Nonlinear optimal control and fllltering
for complex dynamical systems

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1 . Outline

● The reliable functioning of complex nonlinear dynamical systems, such as robotic systems and electric power systems relies on the solution of the associated nonlinear control and state estimation problems

● The main approaches followed towards the solution of nonlinear control problem are as follows: (i) **control with global linearization** methods (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods** (adaptive control methods) when the dynamic or kinematic model of the complex nonlinear Systems is unknown

● The main approaches followed towards the solution of the nonlinear state estimation problems are as follows: (i) state estimation with methods global linearization (ii) state estimation with methods of approximate (asymptotic) linearization

● Factors of major importance for the control loop of complex nonlinear systems are as follows (i) global stability conditions for the related nonlinear control scheme (ii) global stability conditions for the related nonlinear state estimation scheme (iii) global asymptotic stability for the joint control and state estimation scheme

2 . Nonlinear control and state estimation with global linearization

● To this end the differential flatness control theory is used

● The method can be applied to all nonlinear systems which are subject to an input-output linearization and actually such systems posses the property of differential flatness

● The state-space description for the dynamic or kinematic model of the complex nonlinear systems is transformed into a more compact form that is input-output linearized. This is achieved after defining the system's flat outputs

● A system is differentially flat if the following two conditions hold: (i) all state variables and control inputs of the system can be expressed as differential functions of its flat outputs (ii) the flat outputs of the system and their time-derivatives are differentially independent, which means that they are not connected through a relying having the form of an ordinary differential equation

● With the applications of change of variables (diffeomorphisms) that rely on the differential flatness property (i), the state-space description of the complex nonlinear systems is written into the linear canonical form. For the latter state-space description it is possible to solve both the control and the State estimation problem for the complex nonlinear systems

3 . Nonlinear control and state estimation with approximate linearization

- To this end the theory of optimal H-infinity control and the theory of optimal H-infinity state estimation are used

- The nonlinear state-space description of the system undergoes approximate linearization around a temporary operating point which is updated at each iteration of the control and state estimation algorithm

- The linearization relies on first order Taylor series expansion around the temporary operating point and makes use of the computation of the associated Jacobian matrices

- The linearization error which is due to the truncation error of higher-order terms in the Taylor series expansion is considered to be a perturbation that is finally compensated by the robustness of the control algorithm

- For the linearized description of the state-space model an optimal H-infinity controller is designed. For the selection of the controller's feedback gains an algebraic Riccati equation has to be solved at each time step of the control algorithm

- Through Lyapunov stability analysis, the global stability properties of the control method are proven

- For the implementation of the optimal control method through the processing of measurements from a small number of sensors of the complex nonlinear systems, the H-infinity Kalman Filter is used as a robust state estimator

4 . Nonlinear control and state estimation with Lyapunov methods

● By initially proving the differential flatness properties for the complex nonlinear systems and by defining its flat outputs a transformation of Its state-space description into an equivalent input-output linearized form is achieved.

● The unknown dynamics of the complex nonlinear systems is incorporated into the transformed control inputs of the system, which now appear in its equivalent input-output linearized state-space description

● The control problem for the complex nonlinear systems of unknown dynamics in now turned into a problem of indirect adaptive control. The computation of the control inputs of the system is performed simultaneously with the identification of the nonlinear functions which constitute its unknown dynamics.

● The estimation of the unknown dynamics of the complex nonlinear systems is performed through the adaptation of neurofuzzy approximators. The definition of the learning parameters takes place through gradient algorithms of proven convergence, as demonstrated by Lyapunov stability analysis

● The Lyapunov stability method is the tool for selecting both the gains of the stabilizing feedback controller and the learning rate of the estimator of the unknown system's dynamics

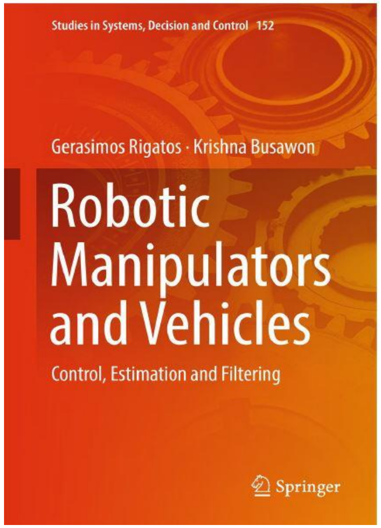
● Equivalently through Lyapunov stability analysis the feedback gains of the state estimators of the complex nonlinear systems are chosen. Such observers are included in the control loop so as to enable feedback control through the processing of a small number of sensor measurements

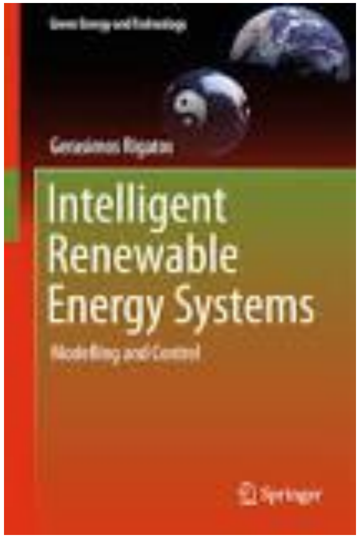
5. Final conclusions

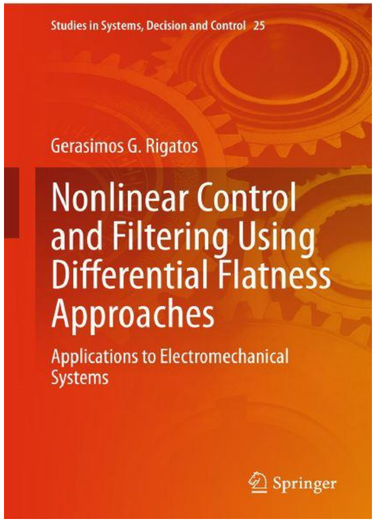
● Methods for nonlinear control and state estimation in complex nonlinear systems have been developed

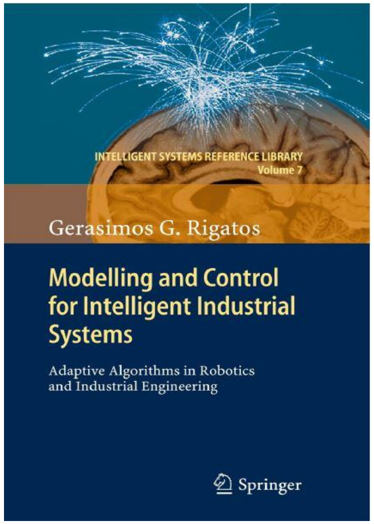
● The main approaches for nonlinear control have been: (i) **control with global linearization** method (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods (adaptive control)** in case that the dynamic or kinematic model of the complex nonlinear systems is unknown

● The main approaches for nonlinear state estimation are: (i) nonlinear state estimation with methods of global linearization (ii) nonlinear state estimation with methods of approximate (asymptotic) linearization









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