing kernel functions $\phi_f^{i,j}(\hat{x}) = \frac{\prod_{j=1}^{n} \mu_{A_j}^{i}(\hat{x}_j)}{\sum_{l=1}^{N} \prod_{j=1}^{n} \mu_{A_j}^{l}(\hat{x}_j)}$

where $\mu_{A_j^i}(\hat{x})$ are fuzzy membership functions

appearing in the antecedent part of the *l-th* fuzzy rule

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the DFRM system

7.5.3. Approximation of functions f(x,t) and g(x,t)

Similarly, **the second of the approximators** of the unknown system dynamics is defined

$$\hat{g}(\hat{x}) = \begin{pmatrix} \hat{g}_1(\hat{x}|\theta_g) & \hat{x}\in R^{4\times 1} & \hat{g}_1(\hat{x}|\theta_g) & \in & R^{1\times 2} \\ \hat{g}_2(\hat{x}|\theta_g) & \hat{x}\in R^{4\times 1} & \hat{g}_2(\hat{x}|\theta_g) & \in & R^{1\times 2} \end{pmatrix}$$

The **values of the weights that result in optimal approximation** are

$$\theta_f^* = arg\ min_{\theta_f \in M_{\theta_f}} [sup_{\hat{x}\in U_{\hat{x}}}(f(x) - \hat{f}(\hat{x}|\theta_f))]$$
$$\theta_g^* = arg\ min_{\theta_g \in M_{\theta_g}} [sup_{\hat{x}\in U_{\hat{x}}}(g(x) - \hat{g}(\hat{x}|\theta_g))]$$

The variation ranges for the weights are given by

$$M_{\theta_f} = \{\theta_f \in R^h : \ ||\theta_f|| \le m_{\theta_f}\}$$
$$M_{\theta_g} = \{\theta_g \in R^h : \ ||\theta_g|| \le m_{\theta_g}\}$$

The **value of the approximation error** that corresponds to the optimal values of the weights vectors is

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f^*)\right) + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g^*)\right)u$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the DFRM system

7.5.3. Approximation of functions f(x,t) and g(x,t)

which is next written as

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f) + \hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)\right) + \\ + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g) + \hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

which can be also written in the following form

$$w = \ (w_a + w_b)$$

with

$$w_a = \{[f(x,t) - \hat{f}(\hat{x}|\theta_f)] + [g(x,t) - \hat{g}(\hat{x}|\theta_g)]\} u$$

and

$$w_b = \{[\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x},\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]\} u$$

Moreover, the following **weights error vectors are defined**

$$\bar{\theta}_f = \theta_f - \theta_f^* \\ \bar{\theta}_g = \theta_g - \theta_g^*$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

The **following Lyapunov function** is considered:

$$V = \frac{1}{2}\hat{e}^T P_1 \hat{e} + \frac{1}{2}\tilde{e}^T P_2 \tilde{e} + \frac{1}{2\gamma_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_2} tr[\tilde{\theta}_g^T \tilde{\theta}_g]$$

The selection of the **Lyapunov function** is based on the following principle of indirect adaptive control

$$\hat{e}: \lim_{t\to\infty} \hat{x}(t) = x_d(t)$$
$$\tilde{e}: \lim_{t\to\infty} \hat{x}(t) = x(t).$$

this results into

$$\lim_{t\to\infty} x(t) = x_d(t)$$

By deriving the **Lyapunov function** with respect to time one obtains:

$$\dot{V} = \frac{1}{2}\dot{\hat{e}}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \dot{\hat{e}} + \frac{1}{2}\dot{\tilde{e}}^T P_2 \tilde{e} + \frac{1}{2}\tilde{e}^T P_2 \dot{\tilde{e}} + \\ + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow$$

$$\dot{V} = \frac{1}{2}\{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \{(A - BK^T)\hat{e} + K_o C^T \tilde{e}\} + \\ + \frac{1}{2}\{(A - K_o C^T)\tilde{e} + Bu_c + B\tilde{d} + Bw\}^T P_2 \tilde{e} + \\ + \frac{1}{2}\tilde{e}^T P_2 \{(A - K_o C^T)\tilde{e} + Bu_c + B\tilde{d} + Bw\} + \\ + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

The equation is rewritten as:

$$\begin{aligned}\dot{V} = &\tfrac{1}{2}\{\hat{e}^T(A-BK^T)^T + \tilde{e}^T C K_o^T\}P_1\hat{e} + \tfrac{1}{2}\hat{e}^T P_1\{(A-BK^T)\hat{e} + K_o C^T\tilde{e}\}+ \\ &+\tfrac{1}{2}\{\tilde{e}^T(A-K_oC^T)^T + u_c^T B^T + w^T B^T + \tilde{d}^T B^T\}P_2\tilde{e}+ \\ &\tfrac{1}{2}\tilde{e}^T P_2\{(A-K_oC^T)\tilde{e} + Bu_c + Bw + B\tilde{d}\} + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \tfrac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g] \Rightarrow\end{aligned}$$

which finally takes the form:

$$\begin{aligned}\dot{V} = &\tfrac{1}{2}\hat{e}^T(A-BK^T)^T P_1\hat{e} + \tfrac{1}{2}\tilde{e}^T C K_o^T P_1\hat{e}+ \\ &+\tfrac{1}{2}\hat{e}^T P_1(A-BK^T)\hat{e} + \tfrac{1}{2}\hat{e}^T P_1 K_o C^T\tilde{e}+ \\ &+\tfrac{1}{2}\tilde{e}^T(A-K_oC^T)^T P_2\tilde{e} + \tfrac{1}{2}(u_c^T + w^T + \tilde{d}^T)B^T P_2\tilde{e}+ \\ &+\tfrac{1}{2}\tilde{e}^T P_2(A-K_oC^T)\tilde{e} + \tfrac{1}{2}\tilde{e}^T P_2 B(u_c + w + \tilde{d})+ \\ &+\tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \tfrac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{aligned}$$

Assumption 1: For given positive definite matrices Q1 and Q2 there exist positive definite matrices P1 and P2, which are the solution of the following **Riccati equations**

$$(A-BK^T)^T P_1 + P_1(A-BK^T) + Q_1 = 0$$

$$\begin{aligned}&(A-K_oC^T)^T P_2 + P_2(A-K_oC^T)- \\ &-P_2B(\tfrac{2}{r} - \tfrac{1}{\rho^2})B^T P_2 + Q_2 = 0\end{aligned}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

By substituting the conditions from the previous Riccati equations into the derivative of the Lyapunov function one gets:

$$\begin{aligned}\dot{V} = {} & \tfrac{1}{2}\hat{e}^T\{(A-BK^T)^T P_1 + P_1(A-BK^T)\}\hat{e} + \tilde{e}^T C K_o^T P_1\hat{e} + \\ & + \tfrac{1}{2}\tilde{e}^T\{(A-K_oC^T)^T P_2 + P_2(A-K_oC^T)\}\tilde{e} + \\ & + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \tfrac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{aligned}$$

or:

$$\begin{aligned}\dot{V} = {} & -\tfrac{1}{2}\hat{e}^T Q_1\hat{e} + \tilde{e}^T C K_o^T P_1\hat{e} - \tfrac{1}{2}\tilde{e}^T\{Q_2 - P_2B(\tfrac{2}{r} - \tfrac{1}{\rho^2})B^T P_2\}\tilde{e} + \\ & + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \tfrac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{aligned}$$

- The **supervisory control term** u_c consists of two terms: u_a and u_b.

$$u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$$

where assuming that the measurable elements of vector $\tilde{e}$ are $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$,

the term Δu_a Is given by

$$-\tfrac{1}{r}\tilde{e}^T P_2 B + \Delta u_a = -\tfrac{1}{r}\begin{pmatrix} p_{11}\tilde{e}_1 + p_{13}\tilde{e}_3 + \cdots + p_{1k}\tilde{e}_k \\ p_{13}\tilde{e}_1 + p_{33}\tilde{e}_3 + \cdots + p_{3k}\tilde{e}_k \\ \cdots\ \cdots\cdots \\ p_{1k}\tilde{e}_1 + p_{3k}\tilde{e}_3 + \cdots + p_{kk}\tilde{e}_k \end{pmatrix}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

- The control term u_b. Is given by

$$u_b = -[(P_2B)^T(P_2B)]^{-1}(P_2B)^T C K_o^T P_1 \hat{e}$$

u_a is an H-infinity control used for the compensation of the approximation error w and the additive disturbance $\tilde{d}$.

Its first component $-\frac{1}{r}\tilde{e}^T P_2 B$ has been chosen so as to compensate for the term $\frac{1}{r}\tilde{e}^T P_2 B B^T P_2 \tilde{e}$, which appears in the previously computed function about $\dot{V}$.

By including also the second component Δu_a one has that u_a is computed based on the feedback only the measurable variables $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$ out of the complete vector $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$

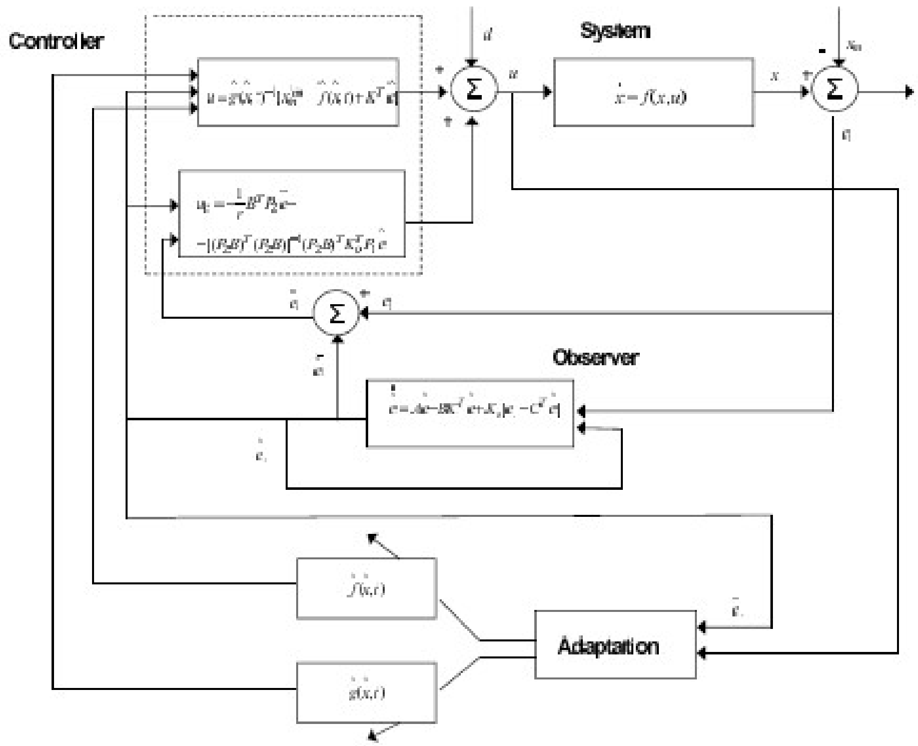
Eq. $u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$ finally rewritten as $u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$.

- u_b is a control used for the compensation of the observation error (the control term has been chosen so as to satisfy the condition

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

The control scheme is depicted in the following diagram



By substituting the supervisory control term in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{r}\bar{e}^T P_2 B B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} +$$
$$+ \bar{e}^T P_2 B u_a + \bar{e}^T P_2 B u_b + \bar{e}^T P_2 B (w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

or equivalently

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \bar{e}^T P_2 B(w + \bar{d}) + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g]$$

Besides, about the adaptation of the weights of the neurofuzzy network it holds

$$\dot{\bar{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f \qquad \dot{\bar{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g.$$

and also

$$\dot{\theta}_f = -\gamma_1 \Phi(\hat{x})^T B^T P_2 \bar{e} \\ \dot{\theta}_g = -\gamma_2 \Phi(\hat{x})^T B^T P_2 \bar{e} u^T$$

By substituting the above relations in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \bar{d}) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[u \bar{e}^T P_2 B \Phi(\hat{x})(\theta_g - \theta_g^*)]$$

or

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \bar{d}) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[u \bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Taking into account that $u \in R^{2\times 1}$ and $\bar{e}^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1\times 2}$

one gets

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w+\tilde{d}) + \\ +\frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ +\frac{1}{\gamma_2}(-\gamma_2) tr[\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u]$$

Since $\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \in R^{1\times 1}$

it holds

$$tr(\bar{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)u) = \\ = \bar{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u$$

Therefore, one finally obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w+\tilde{d}) + \\ +\frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ +\frac{1}{\gamma_2}(-\gamma_2)\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u$$

Next, the following **approximation error** is defined

$$w_\alpha = [\hat{f}(\hat{x}|\theta_f^*) - \hat{f}(\hat{x}|\theta_f)] + [\hat{g}(\hat{x}|\theta_g^*) - \hat{g}(\hat{x}|\theta_g)]u$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Thus, one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + B^T P_2 \bar{e}(w + \tilde{d}) + \bar{e}^T P_2 B w_a$$

Denoting the **aggregate approximation e**rror and disturbances vector as

$$w_1 = w + \tilde{d} + w_a$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \bar{e}^T P_2 B w_1$$

which in turn is written as

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e}$$

Lemma: The following inequality holds

$$\frac{1}{2}\bar{e}^T P_2 B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \\ \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Proof:

The binomial $(\rho a-\frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2+\frac{1}{\rho^2}b^2-2ab \geq 0 \Rightarrow$$
$$\frac{1}{2}\rho^2 a^2+\frac{1}{2\rho^2}b^2-ab \geq 0 \Rightarrow$$
$$ab-\frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow$$
$$\frac{1}{2}ab+\frac{1}{2}ab-\frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$

By substituting $a=w_1$ and $b=\bar{e}^T P_2 B$ one gets

$$\frac{1}{2}w_1^T B^T P_2 \bar{e}+\frac{1}{2}\bar{e}^T P_2 B w_1-\frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Moreover, by substituting the above inequality into the derivative of the Lyapunov function one gets

$$\dot{V} \leq -\frac{1}{2}\hat{e}^T Q_1 \hat{e}-\frac{1}{2}\bar{e}^T Q_2 \bar{e}+\frac{1}{2}\rho^2 w_1^T w_1$$

which is also written as $\dot{V} \leq -\frac{1}{2}E^T Q E+\frac{1}{2}\rho^2 w_1^T w_1$

with $E=\begin{pmatrix}\hat{e}\\ \bar{e}\end{pmatrix}, \quad Q=\begin{pmatrix}Q_1 & 0\\ 0 & Q_2\end{pmatrix}=diag[Q_1,Q_2]$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Hence, the H_∞ performance criterion is derived. For sufficiently small ρ the inequality will be true and the H_∞ tracking criterion will be satisfied. In that case, the integration of ˙V from 0 to T gives

$$\int_0^T \dot{V}(t)dt \le -\frac{1}{2}\int_0^T ||E||^2 dt + \frac{1}{2}\rho^2 \int_0^T ||w_1||^2 dt \Rightarrow$$
$$2V(T) - 2V(0) \le -\int_0^T ||E||_Q^2 dt + \rho^2 \int_0^T ||w_1||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||E||_Q^2 dt \le 2V(0) + \rho^2 \int_0^T ||w_1||^2 dt$$

It is assumed that there exists a positive constant $M_w > 0$ such that

$$\int_0^\infty ||w_1||^2 dt \le M_w$$

Therefore for the integral $\int_0^T ||E||_Q^2 dt$ one gets

$$\int_0^\infty ||E||_Q^2 dt \le 2V(0) + \rho^2 M_w$$

Thus, the integral $\int_0^\infty ||E||_Q^2 dt$ is bounded and **according to Barbalat's Lemma**

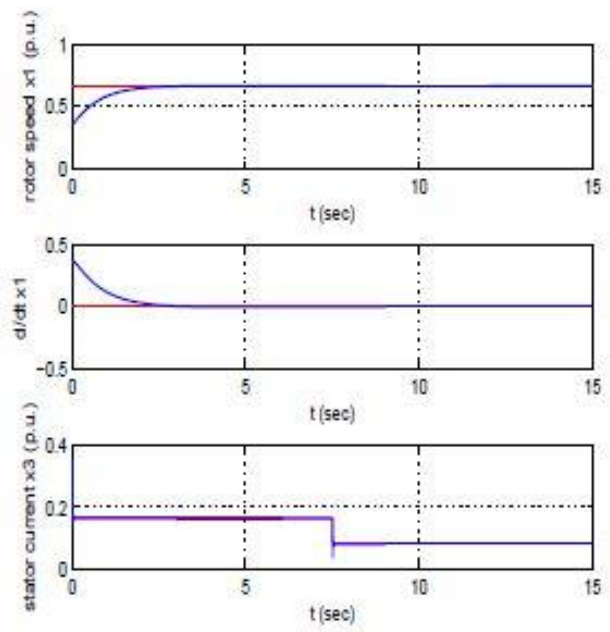
$$\lim_{t\to\infty} e(t) = 0$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

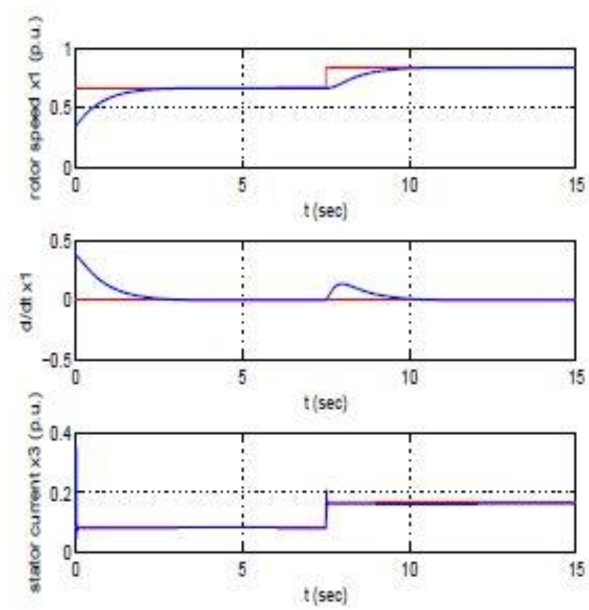
7.7. Simulation tests

The efficiency of the proposed flatness-based control method for doubly-fed reluctance machines has been confirmed with the use of simulation experiments.

The dynamic model of the DFRM was taken to be completely unknown. The system's dynamics were identified with the used of the previously analyzed neurofuzzy approximators.



Tracking of setpoint 1 for the doubly-fed reluctance machine



Tracking of setpoint 2 for the doubly-fed reluctance machine

Fast and accurate tracking of the setpoints was achieved. The transients of the state variables did not exhibit abrupt changes and the variations of the control input were smooth

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.8. Conclusions

● A solution to the problem of model-free adaptive control for brushless doubly-fed synchronous reluctance machines has been proposed

● It was proven that the dynamic model of the DFRM is a differentially flat one. The flat outputs of the model were taken to be the rotor's turn speed and the currents of the secondary (control) winding of the stator.

● By proving differential flatness properties for the machine, the transformation of its model to the linear canonical form was achieved.

● In this new linearized description the control inputs comprised nonlinear terms which were related to the system's unknown dynamics.

● These terms were dynamically identified with the use of neurofuzzy approximators. These estimates of the unknown dynamics were used in turn in the computation of a feedback control input, thus establishing an indirect adaptive control scheme.

● It was also assumed that only the output of the DFRM could be directly measured and that the rest of the state vector elements of the machine had to be computed with the use of a state-observer.

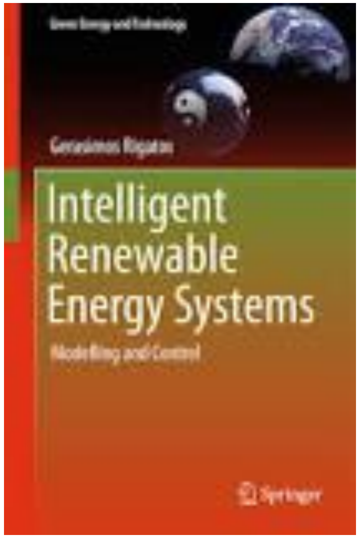
● The stability of the control loop was proven with the use of Lyapunov analysis.

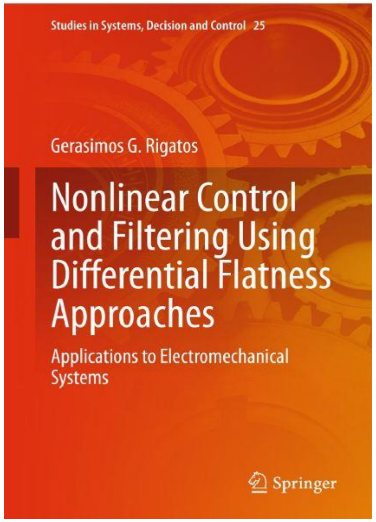
8. Final Conclusions

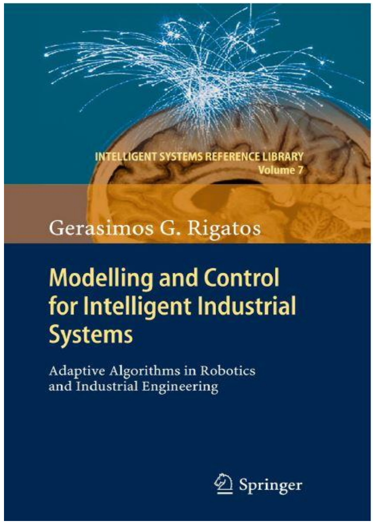
● Methods for nonlinear control and state estimation in electric power systems have been developed

● The main approaches for nonlinear control have been: (i) **control with global linearization** method (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods (adaptive control)** in case that the dynamic model of the electric power system is unknown

● The main approaches for nonlinear state estimation are: (i) nonlinear state estimation with methods of global linearization (ii) nonlinear state estimation with methods of approximate (asymptotic) linearization







grigat@ieee.org