Nonlinear control and estimation for USVs and AUVs: advances in autonomous navigation and optimized propulsion

Part B: Optimized propulsion

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I . Outline

● Optimized propulsion of USVs and AUVs relies on the solution of the associated nonlinear control and state estimation problems

● The main approaches followed towards the solution of nonlinear control problem are as follows: (i) **control with global linearization** methods (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods** (adaptive control methods) when the dynamic model of the USVs and AUVs is unknown.

● The main approaches followed towards the solution of the nonlinear state estimation problems are as follows: (i) state estimation with methods global linearization (ii) state estimation with methods of approximate (asymptotic) linearization

● Factors of major importance for the control loop of USVs and AUVs, in optimized propulsion problems, are as follows (i) global stability conditions for the related nonlinear control scheme (ii) global stability conditions for the related nonlinear state estimation scheme (iii) global asymptotic stability for the joint control and state estimation scheme

II . Nonlinear control and state estimation with global linearization

● To this end the differential flatness control theory is used

● The method can be applied to all nonlinear systems which are subject to input-output linearization and actually such systems posses the property of differential flatness

● The state-space description for the propulsion model of the USVs and AUVs is transformed into a more compact form that is input-output linearized. This is achieved after defining the system's flat outputs

● A system is differentially flat if the following two conditions hold: (i) all state variables and control inputs of the system can be expressed as differential functions of its flat outputs (ii) the flat outputs of the system and their time-derivatives are differentially independent, which means that they are not connected through a relation having the form of an ordinary differential equation

● With the application of change of variables (diffeomorphisms) that rely on the differential flatness property (i), the state-space description of the USVs and AUVs propulsion system is written into the linear canonical form. For the latter state-space description it is possible to solve both the control and the state estimation problem for the USVs and UAVs propulsion system

III . Nonlinear control and state estimation with approximate linearization

● To this end the theory of optimal H-infinity control and the theory of optimal H-infinity state estimation are used

● The nonlinear state-space description of the USVs and AUVs propulsion undergoes approximate linearization around a temporary operating point which is updated at each iteration of the control and state estimation algorithm

● The linearization relies on first order Taylor series expansion around the temporary operating point and makes use of the computation of the associated Jacobian matrices

● The linearization error which is due to the truncation error of higher-order terms in the Taylor series expansion is considered to be a perturbation that is finally compensated by the robustness of the control algorithm

● For the linearized description of the state-space model an optimal H-infinity controller is designed. For the selection of the controller's feedback gains an algebraic Riccati equation has to be solved at each time step of the control algorithm

● Through Lyapunov stability analysis, the global stability properties of the control method are proven

● For the implementation of the optimal control method through the processing of measurements from a small number of sensors in the USVs and AUVs propulsion system, the H-infinity Kalman Filter is used as a robust state estimator

IV . Nonlinear control and state estimation with Lyapunov methods

● By proving differential flatness properties for the USVs and AUVs propulsion and by defining flat outputs a transformation of the related state-space model into an equivalent input-output linearized form is achieved.

● The unknown dynamics of the USVs and AUVs propulsion is incorporated into the transformed control inputs of the system, which now appear in its equivalent input-output linearized state-space description

● The control problem for USVs and AUVs of unknown propulsion dynamics in now turned into a problem of indirect adaptive control. The computation of the control inputs of the system is performed simultaneously with the identification of the nonlinear functions which constitute its unknown dynamics.

● The estimation of the unknown propulsion dynamics of the USVs and AUVs is performed through the adaptation of neurofuzzy approximators. The definition of the learning parameters takes place through gradient algorithms of proven convergence, as demonstrated by Lyapunov stability analysis

● The Lyapunov stability method is the tool for selecting both the gains of the stabilizing feedback controller and the learning rate of the estimator of the unknown system's dynamics

● Equivalently through Lyapunov stability analysis the feedback gains of the state estimators of the USVs and AUVs propulsion system are chosen. Such observers are included in the control loop so as to enable feedback control through the processing of a small number of sensor measurements

Example 1: Nonlinear control and state estimation using global linearization

1. Control of turbocharged ship diesel engines

• The development of efficient **control for turbocharged ship Diesel engines**, requires **elaborated nonlinear control and filtering methods**

• To this end, **nonlinear control for turbocharged Diesel engines** is developed with the use of **Differential flatness theory** and **the Derivative-free nonlinear Kalman Filter**.

• It is shown that the dynamic model of the turbocharged Diesel engine is **differentially flat** and admits dynamic feedback linearization.

• It is also shown that the dynamic model can be written in the **linear Brunovsky canonical form** for which a **state feedback controller** can be easily designed.

• To compensate for **modeling errors** and **external disturbances** the Derivative-free nonlinear Kalman Filter is used and redesigned as a **disturbance observer**.

• The filter consists of the **Kalman Filter recursion** on the **linearized equivalent model of the Diesel engine model** and of an **inverse transformation** based on differential flatness theory which enables to obtain estimates for the state variables of the initial nonlinear model.

• Once the disturbances variables are identified it is possible to compensate them by including an additional control term in the feedback loop.

Example 1: Nonlinear control and state estimation using global linearization

2. Dynamic model of the Diesel engine

The **basic parameters of the Diesel engine** are:

(i) Gas pressure in the intake manifold p_1
(ii) Gas pressure in the exhaust manifold p_2
(iii) Turbine power P_t
(iv) Compressor power P_c

Additional variables of importance are:

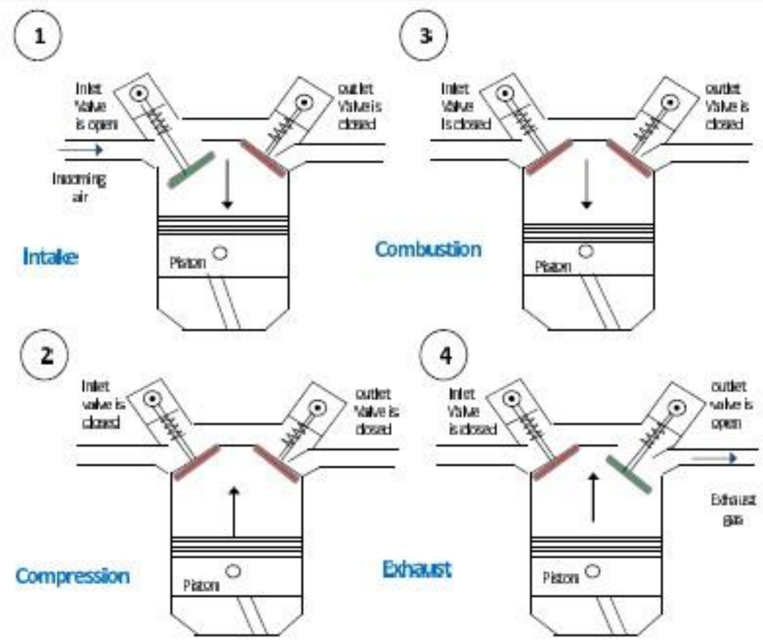
W_c which is the compressor's mass flow rate

T_1 which is the intake manifold temperature

T_2 which is the exhaust manifold temperature

W_t which is the turbine mass-flow rate

W_{EGR} which is the exhaust gas recirculation flow rate



Four-stroke cycle of an internal combustion diesel engine

Example 1: Nonlinear control and state estimation using global linearization

2. Dynamic model of the Diesel engine

The **basic relations of the Diesel-engine's dynamics** are:

$$\dot{p}_1 = K_1(W_c + u_1 - K_e p_1)$$
$$\dot{p}_2 = K_2(K_e p_1 - u_1 - u_2)$$
$$\dot{P}_c = \frac{1}{\tau}(\eta_m P_t - P_c)$$

The **control inputs** to this model are:

(i) The **exhaust-gas recirculation** (EGR) flow rate $u_1 = W_{EGR}$

(ii) The **turbine's mass flow rate** $u_2 = W_t$

Moreover, it holds that: $W_c = P_c \frac{K_c}{p_1^{\mu} - 1}$ $\quad P_t = K_t(1 - p_2{}^{\mu})u_2$

The **description of the Diesel engine in state-space form** is given by:

$$\dot{x} = f(x) + g_a(x)u_1 + g_b(x)u_2$$

where:

$$f(x) = \begin{pmatrix} K_1 K_c \frac{P_c}{p_1^{\mu}} - K_1 K_e p_1 \\ K_2 K_e p_1 \\ -\frac{P_c}{\tau} \end{pmatrix} \quad g_a(x) = \begin{pmatrix} K_1 \\ -K_2 \\ 0 \end{pmatrix} \quad g_b(x) = \begin{pmatrix} 0 \\ -K_2 \\ K_o(1 - p_2{}^{-\mu}) \end{pmatrix}$$

Example 1: Nonlinear control and state estimation using global linearization

2. Dynamic model of the Diesel engine

The **output variables of the Diesel engine model** are:

$$y = \begin{pmatrix} p_1 \\ W_c \end{pmatrix} = \begin{pmatrix} p_1 \\ P_c \frac{K_c}{p_1^{\mu}-1} \end{pmatrix}$$

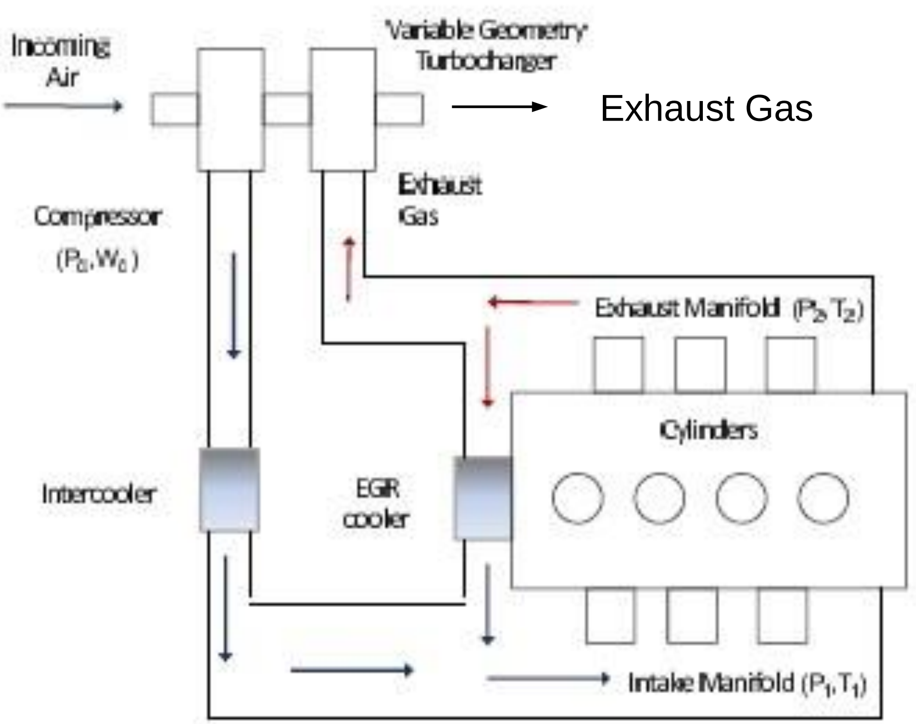


Fig.1 : Diagram of the turbocharged Diesel engine

Example 1: Nonlinear control and state estimation using global linearization

3. Lie algebra-based control

Dynamic feedback linearization is applied to the Diesel engine's model:

The state vector of the system is extended by considering as **additional state variables the control inputs**

The transformed control inputs which appear in the linearized equivalent of the system are functions of not only the initial control variables u_1, u_2 but also of their derivatives $\dot{u}_1, \dot{u}_2$.

The **extended state vector of the diesel engine** becomes:

$$x = [x_1, x_2, x_3, x_4]^T = [p_1, p_2, P_c, z]^T$$

The **control inputs to the linearized** model of the Diesel engine become:

$$v_1 = \dot{u}_1 = \dot{z},\ v_2 = u_2$$

where $u_1 = W_{EGR}$ $u_2 = W_t$

The control inputs which are finally applied *to the system* contain an integral action:

$$u_1 = \int v_1 dt,\ u_2 = v_2$$

Example 1: Nonlinear control and state estimation using global linearization

3. Lie algebra-based control

The **extended state-space description of the Diesel engine** becomes:

$$\begin{gathered}
\dot{x}_1 = K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4 \\
\dot{x}_2 = K_2 K_e x_1 - K_2 x_4 - K_2 v_2 \\
\dot{x}_3 = -\frac{x_3}{\tau} + K_o \left(1 - x_2{}^{-\mu}\right) \\
\dot{x}_4 = v_1
\end{gathered}$$

Consequently, in matrix form one has:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4 \\ K_2 K_e x_1 - K_2 x_4 \\ -\frac{x_3}{\tau} + K_o (1 - x_2^{\mu}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} v_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} v_2$$

The **system's outputs** are chosen to be:

$$\begin{gathered}
y_1 = x_1 = p_1 \\
y_2 = P_c \frac{K_c}{p_1^{\mu}-1} \Rightarrow y_2 = x_3 \frac{K_c}{x_1^{\mu}-1}
\end{gathered}$$

Example 1: Nonlinear control and state estimation using global linearization

3. Lie algebra-based control

Linearization of the system's dynamics is performed using the following state variables:

$$z_1^1 = h_1(x) \qquad z_2^1 = L_f h_1(x) \qquad \dot{z}_2^1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$

and equivalently:

$$z_1^2 = h_2(x) \qquad z_2^2 = L_f h_2(x) \qquad \dot{z}_2^2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

After intermediate computations one obtains:

$$L_f^2 h_1(x) = L_f z_2^1 \Rightarrow L_f^2 h_1(x) = \frac{\partial z_2^1}{\partial x_1} f_1 + \frac{\partial z_2^1}{\partial x_1} f_2 + \frac{\partial z_2^1}{\partial x_3} f_3 \Rightarrow$$

$$L_f^2 h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right) f_1 + 0 f_2 + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right) f_3 + K_1 f_4 \Rightarrow$$

$$L_f^2 h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right)\left(K_1 K_3 \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right)\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right)$$

and also

$$L_{g_a} L_f h_1(x) = L_{g_a} z_2^1 \Rightarrow L_{g_a} L_f h_1(x) = \frac{\partial z_2^1}{\partial x_1} g_{a_1} + \frac{\partial z_2^1}{\partial x_2} g_{a_2} + \frac{\partial z_2^1}{\partial x_3} g_{a_3} + \frac{\partial z_2^1}{\partial x_4} g_{a_4} \Rightarrow$$

$$L_{g_a} L_f h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right) g_{a_1} + 0 g_{a_2} + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right) g_{a_3} + K_1 g_{a_4} \Rightarrow$$

$$L_{g_a} L_f h_1(x) = K_1$$

and

$$L_{g_b} L_f h_1(x) = L_{g_b} z_2^1 \Rightarrow L_{g_b} L_f h_1(x) = \frac{\partial z_2^1}{\partial x_1} g_{b_1} + \frac{\partial z_2^1}{\partial x_2} g_{b_2} + \frac{\partial z_2^1}{\partial x_3} g_{b_3} + \frac{\partial z_2^1}{\partial x_4} g_{b_4} \Rightarrow$$

$$L_{g_b} L_f h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right) g_{b_1} + 0 g_{b_2} + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right) g_{b_3} + K_1 g_{b_4} \Rightarrow$$

$$L_{g_b} L_f h_1(x) = 0$$

Example 1: Nonlinear control and state estimation using global linearization

3. Lie algebra-based control

In a similar manner one obtains:

$$L_f^2 h_2(x) = \frac{\partial z_2^2}{\partial x_1} f_1 + \frac{\partial z_2^2}{\partial x_2} f_2 + \frac{\partial z_2^2}{\partial x_3} f_3 + \frac{\partial z_2^2}{\partial x_4} f_4 \Rightarrow$$

$$L_f^2 h_2(x) = \{x_3 \frac{(-K_c \mu \mu - 1 x_1^{\mu-2})(x_1^{\mu}-1)^2 - (-K_c \mu x_1^{\mu-1}) 2 (x_1^{\mu}-1) \mu x_1^{\mu-1}}{x_1^{\mu}-1^4} \left(\frac{K_1 K_c x_3}{x_1^{\mu}-1-K_1 K_c + K_1 x_4}\right) +$$
$$x_3 \frac{-K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1} \left(\frac{K_1 K_c - x_3 \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2} - K_1 K_c\right) +$$
$$+ \frac{-K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1^2} \left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right)\} \left(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) +$$
$$+ \frac{K_c}{x_1^{\mu}-1} K_o \mu x_2^{-\mu-1} (K_2 K_e x_1 - K_2 x_4) +$$
$$+ \{x_3 \frac{-K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1^2} \frac{K_1 K_c}{x_1^{\mu}-1} + \frac{K_c}{x_1^{\mu}-1}\left(-\frac{1}{\tau}\right)\} \left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right) +$$
$$\{x_3 \frac{-K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1^2} K_1\} 0$$

and also

$$L_{g_a} L_f h_2(x) = \frac{\partial z_2^2}{\partial x_1} g_{a_1} + \frac{\partial z_2^2}{\partial x_2} g_{a_2} + \frac{\partial z_2^2}{\partial x_3} g_{a_3} + \frac{\partial z_2^2}{\partial x_4} g_{a_4} \Rightarrow$$
$$L_{g_a} L_f h_2(x) = \frac{\partial z_2^2}{\partial x_4} 1 \Rightarrow L_{g_a} L_f h_2(x) = x_3 \frac{-K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1^2)} K_1$$

and

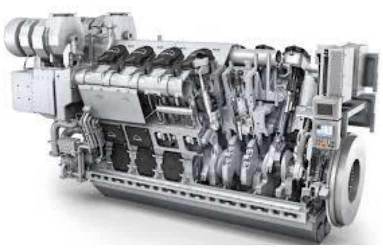
$$L_{g_b} L_f h_2(x) = \frac{\partial z_2^2}{\partial x_1} g_{b_1} + \frac{\partial z_2^2}{\partial x_2} g_{b_2} + \frac{\partial z_2^2}{\partial x_3} g_{b_3} + \frac{\partial z_2^2}{\partial x_4} g_{b_4} \Rightarrow$$
$$L_{g_b} L_f h_2(x) = \frac{\partial z_2^2}{\partial x_2} 1 \Rightarrow L_{g_b} L_f h_2(x) = \frac{K_c}{x_1^{\mu}-1} K_o \mu x_2^{-\mu-1}$$

Example 1: Nonlinear control and state estimation using global linearization

3. Lie algebra-based control

Thus, after the change of coordinates **the following description of the system is obtai**ned

$$
\begin{gathered}
\dot{z}_1^1 = z_2^1 \\
\dot{z}_2^1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) v_1 + L_{g_b} L_f h_1(x) v_2 \\
\dot{z}_1^2 = z_2^2 \\
\dot{z}_2^2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) v_1 + L_{g_b} L_f h_2(x) v_2
\end{gathered}
$$



which is also written in the **state-space form**

$$
\begin{pmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{pmatrix} = \begin{pmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} + \begin{pmatrix} L_{g_a} L_f h_1(x) & L_{g_b} L_f h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}
$$

or also in the more **compact form**

$$
\ddot{z} = \bar{f}_a + \bar{M}_a v
$$

Example 1: Nonlinear control and state estimation using global linearization

3. Lie algebra-based control

Moreover, by defining the **control inputs**

$$v_{in}^1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) v_1 + L_{g_b} L_f h_1(x) v_2$$
$$v_{in}^2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) v_1 + L_{g_b} L_f h_2(x) v_2$$

the system's description comes to the following **canonical form**:

$$\begin{pmatrix} \dot{z}_1^1 \\ \dot{z}_2^1 \\ \dot{z}_1^2 \\ \dot{z}_2^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^1 \\ z_1^2 \\ z_1^2 \\ z_2^2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_{in}^1 \\ v_{in}^2 \end{pmatrix}$$

The selection of the **state feedback control law**, which assures elimination of the tracking error is:

$$v_{in}^1 = \ddot{z}_{1,d}^1 - K_d^1(\dot{z}_1^1 - \dot{z}_{1,d}^1) - K_p^1(z_1^1 - z_{1,d}^1)$$
$$v_{in}^2 = \ddot{z}_{1,d}^2 - K_d^2(\dot{z}_1^2 - \dot{z}_{1,d}^2) - K_p^2(z_1^2 - z_{1,d}^2)$$

The **control input** that is finally applied to the system is:

$$\bar{v}_{in} = \bar{f}_a + \bar{M}_a \bar{v} \Rightarrow \bar{v} = \bar{M}_a^{-1}(\bar{v}_{in} - \bar{f}_a)$$

Example 1: Nonlinear control and state estimation using global linearization

4. Nonlinear control of the diesel engine using differential flatness theory

The results about **dynamic state feedback system linearization** can be obtained with the computation of time derivatives and *differential flatness theory*. The following **differentially flat system outputs** are considered

$$y_1 = p_1 = x_1$$
$$y_2 = P_c \frac{K_c}{p_1^{\mu} - 1} \Rightarrow y_2 = x_3 \frac{K_c}{x_1^{\mu} - 1}$$

The **dynamics of the extended system** are:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} K_1 K_c \frac{x_3}{x_1^{\mu} - 1} - K_1 K_e x_1 + K_1 x_4 \\ K_2 K_e x_1 - K_2 x_4 \\ \frac{-x_3}{\tau} + K_o (1 - x_2^{\mu}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} v_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} v_2$$

It holds that $y_1 = x_1$ therefore:

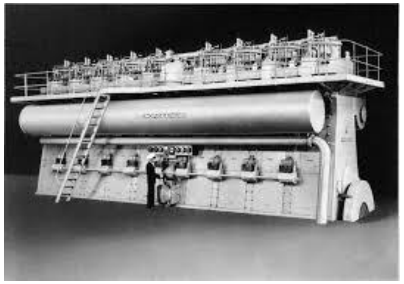
$$x_1 = q_1(y, \dot{y})$$
$$y_2 = x_3 \frac{K_c}{x_1^{\mu} - 1} \Rightarrow x_3 = \frac{y_2 (x_1^{\mu} - 1)}{K_c} \Rightarrow x_3 = \frac{y_2 (y_1^{\mu} - 1)}{K_c}$$

which means that variable x_3 is also a function of the flat output and its derivatives.

Example 1: Nonlinear control and state estimation using global linearization

4. Nonlinear control of the diesel engine using differential flatness theory

Moreover, from the first row of the state-space equations one obtains:



$$\dot{x}_1 = K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4 \Rightarrow$$

$$x_4 = \frac{\dot{x}_1 - K_1 K_c \frac{x_3}{x_1^{\mu}-1} + K_1 K_e x_1}{K_1} \Rightarrow x_4 = q_4(y, \dot{y})$$

which means that variable x_4 is also a function of the flat output and its derivatives.

Additionally, from the fourth row of the state-space equations one obtains:

$$\dot{x}_4 = v_1 \Rightarrow v_1 = q_5(y, \dot{y})$$

This means that the control input v_1 is also a function of the flat output and its derivatives.

Similarly, from the third row of the state-space equations one has:

$$\dot{x}_3 = -\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu}) \Rightarrow$$

$$x_2^{-\mu} = -\frac{\dot{x}_3 + \frac{x_3}{\tau}}{K_o} \Rightarrow x_2 = \left(\frac{-\dot{x}_3 + \frac{x_3}{\tau}}{K_o}\right)^{\mu}$$

Example 1: Nonlinear control and state estimation using global linearization

4. Nonlinear control of the diesel engine using differential flatness theory

while, from the second row of the state-space equations one obtains:

$$
\begin{gathered}
\dot{x}_2 = K_2 K_g x_1 - K_2 x_4 + v_2 \Rightarrow \\
v_2 = \dot{x}_2 - K_2 K_g x_1 + K_2 x_4 \Rightarrow \\
v_2 = q_6(y, \dot{y})
\end{gathered}
$$

Therefore, **all state variables of the system** and the **control inputs** can be written as **functions of the flat output and its derivatives**. Therefore, the system of the diesel engine is differentially flat and can be subjected to dynamic feedback linearization.

Next, by considering the **flat outputs** and by **differentiating with respect to time** one obtains:

$$
\begin{gathered}
y_1 = x_1 \\
\dot{y}_1 = \dot{x}_1 \Rightarrow \dot{y}_1 = K_1 K_c \frac{x_3}{x_1^{\mu} - 1} - K_1 K_c x_1 + K_1 x_4
\end{gathered}
$$

By differentiating once more with respect to time one gets:

$$
\begin{gathered}
\ddot{y}_1 = \left(K_1 K_c \frac{-x_3 \mu x_1^{\mu-1}}{{x_1^{\mu} - 1}^2} - K_1 K_c\right)\left(K_1 K_c \frac{x_3}{x_1^{\mu} - 1} - K_1 K_e x_1 + K_1 x_4\right) + \\
+ \left(K_1 K_c \frac{1}{x_1^{\mu} - 1}\right)\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right) + K_1 v_1
\end{gathered}
$$

Example 1: Nonlinear control and state estimation using global linearization

4. Nonlinear control of the diesel engine using differential flatness theory

In a similar manner one has:

$$y_2 = x_3 \frac{K_c}{x_1^{\mu}-1}$$

$$\dot{y}_2 = \frac{x_3 K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1^2}\dot{x}_1 + \frac{K_c}{x_1^{\mu}-1}\dot{x}_3 \Rightarrow$$

$$\dot{y}_2 = \frac{x_3 K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1^2}\dot{x}_1 + \frac{K_c}{x_1^{\mu}-1}\dot{x}_3 \Rightarrow$$

while by differentiating once more with respect to time one obtains:

$$\begin{aligned}\ddot{y}_2 = \{&\frac{x_3 K_c \mu(\mu-1)x_1^{\mu-2}(x_1^{\mu}-1)^2 - x_3 K_c \mu x_1^{\mu-1} 2(x_1^{\mu}-1)\mu x_1^{\mu-1}}{(x_1^{\mu}-1)^4}(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4) + \\ &+\frac{-K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}(-\frac{x_3}{\tau} + K_o(1-x_2^{\mu}))\}(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4) + \frac{K_c}{x_1^{\mu}-1}(K_o \mu x_2^{\mu-1})(K_2 K_e x_1 - K_2 x_4 + v_2) + \\ &+\{\frac{K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}(K_1 K_c \frac{x_3}{x_1^{\mu}-1)} - K_1 K_e x_1 + K_1 x_4 + \frac{K_c}{x_1^{\mu}-1}(-\frac{1}{\tau})\}(-\frac{x_3}{\tau} + K_o(1-x_2^{-\mu})) + \\ &+\frac{x_3 K_c \mu x_1^{\mu-1}}{x_1^{\mu}-1^2}K_1\}v_1\end{aligned}$$

Thus one arrives at a representation of the system's dynamics that is analogous to the one obtained by applying the Lie algebra-based approach:

Example 1: Nonlinear control and state estimation using global linearization

4. Nonlinear control of the diesel engine using differential flatness theory

$$\ddot{y}_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$
$$\ddot{y}_2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

where:

$$L_f^2 h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right)\left(K_1 K_3 \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right)\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right)$$

$$L_{g_a} L_f h_1(x) = K_1 \quad \text{and} \quad L_{g_b} L_f h_1(x) = 0$$

and also:

$$L_f^2 h_2(x) = \Big\{ \frac{x_3 K_c \mu(\mu-1) x_1^{\mu-2} (x_1^{\mu}-1)^2 - x_3 K_c \mu x_1^{\mu-1} 2(x_1^{\mu}-1)\mu x_1^{\mu-1}}{(x_1^{\mu}-1)^4} \left(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) +$$
$$+ \frac{-K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{\mu})\right)\Big\}\left(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \frac{K_c}{x_1^{\mu}-1}\left(K_o \mu x_2^{\mu-1}\right)\left(K_2 K_e x_1 - K_2 x_4\right) +$$
$$+ \Big\{ \frac{K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}\left(K_1 K_c \frac{x_3}{x_1^{\mu}-1}\right) - K_1 K_e x_1 + K_1 x_4 + \frac{K_c}{x_1^{\mu}-1}\left(-\frac{1}{\tau}\right)\Big\}\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right)$$

$$L_{g_a} L_f h_2(x) = \frac{x_3 K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2} K_1 \quad \text{and} \quad L_{g_b} L_f h_2(x) = \frac{K_c}{x_1^{\mu}-1}\left(K_o \mu x_2^{\mu-1}\right)$$

The design of the state feedback controller proceeds as in case of linearization with the use of Lie algebra-based computations

Example 1: Nonlinear control and state estimation using global linearization

5. Disturbances compensation using the Derivative-free nonlinear Kalman Filter

It is assumed that **model uncertainty effects** and **external perturbation terms** are described in the model of the diesel engine as additive disturbance inputs which appear in the linearized equivalent. Thus, one has the dynamics

$$\ddot{y}_1 = L_f^2 h_1(x) + L_{g_a} h_1(x) v_1 + L_{g_b} L_f h_2(x) v_2 + \tilde{d}_1$$
$$\ddot{y}_2 = L_f^2 h_2(x) + L_{g_a} h_2(x) v_1 + L_{g_b} L_f h_2(x) v_2 + \tilde{d}_2$$

and after defining

$$v_1 = L_f^2 h_1(x) + L_{g_a} h_1(x) v_1 + L_{g_b} L_f h_2(x) v_2$$

$$v_2 = L_f^2 h_2(x) + L_{g_a} h_2(x) v_1 + L_{g_b} L_f h_2(x) v_2$$

one gets

$$\ddot{y}_1 = v_{in}^1 + \tilde{d}_1$$
$$\ddot{y}_2 = v_{in}^2 + \tilde{d}_2$$

while the disturbances are considered to be described by the associated 2nd order derivative

$$\ddot{\tilde{d}}_1 = f_{d_1}$$
$$\ddot{\tilde{d}}_2 = f_{d_2}$$

In the latter case, the system's dynamics can be extended by **considering as additional state variables** the **disturbance terms and their deri**vatives. Thus one obtains

$$\dot{z} = Az + B\tilde{v}_{in}$$
$$z^m = Cz$$

Example 1: Nonlinear control and state estimation using global linearization

5. Disturbances compensation using the Derivative-free nonlinear Kalman Filter

where $z = [z_1, z_2, \cdots, z_8]^T$, $\tilde{v}_{in} = [v_1, v_2, f_{d_1}, f_{d_2}]^T$

and matrices A,B,C are defined as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad C^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In the design of the **Kalman Filter-based disturbances estimator** it is assumed that the disturbances' dynamics is completely unknown. Thus, the considered dynamics now is

$$\dot{\hat{z}} = A_o \hat{z} + B_o v_{in} + K[z^m - \hat{z}^m] \Rightarrow$$
$$\dot{\hat{z}} = A_o \hat{z} + B_o v_{in} + K[Cz - C\hat{z}]$$

where $v_{in} = [v_1, v_2]^T$, $A_o = A$, $C_o = C$ and

$$B_o^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Example 1: Nonlinear control and state estimation using global linearization

5. Disturbances compensation using the Derivative-free nonlinear Kalman Filter

For the above definition of dynamics of the disturbances estimator, the **selection of the observer's gain** K can be performed using the standard Kalman Filter recursion. Prior to this, matrices A_o, B_o and C_o are brought to the discrete-time form A_d, B_d and C_d using common discretization methods. The discrete-time Kalman Filter recursion is

measurement update:

$$K(k) = P^{-}(k)C_d^T[C_d P^{-}(k)C_d^T + R]^{-1}$$
$$\hat{z}(k) = \hat{x}^{-}(k) + K(k)[z^m(k) - \hat{z}^m(k)]$$
$$P(k) = P^{-}(k) - K(k)C_d P^{-}(k)$$

time update:

$$P^{-}(k+1) = A_d P(k) A_d^T + Q$$
$$\hat{z}(k+1) = A_d \hat{z}(k) + B_d v_{in}(k)$$

From the previous estimation procedure one **can reconstruct the state vector of the initial nonlinear model** of the diesel engine

$$\hat{x}_1 = \hat{y}_1 \qquad \hat{x}_3 = \hat{y}_2 \frac{(\hat{y}_1^{\mu} - 1)}{K_e}$$

$$\hat{x}_2 = -\left(\frac{\hat{x}_3 + \frac{\hat{x}_3}{\tau}}{K_o}\right)^{\mu} \qquad \hat{x}_4 = \frac{\hat{y}_1 - K_1 K_e \frac{\hat{x}_3}{\hat{x}_1^{\mu} - 1} + K_1 K_e \hat{x}_1}{K_1}$$

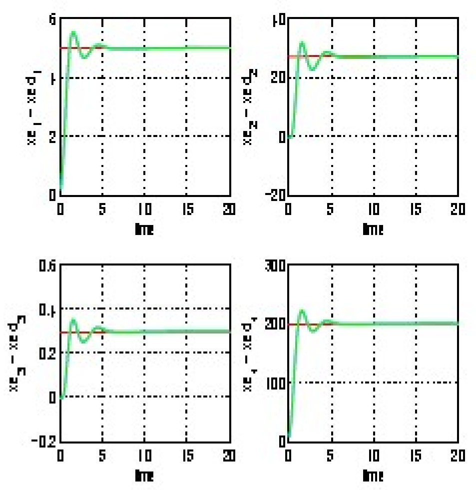
The **control signal** that enables the disturbances compensation is

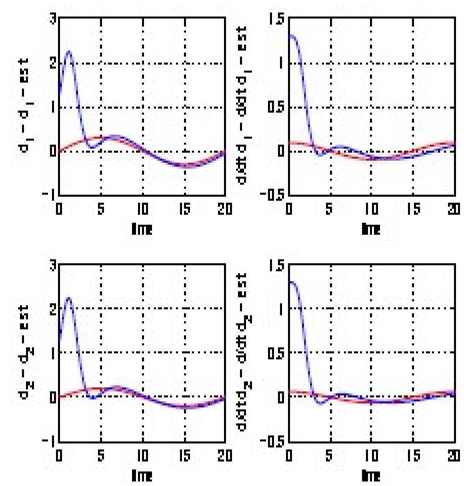
$$v_{in}^{1,*} = v_{in}^{1} - \hat{z}_5$$
$$v_{in}^{2,*} = v_{in}^{2} - \hat{z}_7$$

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

Through simulation experiments it has been confirmed that the proposed control and Kalman Filter-based estimation scheme can (i) **succeed convergence** of the elements of the state vector of the turbocharged diesel engine to the **desirable setpoints**, (ii) **estimate non-measurable elements of the state vector** as well as disturbance terms that affect the engine's dynamics.





Fig, 2(a) Convergence of the state variables to the **associated setpoints 1** (red line: setpoint, blue line: real value, green line: estimated value)

Fig, 2(b) Estimation (blue line) of perturbation terms (red line) affecting the diesel engine

Example 1: Nonlinear control and state estimation using global linearization

6. Simulation tests

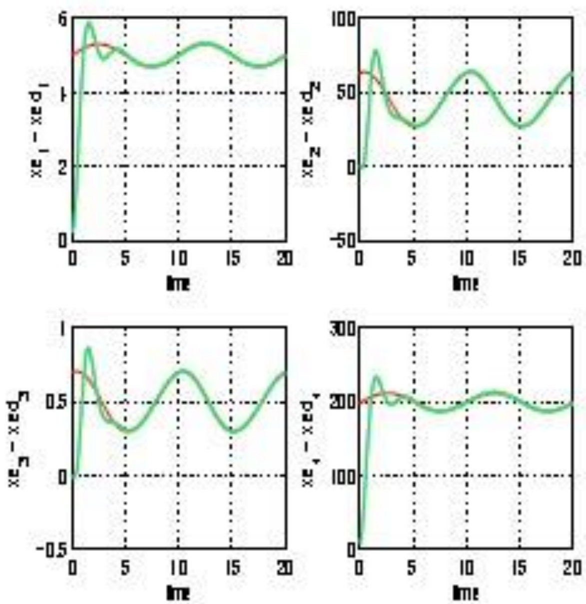


Fig 2(a): Convergence of the state variables to the **associated setpoints 2** (red line: setpoint, blue line: real value, green line: estimated value),

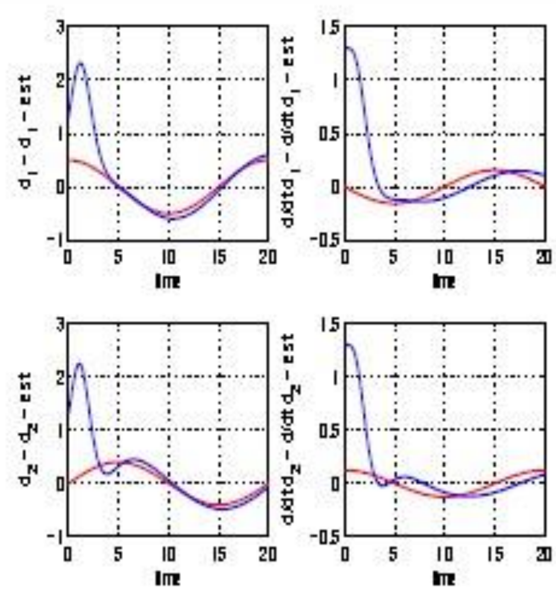


Fig. 2(b) estimation (blue line) of perturbation terms (red line) affecting the diesel engine

Example 1: Nonlinear control and state estimation using global linearization

7. Conclusions

• The **Diesel engine's model** does not admit static feedback linearization and this increases the degree of difficulty of the associated nonlinear control problem.

• To handle this, it has been proposed to apply **dynamic feedback linearization** which is based on extending the state-space description of the engine with the **inclusion of additional state variables representing the derivatives of the control inputs**.

• The extended state-space model of the turbocharged diesel engine satisfies **differential flatness properties** and can be finally transformed into **MIMO canonical (Brunovsky) form**.

• The latter description facilitates the design of a **state feedback controller** and assures that the elements of the state vector of the engine will converge asymptotically to the desirable setpoints.

• To compensate for **modeling errors and external disturbances** the Derivative-free nonlinear Kalman Filter has been used and redesigned as a **disturbance observer**.

• The filter consists of the **Kalman Filter recursion** on the linearized equivalent model of the Diesel engine model and of an **inverse transformation** based on differential flatness theory which enables to obtain estimates for the state variables of the initial nonlinear model.

Example 2: Nonlinear control and state estimation using approximate linearization

1 . Control of turbocharged ship diesel engines

● A **nonlinear optimal (H-infinity) control** approach is proposed for **turbocharged diesel engines** with potential use in **ship propulsion**.

● The **dynamic model of the diesel engine** undergoes **approximate linearization** round a temporary operating point.

● This is defined at each time instant by the present value of the system's state vector and the last sampled value of the control inputs vector.

● The **linearization is based on Taylor series expansion** and on the associated **Jacobians**. For the linearized model an **H-infinity feedback controller** is computed.

● The **controller's gain** is calculated by solving an **algebraic Riccati equation** at each iteration of the control method.

● The **asymptotic stability** of the control approach is proven through **Lyapunov analysis**.

● This assures that the state variables of the diesel engine will finally converge to the designated reference values.

● **Optimal functioning** of the diesel engine signifies **improved power**, **reduced polluting emissions** and **reduced fuel consumption**

Example 2: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the turbo-charged diesel engine

A. Nonlinear dynamics of the diesel engine

The **basic parameters of the Diesel engine** are:

(i) **Gas pressure in the intake manifold** p_1,
(ii) **Gas pressure in the exhaust manifold** p_2,
(iii)**Turbine power** P_t, (iv) **Compressor power** P_c
.

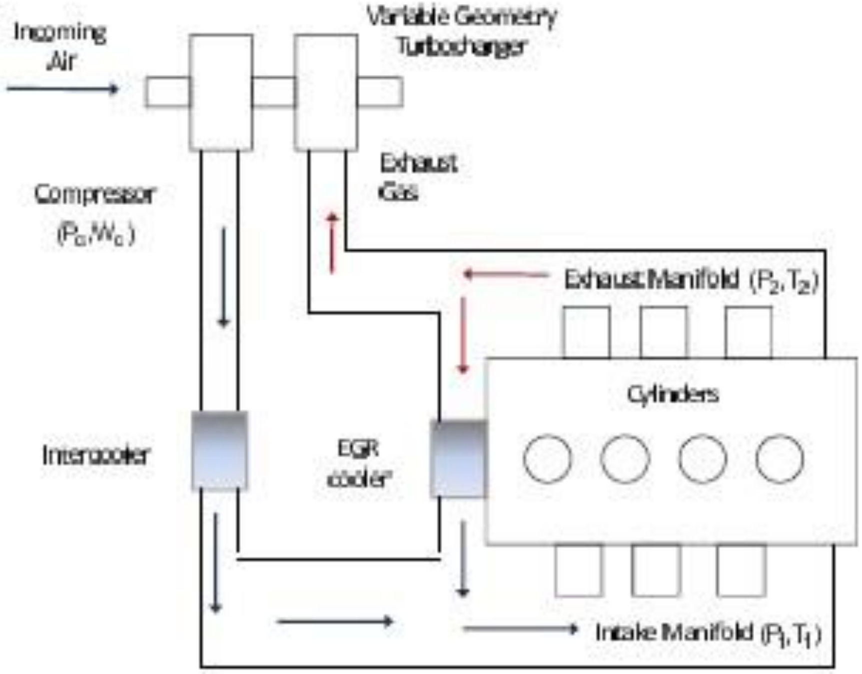


Fig. 1. Diagram of the turbocharged Diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the turbo-charged diesel engine

2.1. Nonlinear dynamics of the diesel engine

Additional variables of importance are

W_c which is **the compressor mass flow rate**, T_1 **the intake manifold temperature**, T_2 is the **exhaust manifold temperature**, W_t is the **turbine mass flow rate** W_{EGR} is **the exhaust gas recirculation flow rate**

The **basic relations of the diesel engine's dynamics** are:

$$\begin{gathered} \dot{p}_1 = K_1(W_c + u_1 - K_e p_1) + \frac{\dot{T}_1}{T_1} p_1 \\ \dot{p}_2 = K_2(K_e p_1 - u_1 - u_2) + \frac{\dot{T}_2}{T_2} p_2 \\ \dot{P}_c = \frac{1}{\tau}(\eta_m P_t - P_c) \end{gathered} \quad (1)$$

The **control inputs** to this model are the **exhaust gas recirculation (EGR) flow rate** $u_1 = W_{EGR}$ and **the turbine's mass flow rate** $u_2 = W_t$. Moreover, it holds that

$$W_c = P_c \frac{K_c}{p_1^{\mu} - 1}$$

$$P_t = K_t(1 - p_2^{-\mu}) u_2 \quad (2)$$

Example 2: Nonlinear control and state estimation using approximate linearization

2. Dynamic model of the turbo-charged diesel engine

2.1. Nonlinear dynamics of the diesel engine

The model is simplified by setting. $\dot{T}_1 = 0$ and $\dot{T}_2 = 0$. In such a case the associated **state-space equations** are given by

$$\begin{aligned} \dot{p}_1 &= K_1(W_c + u_1 - K_e p_1) \\ \dot{p}_2 &= K_2(K_e p_1 - u_1 - u_2) \\ \dot{P}_c &= \frac{1}{\tau}(\eta_m P_t - P_c) \end{aligned} \qquad (3)$$

The description of the **diesel engine in state-space form** is given by

$$\dot{x} = f(x) + g_a(x)u_1 + g_b(x)u_2$$

where

$$f(x) = \begin{pmatrix} K_1 K_c \frac{P_c}{p_1^{\mu}-1} - K_1 K_e p_1 \\ K_2 K_e p_1 \\ -\frac{P_c}{\tau} \end{pmatrix} \qquad g_a(x) = \begin{pmatrix} K_1 \\ -K_2 \\ 0 \end{pmatrix} \quad g_b(x) = \begin{pmatrix} 0 \\ -K_2 \\ K_o(1 - p_2{}^{-\mu}) \end{pmatrix} \qquad (4)$$

With respect to the control, the **variables of the output** are: (i) **the input manifold pressure** p_1 and (ii) the **compressor mass flow rate** W_c

$$y = \begin{pmatrix} p_1 \\ W_c \end{pmatrix} = \begin{pmatrix} p_1 \\ P_c \frac{K_c}{p_1^{\mu} - 1} \end{pmatrix} \qquad (5)$$

Example 2: Nonlinear control and state estimation using approximate linearization

2. Dynamic model of the turbo-charged diesel engine

2.2. Dynamic extension of the diesel engine's state-space description

Dynamic extension is performed which means that **the state vector** of the diesel engine is extended by **considering as additional state variables** specific **control inputs** $u_i,\ i=1,2$ and **their derivatives** $\dot{u}_i,\ i=1,2$.

The purpose of dynamic extension is to select feasible reference setpoints for the system's nonlinear optimal controller.

In the **state-space description that is obtained after dynamic extension**, one has that the **transformed control inputs**

$$v_1 = f_1(u_1, \dot{u}_1) \qquad v_2 = f_2(u_2, \dot{u}_2) \qquad (6)$$

are applied to the diesel engine's model. Equivalently, this means that the control inputs u_1, u_2 which are finally applied to the real system depend on v_1, v_2 through an **integration relation**, that is

$$u_1 = \int v_1 dt, \ u_2 = v_2 \qquad (7)$$

Example 2: Nonlinear control and state estimation using approximate linearization

2. Dynamic model of the turbo-charged diesel engine

2.2. Dynamic extension of the diesel engine's state-space description

The dynamical system of the diesel engine is written in an **extended form** using the variables

$$v_1 = \dot{u}_1 = \dot{z}, \quad v_2 = u_2$$

which means

$$u_1 = \int v_1 dt, \quad u_2 = v_2$$

Thus, using the previous **state-space description** of the system and by substituting

$$u_1 = z_1 \text{ and } \dot{z} = v_1 \qquad (8)$$

as **intermediate state variable** it holds

$$\begin{aligned} \dot{p}_1 &= K_1 K_c P_c p_1^{\mu} - 1 - K_1 K_e p_1 + K_1 z \\ \dot{p}_2 &= K_2 K_e p_1 - K_2 z - K_2 v_2 \\ \dot{P}_c &= -\frac{P_c}{\tau} + K_c(1 - p_2^{-\mu}) \\ \dot{z} &= v_1 \end{aligned} \qquad (9)$$

therefore, by defining the **state vector** $x = [x_1, x_2, x_3, x_4]^T = [p_1, p_2, P_c, z]^T$

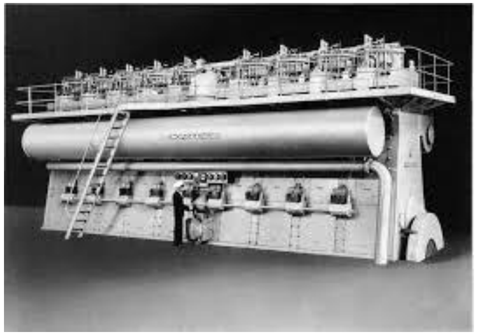
Example 2: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the turbo-charged diesel engine

2.2. Dynamic extension of the diesel engine's state-space description

therefore, by **defining the state vector** $x=[x_1,x_2,x_3,x_4]^T=[p_1,p_2,P_c,\dot{z}]^T$

$$
\begin{aligned}
\dot{x}_1 &= K_1K_c\frac{x_3}{x_1^{\mu}-1}-K_1K_ex_1+K_1x_4\\
\dot{x}_2 &= K_2K_ex_1-K_2x_4-K_2v_2\\
\dot{x}_3 &= -\frac{x_3}{\tau}+K_c(1-x_2{}^{-\mu})\\
\dot{x}_4 &= v_1
\end{aligned} \tag{10}
$$



Consequently, in **matrix form one has**

$$
\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix}=\begin{pmatrix}K_1K_c\frac{x_3}{x_1^{\mu}-1}-K_1K_ex_1+K_1x_4\\ K_2K_ex_1-K_2x_4\\ -\frac{x_3}{\tau}+K_c(1-x_2^{-\mu})\\ 0\end{pmatrix}+\begin{pmatrix}0\\0\\0\\1\end{pmatrix}v_1+\begin{pmatrix}0\\1\\0\\0\end{pmatrix}v_2 \tag{11}
$$

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the turbo-charged diesel engine

After **dynamic extension**, the state-space model of the diesel engine was brought to the form of (10) where the state variables of the model are

$$x_1 = p_1,\ x_2 = p_2,\ x_3 = P,\ x_4 = u_1$$

while the **control inputs** are. $v_1 = \dot{u}_1$ and $v_2 = u_2$.

The above state-space model of the diesel engine undergoes **approximate linearization** around the **temporary equilibrium** (x^*, u^*),

x^* is the **present value of the system's state vector**

$v^* = [v_1^*, v_2^*]$ is the **last value of the control input** that was exerted on the system.

The **state-space model of the diesel engine** can be also written in the matrix form

$$\dot{x} = Ax + Bu + \tilde{d} \qquad (12)$$

where matrices **A and B are described by the system's Jacobians**

$$A = \nabla_x\{f(x) + [g_1(x), g_2(x)]v\} \Rightarrow A = \nabla_x f(x)$$
$$B = \nabla_v\{f(x) + [g_1(x), g_2(x)]v\} \Rightarrow B = [g_1(x), g_2(x)] \qquad (13)$$

Thus about **matrices A and B of the linearized m**odel one has

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the turbo-charged diesel engine

The initial nonlinear system of the diesel engine is in the form

$$\dot{x} = f(x,u) \quad x \in R^n, \ u \in R^m$$

Linearization of the system is performed at each iteration of the control algorithm around its present operating point

$$(x^*, u^*) = (x(t), u(t - T_s)).$$

The **linearized equivalent of the system** is described by

$$\dot{x} = Ax + Bu + Ld \quad x \in R^n, \ u \in R^m, \ d \in R^q \qquad (14)$$

where matrices A and B are obtained from the **computation of the Jacobians**

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Big|_{(x^*, u^*)} \qquad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Big|_{(x^*, u^*)}$$

and vector d denotes **disturbance terms due to linearization errors**.

Example 2: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the turbo-charged diesel engine

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}$$

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For the **Jacobian matrix** $\nabla_x f$ it holds

1st row of the Jacobian matrix $\nabla_x f$: $\frac{\partial f_1}{\partial x_1} = K_1 K_e \frac{-x_3 \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2} - K_1 K_e$, $\frac{\partial f_1}{\partial x_2} = 0$, $\frac{\partial f_1}{\partial x_3} = K_1 K_e \frac{1}{x_1^{\mu}-1}$, $\frac{\partial f_1}{\partial x_4} = K_1$.

2nd row of the Jacobian matrix $\nabla_x f$: $\frac{\partial f_2}{\partial x_1} = K_2 K_e$, $\frac{\partial f_2}{\partial x_2} = 0$, $\frac{\partial f_2}{\partial x_3} = 0$, $\frac{\partial f_2}{\partial x_4} = -K_2$.

3rd row of the Jacobian matrix $\nabla_x f$: $\frac{\partial f_3}{\partial x_1} = 0$, $\frac{\partial f_3}{\partial x_2} = K_0 \mu x_2^{-\mu-1}$, $\frac{\partial f_3}{\partial x_3} = -\frac{1}{\tau}$, $\frac{\partial f_3}{\partial x_4} = 0$.

4th row of the Jacobian matrix $\nabla_x f$: $\frac{\partial f_4}{\partial x_1} = 0$, $\frac{\partial f_4}{\partial x_2} = 0$, $\frac{\partial f_4}{\partial x_3} = 0$, $\frac{\partial f_4}{\partial x_4} = 0$.

Example 2: Nonlinear control and state estimation using approximate linearization

4. Design of an H-infinity nonlinear feedback controller

The **state vector** of the turbocharged diesel engine is denoted as $x = [x_1, x_2, x_3, x_4]^T$

The **input vector** of the turbocharged diesel engine is denoted as $v = [v_1, v_2]^T$

After **linearization** round its current position, the diesel engine's dynamic **model** is written as

$$\dot{x} = Ax + Bu + d_1 \qquad (16)$$

Parameter d_1 stands for the **linearization error** in the diesel engine's dynamic model

The **reference setpoint** of the turbocharged diesel engine is denoted ,

$$x_d = [x_1^d, x_2^d, x_3^d, x_4^d]^T$$

Tracking of this reference setpoint is achieved after applying the control input u^*

At every time instant the control input u^* is assumed to differ from the control input u appearing in (16) by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \qquad (17)$$

Example 2: Nonlinear control and state estimation using approximate linearization

4. Design of an H-infinity nonlinear feedback controller

The dynamic model of the system of Eq. **16** can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad \textbf{(18)}$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad \textbf{(19)}$$

By subtracting Eq. **19** from Eq. **16** one has

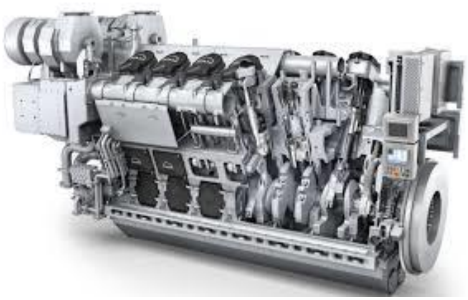
$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad \textbf{(20)}$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$ the **tracking error dynamics** becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad \textbf{(21)}$$

Example 2: Nonlinear control and state estimation using approximate linearization

4. Design of an H-infinity nonlinear feedback controller



The problem of **disturbance rejection** for the linearized model that is described by

$$\dot{x} = Ax + Bu + Ld$$
$$y = Cx$$

where $x \in R^n$, $u \in R^m$, $d \in R^q$ and $y \in R^p$ cannot be handled efficiently if the classical LQR control scheme is applied. This because of the existence of the perturbation term d.

In the $H\infty$ control approach, a **feedback control scheme** is designed for **trajectory tracking** by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner

The disturbances' effect are incorporated in the following **quadratic cost function**

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + \\ + r u^T(t)u(t) - \rho^2 d^T(t)d(t)]dt, \quad r, \rho > 0 \qquad (22)$$

Coefficient r determines **the penalization of the control input** and the weight coefficient ρ determines the **reward of the disturbances**' effects. It is assumed that

Example 2: Nonlinear control and state estimation using approximate linearization

4. Design of an H-infinity nonlinear feedback controller

Then, the **optimal feedback control** law is given by

$$u(t) = -Kx(t) \quad (23)$$

with

$$K = \frac{1}{r}B^T P$$

where P **is a positive semi-definite symmetric matrix** which is obtained from the solution of a **Riccati equation** of the form

$$A^T P + PA + Q - P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P = 0 \quad (24)$$

where Q is also a positive definite symmetric matrix.

The parameter ρ in Eq. (15), is an **indication of the closed-loop system robustness**. If the values of $\rho > 0$ are excessively decreased with respect to r, then the solution of the Riccati equation is no longer a positive definite matrix. Consequently, there is a lower bound ρ_{min} of for which the H-infinity control problem has a solution.

Example 2: Nonlinear control and state estimation using approximate linearization

4. Design of an H-infinity nonlinear feedback controller

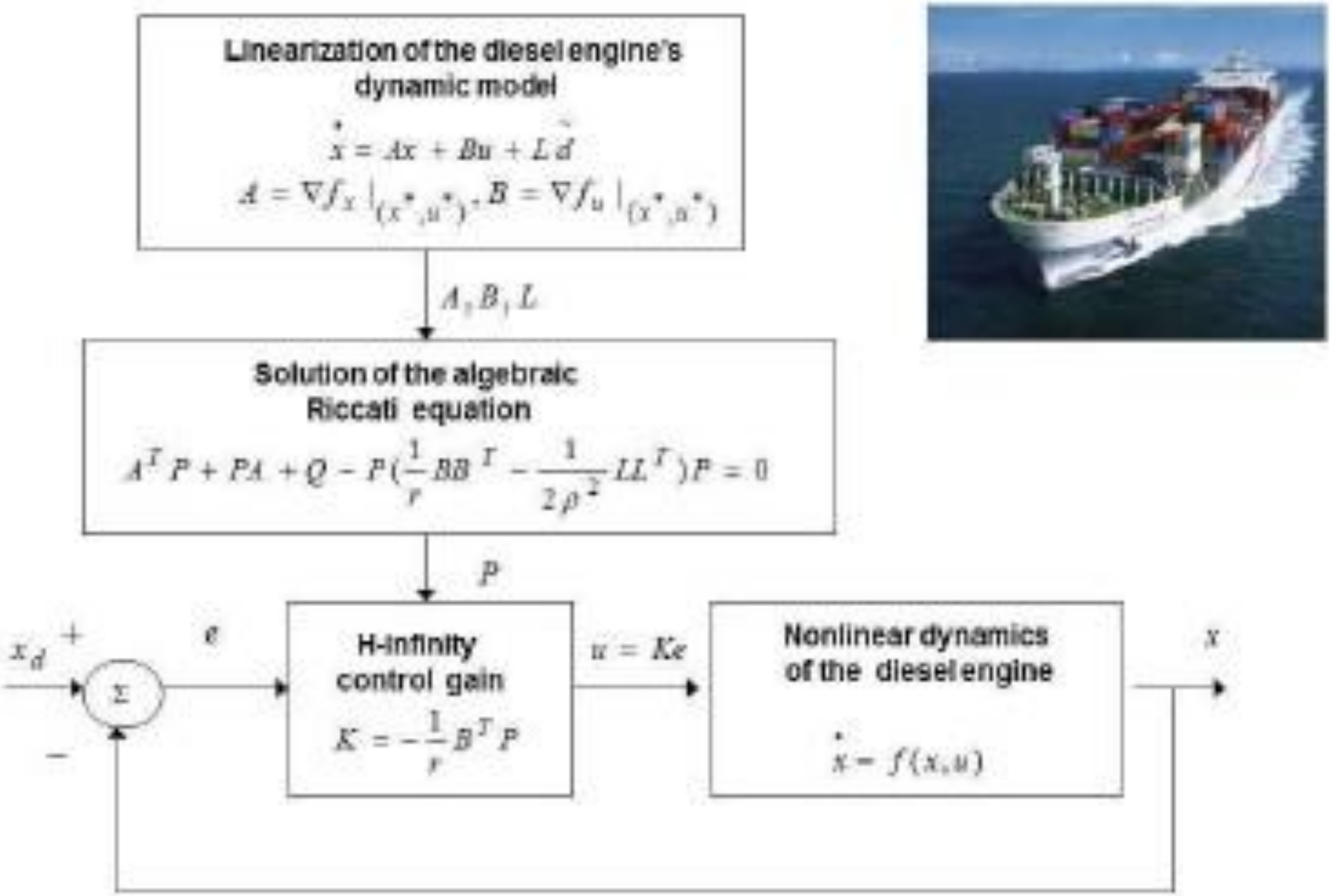


Fig. 2. Diagram of the control scheme for the turbocharged ship's diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

The **tracking error dynamics** for the diesel engine is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d}$$

where for the three-phase voltage source converter example $L \in R^4$ with I being the identity matrix. The following **Lyapunov function** is considered

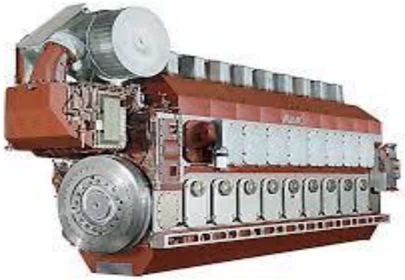
$$V = \frac{1}{2}e^T P e \quad (25)$$

with $e = x - x_d$ to be the tracking error

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}eP\dot{e} \Rightarrow$$

$$\dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$



$$\dot{V} = \frac{1}{2}e^T A^T Pe + \frac{1}{2}u^T B^T Pe + \frac{1}{2}\tilde{d}^T L^T Pe + \\ \frac{1}{2}e^T PAe + \frac{1}{2}e^T PBu + \frac{1}{2}e^T PL\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V}=\frac{1}{2}e^T(A^TP+PA)e+(\frac{1}{2}u^TB^TPe+\frac{1}{2}e^TPBu)+ \\ +(\frac{1}{2}\tilde{d}^TL^TPe+\frac{1}{2}e^TPL\tilde{d})$$

Assumption: For given positive definite matrix Q *and coefficients* r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP+PA=-Q+P(\frac{2}{r}BB^T-\frac{1}{\rho^2}LL^T)P \qquad 26$$

Moreover, the following **feedback control law** is applied to the system

$$u=-\frac{1}{r}B^TPe \qquad 27$$

By substituting Eq. 27 and Eq. 26 one obtains

$$\dot{V}=\frac{1}{2}e^T[-Q+P(\frac{2}{r}BB^T-\frac{1}{\rho^2}LL^T)P]e+ \\ +e^TPB(-\frac{1}{r}B^TPe)+e^TPL\tilde{d}\Rightarrow$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Continuing with computations one obtains

$$\dot{V} = -\frac{1}{2}e^T Qe + \frac{1}{r}e^T PBB^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \\ -\frac{1}{r}e^T PBB^T Pe + e^T PL\tilde{d}$$

which next gives

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe + e^T PL\tilde{d}$$

or equivalently

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe + \\ +\frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}^T L^T Pe$$

28

Lemma: The following inequality holds

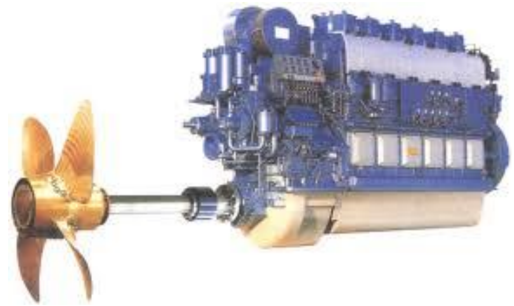
$$\frac{1}{2}e^T L\tilde{d} + \frac{1}{2}\tilde{d}L^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Proof : *The binomial* $(\rho a - \frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$



The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d} - \frac{1}{2\rho^2}e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (29)$$

Eq. (29) is substituted in Eq. (28) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (30)$$

Eq. (30) shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T ||\bar{d}||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||\bar{d}||^2 dt$$

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\bar{d}||^2 dt \leq M_d$$

then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded.

Moreover, V(T) is bounded and from the definition of the Lyapunov function V it becomes clear that **e(t) will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}.$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$\lim_{t \to \infty} e(t) = 0. \qquad (31)$$

This completes the stability proof.

Example 2: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

- Elaborating on the above, it can be noted that the proof of **global asymptotic stability** for the control loop of the turbocharged diesel engine is based on

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \qquad (30)$$

and on the application of Barbalat's Lemma. It uses the condition $\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d$

about the boundedness of the square of the aggregate disturbance and modelling error term $\tilde{d}$ that affects the model.

- However, the proof of global asymptotic stability is not restricted by this condition. By **selecting the attenuation coefficient** to be sufficiently small and in particular to satisfy

$$\rho^2 < ||e||_Q^2 / ||\tilde{d}||^2$$

one has that the first derivative of the **Lyapunov function is upper bounded by 0**.

- Therefore for the i-th time interval it is proven that the Lyapunov function defined in

$$V = \frac{1}{2}e^T P e \qquad (25)$$

is a decreasing one. This also assures the **Lyapunov function** of the system defined in will always have a **negative first-order derivative**.

Example 2: Nonlinear control and state estimation using approximate linearization

6. Robust state estimation with the H-infinity Kalman Filter

- The control loop has to be implemented with the use of information provided by a **small number of sensors** and by processing only a small **number of state variables**.

- To reconstruct the missing information about the state vector of the turbocharged diesel engine it is proposed to use a **filtering scheme** and based on it to apply **state estimation-based control**

- The recursion of the **H-infinity Kalman Filter**, for the model of the diesel engine, can be formulated in terms of a **measurement update** *and a* **time update part**

Measurement update:

$$\begin{aligned} D(k) &= [I - \theta W(k) P^-(k) + C^T(k) R(k)^{-1} C(k) P^-(k)]^{-1} \\ K(k) &= P^-(k) D(k) C^T(k) R(k)^{-1} \\ \hat{x}(k) &= \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)] \end{aligned} \quad (32)$$

Time update:

$$\begin{aligned} \hat{x}^-(k+1) &= A(k)x(k) + B(k)u(k) \\ P^-(k+1) &= A(k)P^-(k)D(k)A^T(k) + Q(k) \end{aligned} \quad (33)$$

where it is assumed that parameter θ is sufficiently small to assure that the following covariance matrix will be positive definite

$$P^-(k)^{-1} - \theta W(k) + C^T(k) R(k)^{-1} C(k)$$

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

- The performance of the proposed **nonlinear H-infinity control** for the **turbocharged diesel engine** has been evaluated through simulation experiments.
- The computation of the **feedback control gain** was based on the solution of the **algebraic Riccati equation** given in the related Riccati equation, through a procedure that was repeated at each iteration of the control method.

$$A^T P + PA = -Q + P\left(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T\right)P \qquad (26)$$

- It can be confirmed that **fast and accurate convergence** of the state variables of the diesel engine to the reference setpoints was achieved.
- Moreover, it can be seen that **the variation of the control inputs remained smooth** and within moderate ranges.
- Despite nonlinearities, the control method’s performance was very satisfactory and **precise tracking of the reference setpoints** was achieved.
- In the presented simulation experiments **state estimation-based control** has been implemented. Out of the 3 state variables of the turbocharged diesel only 1 was considered to be measurable.
- The only **measurable state variable** was the **gas pressure p_1 in the intake manifold**, The rest of the state variables, were indirectly estimated usingf the **H-infinity Kalman Filter.**

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

- The real value of each state variable has been plotted in blue, the estimated value has been plotted in green, while the associated reference setpoint has been plotted in red.

- It can be noticed that despite model uncertainty the H-infinity Kalman Filter **achieved accurate estimation of the real values of the state vector** elements.

- In this manner the **robustness of the state estimation-based H-infinity control scheme** was also improved

- Comparing to the control of diesel engines that can be based on global linearization methods **the following features can be attributed to the nonlinear H-infinity control scheme**

(i) it is **applied directly on the nonlinear dynamical model** of the turbocharged diesel engine and does not require the computation of diffeomorphisms (change of variables) that will bring the system into an equivalent linearized form

(ii) the computation of the feedback control signal **does not require inverse transformations thus avoiding the appearance of singularities**

(iii) the method retains the known advantages of linear optimal control, that is **accurate tracking of the reference setpoints** under **moderate variations of the control inputs**

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

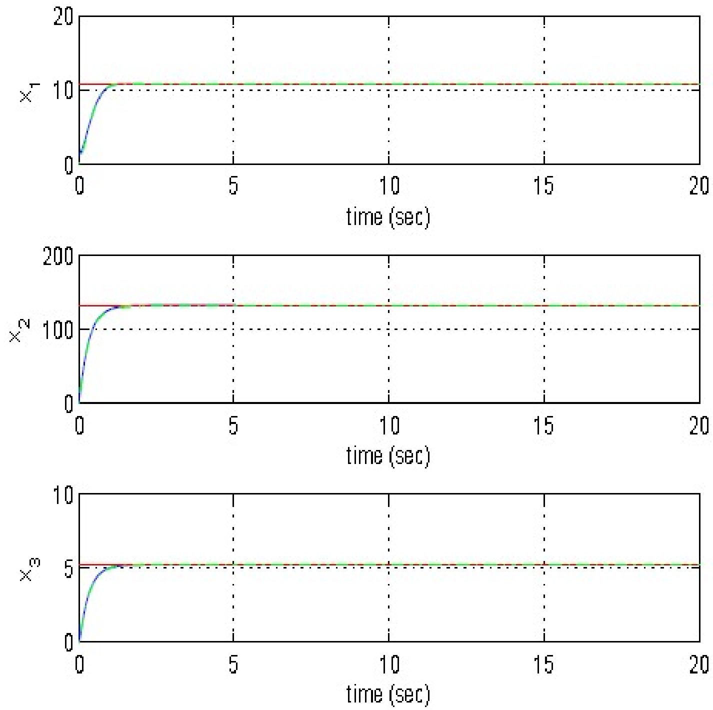


Fig. 3. (a) Tracking of set-point 1 (red lines) by states $x_i = 1,..,3$ (blue line: real values, green line: estimated values)

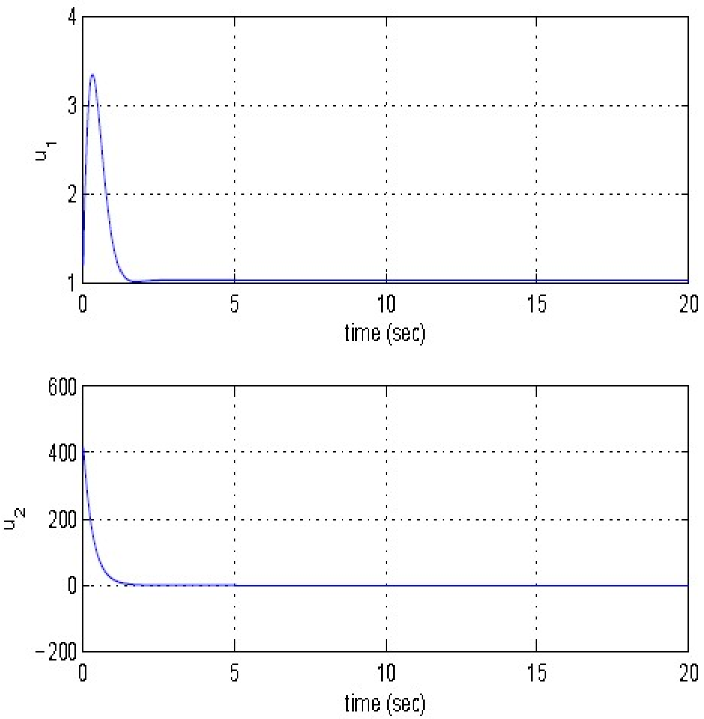


Fig. 3. (b) Control inputs $u_i = 1,2$ applied to the diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

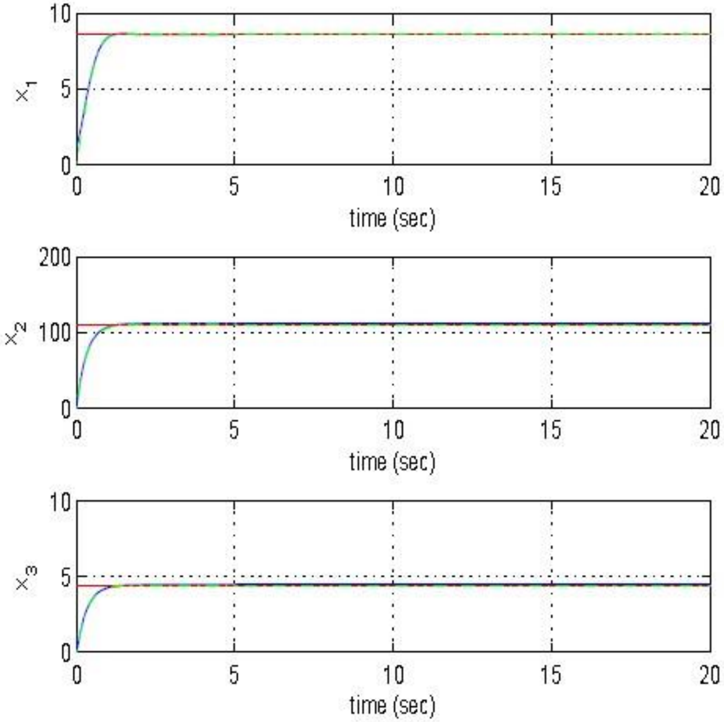


Fig. 4. (a) Tracking of set-point 2 (red lines) by states $x_i = 1,,..,3$ (blue line: real values, green line: estimated values)

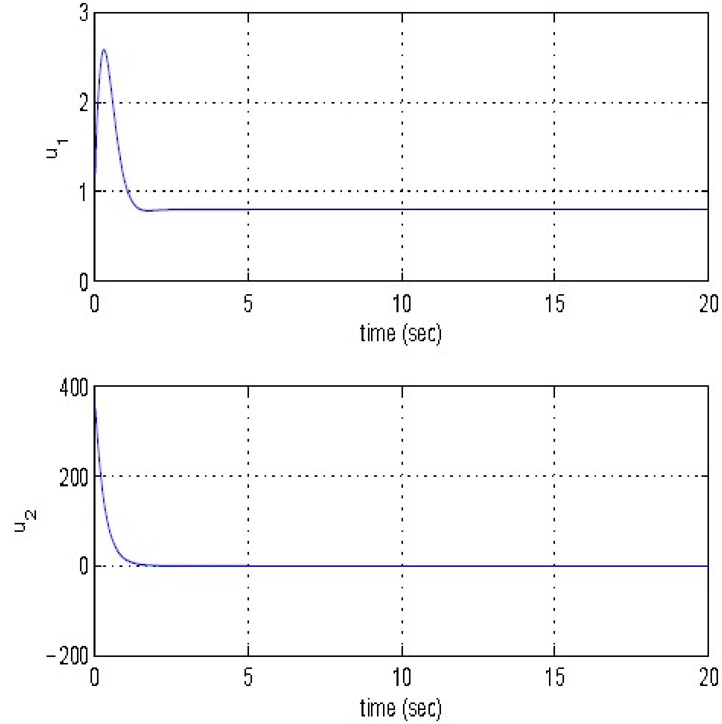


Fig. 4. (b) Control inputs $u_i = 1,2$ applied to the diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

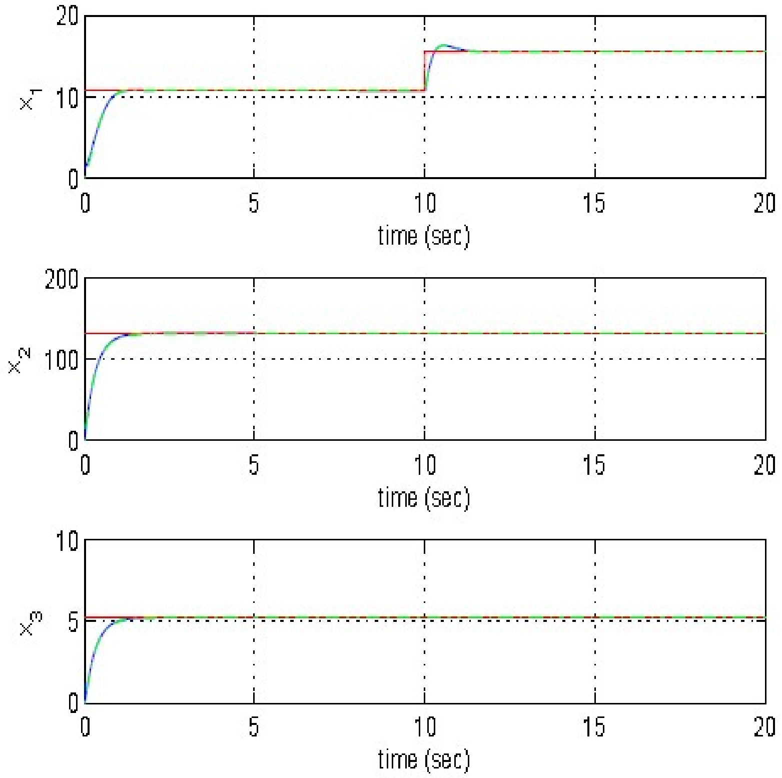


Fig. 5. (a) Tracking of set-point 3 (red lines) by states $x_i = 1,..,3$ (blue line: real values, green line: estimated values)

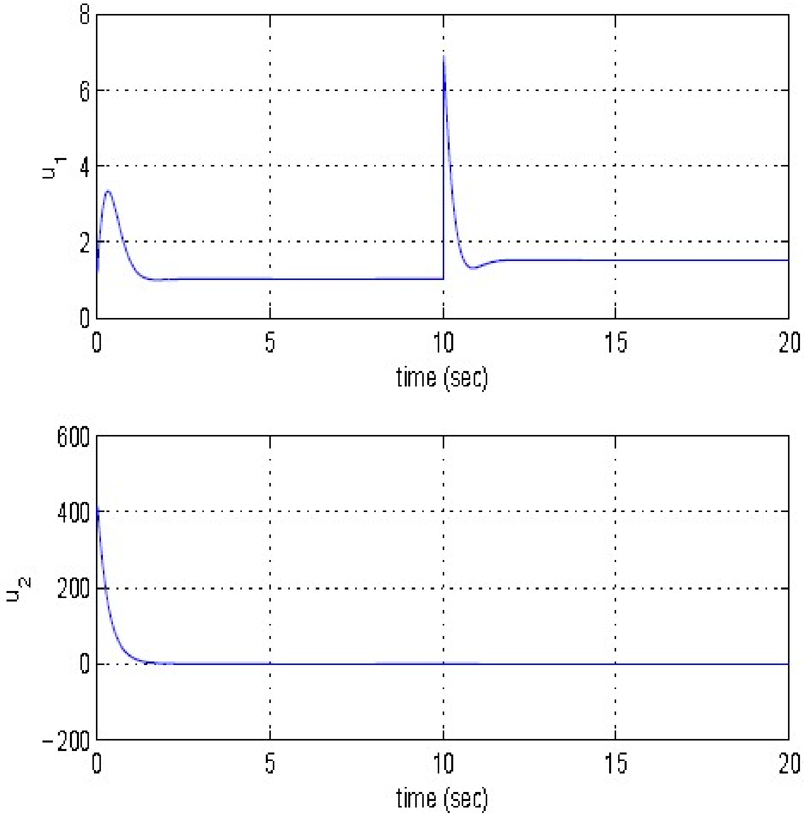


Fig. 5. (b) Control inputs $u_i = 1,2$ applied to the diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

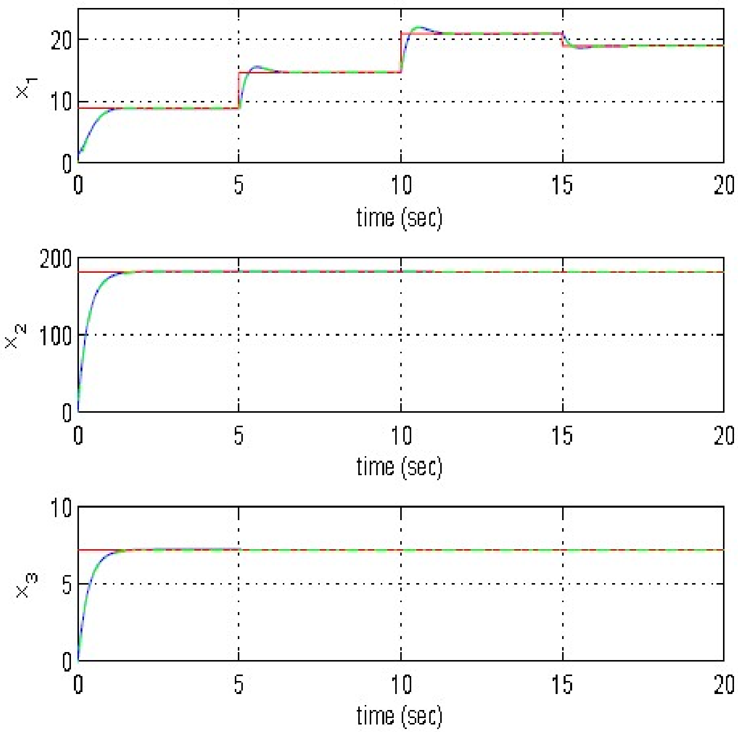


Fig. 6 (a) Tracking of set-point 4 (red lines) by states $x_i = 1,,..,3$ (blue line: real values, green line: estimated values)

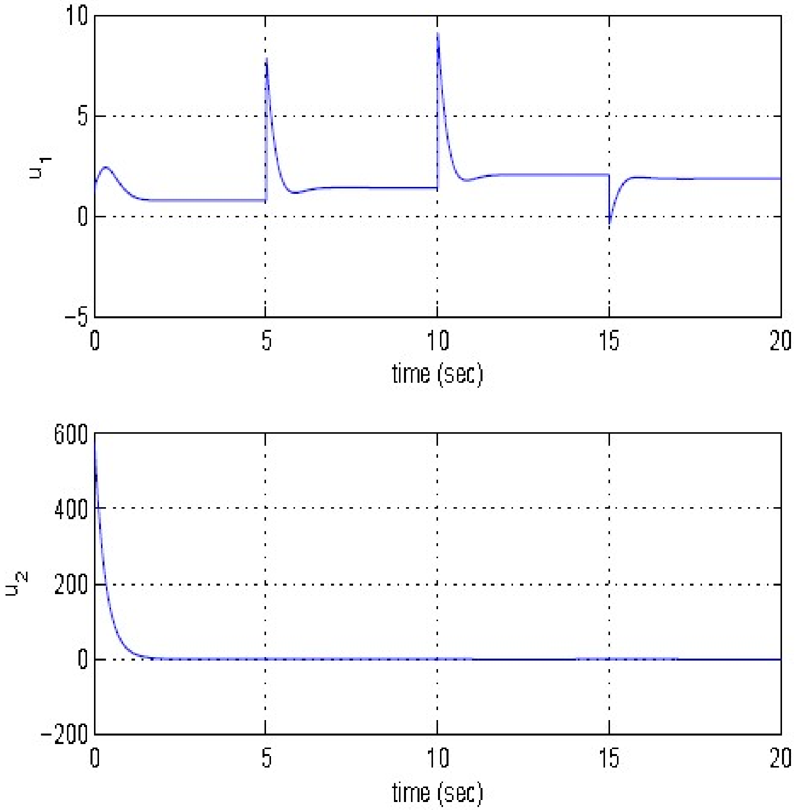
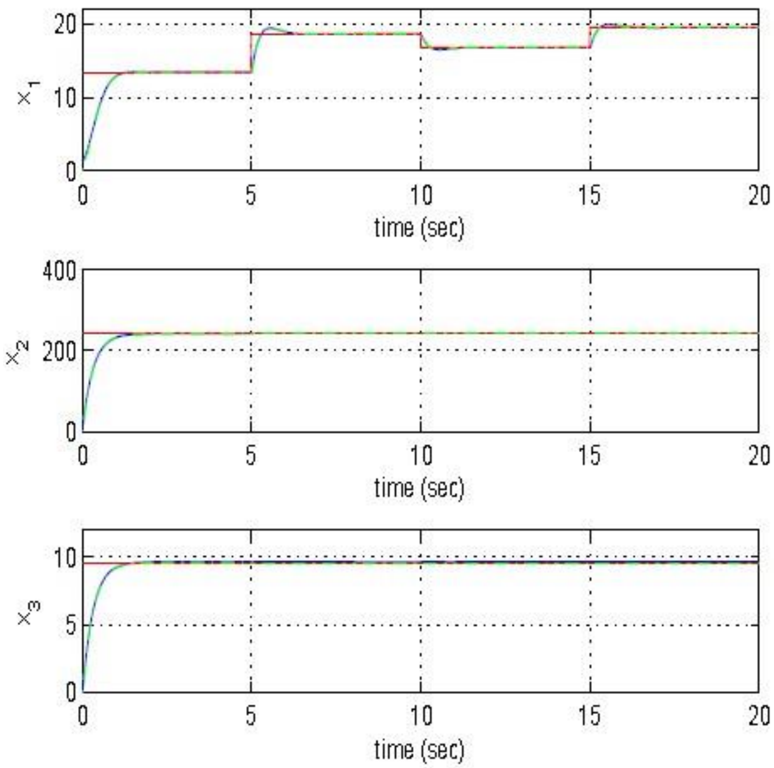


Fig. 6. (b) Control inputs $u_i = 1,2$ applied to the diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests



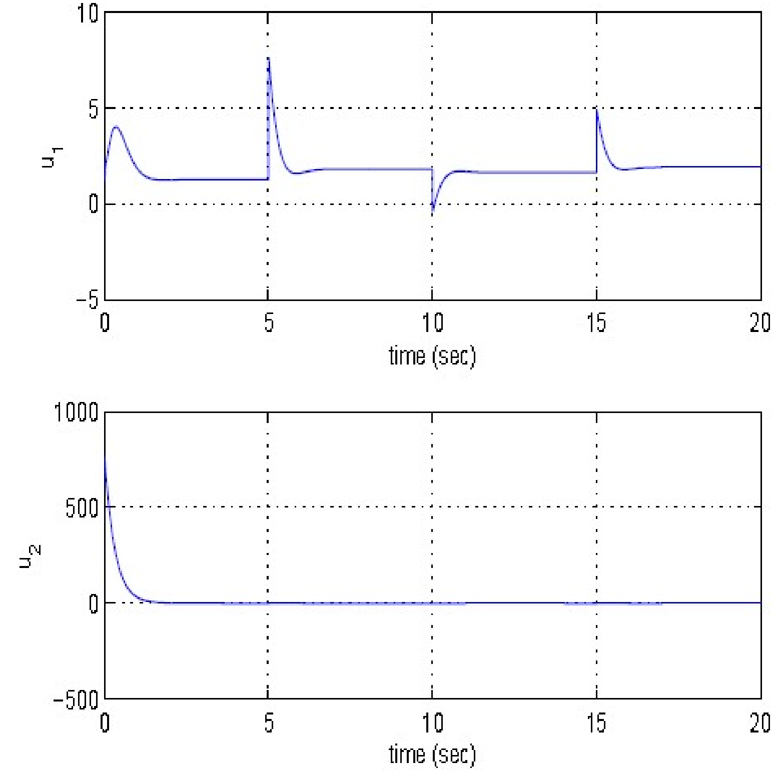


Fig. 7. (a) Tracking of set-point 5 (red lines) by states $x_i = 1,,..,3$ (blue line: real values, green line: estimated values)

Fig. 7. (b) Control inputs $u_i = 1,2$ applied to the diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

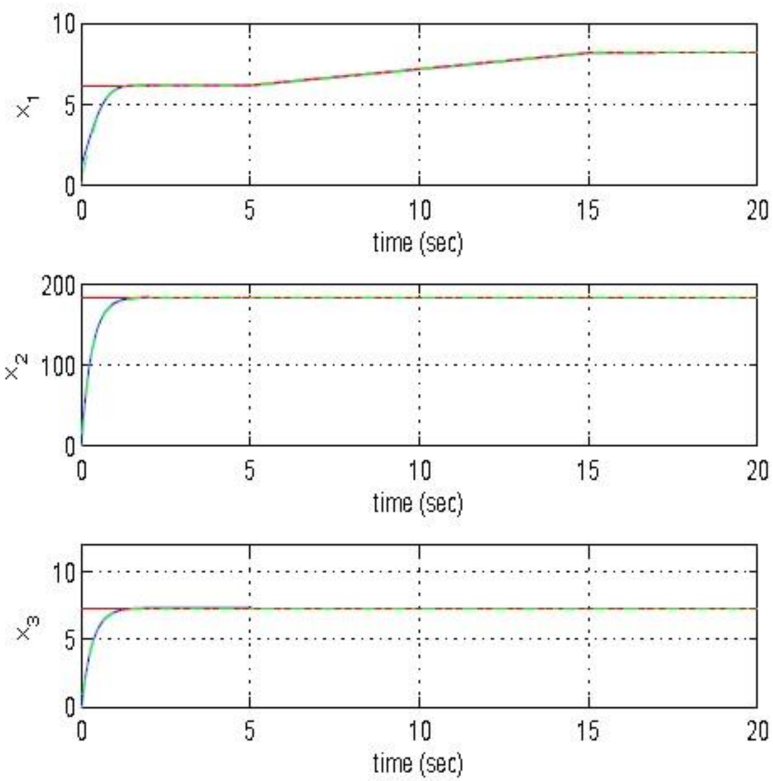


Fig. 8. (a) Tracking of set-point 6 (red lines) by states $x_i = 1,..,3$ (blue line: real values, green line: estimated values)

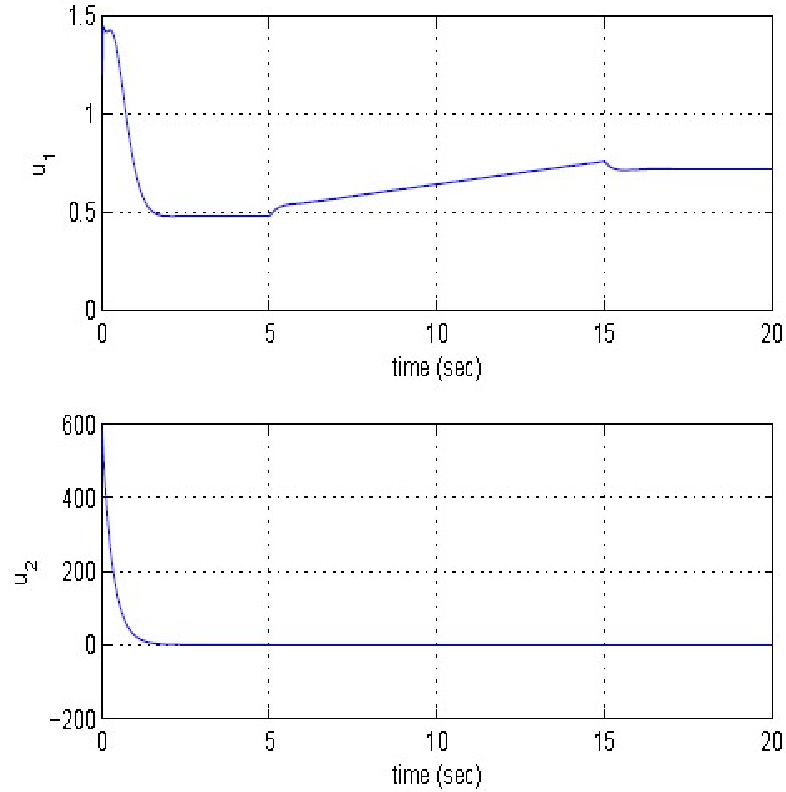


Fig. 8. (b) Control inputs $u_i = 1,2$ applied to the diesel engine

Example 2: Nonlinear control and state estimation using approximate linearization

8 . Conclusions

● In this article, a **nonlinear optimal (H-infinity) control method** has been developed for **turbocharged ship diesel engines**.

● First, a new **state-space description for the diesel engine** was obtained through **dynamic extension**, that is after **considering specific control inputs** and their **time derivatives** as **additional state variables for the system.**

● Next, the **extended state-space model** of the diesel engine was **subjected to approximate** linearization around a **temporary operating point** (equilibrium) that recomputed at each iteration of the control algorithm.

● This **equilibrium** consisted of the **present value of the engine's state vector** and of the **last ampled value of the control inputs** vector.

● **Linearization was performed through Taylor series expansion** and the computation of the associated Jacobian matrices.

● For the **linearized model of the diesel engine**, the H-infinity control problem (optimal control problem under uncertainty) was solved.

Example 2: Nonlinear control and state estimation using approximate linearization

8 . Conclusions

● The **feedback gain of the controller** was **repetitively computed at each iteration of the control algorithm** through the **solution of an algebraic Riccati equation**. The stability of the control scheme was proven through **Lyapunov analysis**.

● First, it was demonstrated that the control scheme satisfied the **H-infinity tracking performance criterion**.

● Moreover, under moderate conditions the **global asymptotic stability of the control loop** was proven.

● To implement feedback control without need to measure the entire state vector of the diesel engine, t**he H-infinity Kalman Filter** has been proposed.

● Despite its **computational simplicity** the proposed **nonlinear optimal control method** was confirmed to have an excellent performance.

Example 3: Nonlinear control and state estimation using approximate linearization

1 . Control of the electric ship propulsion system

• A **nonlinear optimal (H-infinity) control** method is proposed for **electric ships' propulsion** systems comprising an **induction motor**, a **drivetrain** and a **propeller**.

• The control method relies on **approximate linearization** of the propulsion system's dynamic model using **Taylor-series expansion** and on the computation of the state-space description's **Jacobian matrices**.

• The linearization takes place around a **temporary operating point** which is recomputed at each time-step of the control method.

• For the approximately linearized model of the ship's propulsion system, **an H-infinity (optimal) feedback controller** is developed.

• For the computation of the controller's gains an **algebraic Riccati equation** is solved at each iteration of the control algorithm.

• The **stability properties** of the control method are proven through **Lyapunov analysis**. The method is also robust to model uncertainties and external perturbations

• The proposed control method retains the advantages of linear optimal control, that is **fast and accurate tracking of reference setpoints** under **moderate variations of the control inputs**

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

The **propulsion system of the electric ship**, comprises a **three-phase induction motor**, a **drivetrain (gearbox),** and the **propeller** (Fig. 1).

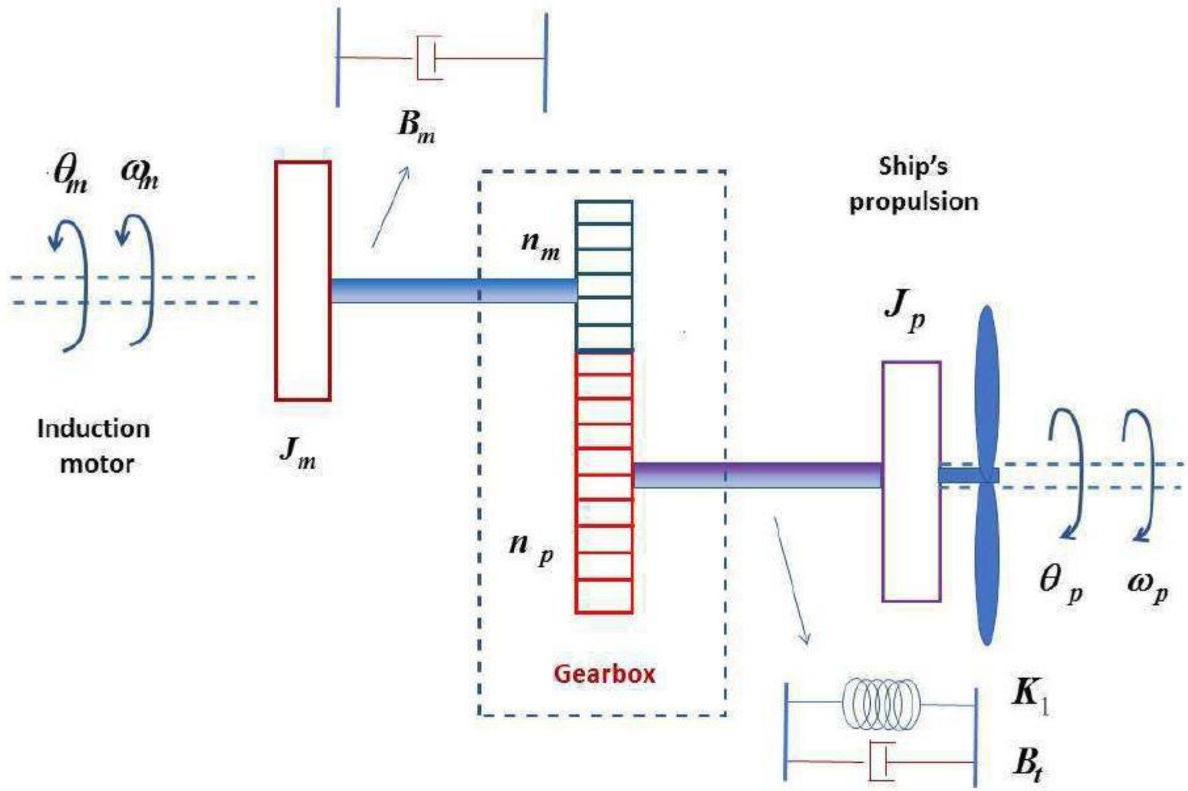


Fig. 1: Diagram of the electric ship propulsion system

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

The **rotational motion of the induction motor** is given by

$$J_m \frac{d\omega_m}{dt} = T_e - T_{hs} - B_m \omega_m \quad (1)$$

T_e is the electromagnetic torque that is developed by the motor,
T_{hs} is the torque developed by the shaft at the motor's side (high-speed),
$B_m\omega_m$ is a friction torque that opposes to the rotational motion of the rotor.

The **rotational motion of the ship's propeller** is given by

$$J_p \frac{d\omega_p}{dt} = T_{ls} - c_{ba} - B_p \omega_p \quad (2)$$

T_{ls} is the torque developed by the shaft at the propeller's side (low-speed),
c_{lba} expresses the mechanical torque that is applied on the propeller
$B_p\omega_p$ is a friction torque that opposes to the rotational motion of the propeller.

B_m is the damping coefficient in the turn motion of the motor,
θ_m is the rotational speed of the motor,
B_p is the damping coefficient in the turn motion of the propeller
θ_p is the rotational speed of the propeller.

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

The **rotational motion of the induction motor** is given by

$$J_m \frac{d\omega_m}{dt} = T_e - T_{hs} - B_m\omega_m \qquad (1)$$

T_e is the electromagnetic torque that is developed by the motor,
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$B_m\omega_m$ is a friction torque that opposes to the rotational motion of the rotor.

The **rotational motion of the ship's propeller** is given by

$$J_p \frac{d\omega_p}{dt} = T_{ls} - c_{ba} - B_p\omega_p \qquad (2)$$

T_{ls} is the torque developed by the shaft at the propeller's side (low-speed),
c_{lba} expresses the mechanical torque that is applied on the propeller
$B_p\omega_p$ is a friction torque that opposes to the rotational motion of the propeller.
B_m is the damping coefficient in the turn motion of the motor,
θm is the rotational speed of the motor,
B_p is the damping coefficient in the turn motion of the propeller
θ_p is the rotational speed of the propeller.

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

A drivetrain which comprises, a gear of n_m teeth at the side of the motor, and a gear of n_p teeth at the side of the propeller, is considered

The relation between the **torque at the motor's side T_{hs}** and the **torque at the propeller's side T_{ls}** is given by

$$\frac{T_{hs}}{T_{ls}} = \frac{\omega_p}{\omega_m} = \frac{n_m}{n_p} \Rightarrow$$

$$T_{hs} = \frac{n_m}{n_p} T_{ls} \qquad (3)$$

The **torque of the shaft is due to torsion** and at the propeller's side is given by

$$T_{ls} = K_1(\theta_p - \theta_m) + D_1(\omega_p - \omega_m) \qquad (4)$$

K_1 is an elasticity coefficient, D_1 is a damping coefficient.

Using that the value of D_1 is significantly smaller that the value of K_1 this result into the following **relation about the shaft's torque at the propeller's side**

$$T_{ls} = K_1(\theta_p - \theta_m) \qquad (5)$$

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

Consequently, the **shaft's torque at the side of the induction motor** is given by

$$T_{hs} = \frac{n_m}{n_p} K_1(\theta_p - \theta_m) \qquad (6)$$

Next, about the mechanical part of the transmission system one can define the **state variables**

$$x_1 = \theta_p, \quad x_2 = \omega_p, \; x_3 = \theta_m, \; x_4 = \omega_m,$$

and the **control input** $u_1 = c_{ba}$ which is related to the propeller's pitch angle

This results into the following **state-space description**:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{K_1}{J_p}(x_1 - x_3) - \frac{B_p}{J_p}x_2 - \frac{1}{J_p}u_1 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{n_m}{n_p}\frac{K_1}{J_m}(x_1 - x_3) - \frac{B_m}{J_n}x_4 \end{aligned} \qquad (7)$$

The dynamics of the electrical part of the propulsion system is dependent on the components of the **currents of the machine's stator** $[i_{sd}, i_{sq}]$ and on the **components of the rotor's magnetic flux** $[\psi_{rd}, \psi_{rq}]$, which are expressed in the asynchronously rotating dq reference frame.

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

By applying the **field orientation concept**, that is by selecting the turn speed of the **asynchronously rotating reference frame**, defined by the derivative of the angle of the rotor's magnetic field

$$\rho = tan^{-1}\left(\frac{\psi_{rb}}{\psi_{ra}}\right)$$

(i) the q-axis component of the magnetic flux vanishes, that is $\psi_{rq} = 0$

(ii) the d-axis component of the magnetic flux becomes equal to the magnitude of the flux vector, that is it becomes equal to $\psi_{rd} = ||\psi|| = \sqrt{\psi_{ra}^2 + \psi_{rb}^2}$,

with $[\psi_{ra}, \psi_{rb}]$ to denote the **magnetic flux coefficients in the non-rotating ab reference frame**. Thus the **electric part of the propulsion system** is

$$\frac{d\psi_{rd}}{dt} = -a\psi_{rd} + aMi_{sd} \quad (8)$$

$$\frac{di_{sd}}{dt} = -\gamma i_{sd} + a\beta\psi_{rd} + n_p\omega_m i_{sq} + \frac{aMi_{sq}^2}{\psi_{rd}} + \frac{1}{\sigma L_s}v_{sd} \quad (9)$$

$$\frac{di_{sq}}{dt} = -\gamma i_{sq} - \beta n_p\omega_m\psi_{rd} - n_p\omega_m i_{sd} - \frac{aMi_{sd}i_{sq}}{\psi_{rd}} + \frac{1}{\sigma L_s}v_{sq} \quad (10)$$

$$\frac{d\rho}{dt} = n_p\omega_m + \frac{aMi_{sq}}{\psi_{rd}} \quad (11)$$

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

The **coefficients of the electric part** of the model are defined as follows:

$$\sigma = \frac{1-M^2}{L_s L_r}, \; a = \frac{R_r}{L_r} \text{ and } \beta = \frac{M}{\sigma L_s L_r}$$

M is the mutual inductance between the stator and the rotor,
L_s is the stator's inductance,
L_r is the rotor's inductance

Taking into account the **field-orientation condition**, the **electromagnetic torque** that is developed by the induction motor is given by

$$T_e = \mu[i_{sq}\psi_{rd} - i_{sd}\psi_{rq}] \Rightarrow$$
$$T_e = \mu(i_{sq}\psi_{rd}) \qquad (12)$$

coefficient μ is dependent on the number of poles of the IM and is defined as $\mu = \frac{n_p M}{J_m L_r}$

The following **state variables are defined for the electric dynamics** of the propulsion system

$$x_5 = \psi_{rd}, \; x_6 = i_{sd}, \; x_7 = i_{sq} \text{ and } x_8 = \rho$$

while the associated **control inputs** are:

$$u_2 = v_{sd} \text{ and } \; v_3 = v_{sq}$$

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

one obtains the following **state-space description for the electrical part** of the propulsion system

$$\begin{aligned}\dot{x}_5 &= -ax_5 + aMx_6 \\ \dot{x}_6 &= -\gamma x_6 + a\beta x_5 + n_p x_4 x_7 + \frac{aMx_7^2}{x_5} + \frac{1}{\sigma L_s}u_2 \\ \dot{x}_7 &= -\gamma x_7 - \beta n_p x_4 x_5 - n_p x_4 x_6 - \frac{aMx_6 x_7}{x_5} + \frac{1}{\sigma L_s}u_3 \\ \dot{x}_8 &= n_p x_4 + \frac{aMx_7}{x_5}\end{aligned} \quad (13)$$

Moreover, using the previous notation of the state variables the electromagnetic torque which is provided by the motor is given by

$$T_e = \mu x_5 x_7 \quad (14)$$

By defining the **entire state vector of the propulsion system** as

$$x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8]^T$$

or $$x = [\theta_p, \omega_p, \theta_m, \omega_m, \psi_{rd}, i_{rd}, i_{sq}, \rho]^T$$

and the **entire control inputs vector** as

$$u = [u_1, u_2, u_3]^T \quad \text{or} \quad u = [c_{ba}, v_{sd}, v_{sq}]^T$$

one has the complete state-space model

Example 3: Nonlinear control and state estimation using approximate linearization

2 . Dynamic model of the electric ship propulsion system

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{K_1}{J_p}(x_1 - x_3) - \frac{B_p}{J_p}x_2 - \frac{1}{J_p}u_1 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \mu x_5 x_7 - \frac{1}{J_m}[\frac{n_m}{n_p}K_1(x_1 - x_3) - B_m x_4] \\
\dot{x}_5 &= -a x_5 + a M x_6 \\
\dot{x}_6 &= -\gamma x_6 + a\beta x_5 + n_p x_4 x_7 + \frac{aMx_7^2}{x_5} + \frac{1}{\sigma L_s}u_2 \\
\dot{x}_7 &= -\gamma x_7 - \beta n_p x_4 x_5 - n_p x_4 x_6 - \frac{aMx_6x_7}{x_5} + \frac{1}{\sigma L_s}u_3 \\
\dot{x}_8 &= n_p x_4 + \frac{aMx_7}{x_5}
\end{aligned} \quad (15)$$

In **vector fields form**, the previous state-space description can be written as:

$\dot{x} = f(x) + g(x)u$ with $x \in R^{8\times 1}$, $f(x) \in R^{8\times 1}$, $g(x) \in R^{8\times 3}$, $u \in R^{3\times 1}$. (16)

where

$$f(x) = \begin{pmatrix} x_2 \\ \frac{K_1}{J_p}(x_1 - x_3) - \frac{B_p}{J_p}x_2 \\ x_4 \\ \mu x_5 x_7 - \frac{1}{J_m}[\frac{n_m}{n_p}K_1(x_1 - x_3) - B_m x_4] \\ -a x_5 + a M x_6 \\ -\gamma x_6 + a\beta x_5 + n_p x_4 x_7 + \frac{aMx_7^2}{x_5} \\ -\gamma x_7 - \beta n_p x_4 x_5 - n_p x_4 x_6 - \frac{aMx_6x_7}{x_5} \\ n_p x_4 + \frac{aMx_7}{x_5} \end{pmatrix} \quad g(x) = \begin{pmatrix} 0 & 0 & 0 \\ -\frac{1}{J_p} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{\sigma L_s} & 0 \\ 0 & 0 & \frac{1}{\sigma L_s} \\ 0 & 0 & 0 \end{pmatrix}$$

Example 3: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the electric ship propulsion system

The **state-space model of the electric ship propulsion system** undergoes **approximate linearization** around the **temporary operating point** (equilibrium) (x*, u*), where

x* is the **present value of the system's state vector** and
u* is **the last sampled value of the control inputs vector**

For the **linearized state-space model** of the system it holds that

$$\dot{x} = Ax + Bu + \tilde{d} \quad (17)$$

where $\tilde{d}$ is **the cumulative disturbance vector** due to approximate linearization and **truncation of higher-order terms in the Taylor series expansion**, and

$$A = \nabla_x[f(x) + g(x)u] \mid_{(x^*,u^*)} \Rightarrow A = \nabla_x[f(x)] \mid_{(x^*,u^*)} \quad (18)$$

$$B = \nabla_u[f(x) + g(x)u] \mid_{(x^*,u^*)} \Rightarrow B = g(x) \mid_{(x^*,u^*)} \quad (19)$$

About the **Jacobian matrix** $\nabla_x[f(x)] \mid_{(x^*,u^*)}$ one has

First row of the **Jacobian matrix** $\nabla_x[f(x)] \mid_{(x^*,u^*)}$

$$\frac{\partial f_1}{\partial x_1} = 0, \; \frac{\partial f_1}{\partial x_2} = 1, \; \frac{\partial f_1}{\partial x_3} = 0, \; \frac{\partial f_1}{\partial x_4} = 0, \quad \frac{\partial f_1}{\partial x_5} = 0, \; \frac{\partial f_1}{\partial x_6} = 0, \; \frac{\partial f_1}{\partial x_7} = 0, \; \frac{\partial f_1}{\partial x_8} = 0.$$

Example 3: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the electric ship propulsion system

Second row of the **Jacobian matrix** $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_2}{\partial x_1}=\frac{K_1}{J_p},\ \frac{\partial f_2}{\partial x_2}=-\frac{B_p}{J_p},\ \frac{\partial f_2}{\partial x_3}=0,\qquad \frac{\partial f_2}{\partial x_4}=0,\ \frac{\partial f_2}{\partial x_5}=0,\ \frac{\partial f_2}{\partial x_6}=0,\ \frac{\partial f_2}{\partial x_7}=0,\ \frac{\partial f_2}{\partial x_8}=0.$$

Third row of the **Jacobian matrix** $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_3}{\partial x_1}=0,\ \frac{\partial f_3}{\partial x_2}=0,\ \frac{\partial f_3}{\partial x_3}=0,\ \frac{\partial f_3}{\partial x_4}=1,\ \frac{\partial f_3}{\partial x_5}=0,\ \frac{\partial f_3}{\partial x_6}=0,\ \frac{\partial f_3}{\partial x_7}=0,\ \frac{\partial f_3}{\partial x_8}=0.$$

Fourth row of the **Jacobian matrix** $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_4}{\partial x_1}=-\frac{1}{J_m}\frac{n_m}{n_p}K_1,\ \frac{\partial f_4}{\partial x_2}=0,$$

$$\frac{\partial f_4}{\partial x_3}=\frac{1}{J_m}\frac{n_m}{n_p}K_1,\ \frac{\partial f_4}{\partial x_4}=\frac{B_m}{J_m},\ \frac{\partial f_4}{\partial x_5}=\mu x_7,\ \frac{\partial f_4}{\partial x_6}=0,\ \frac{\partial f_4}{\partial x_7}=\mu x_5,\ \frac{\partial f_4}{\partial x_8}=0.$$

Fifth row of the **Jacobian matrix** $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_5}{\partial x_1}=0,\ \frac{\partial f_5}{\partial x_2}=0,\ \frac{\partial f_5}{\partial x_3}=0,\ \frac{\partial f_5}{\partial x_4}=0,\ \frac{\partial f_5}{\partial x_5}=-a,\ \frac{\partial f_5}{\partial x_6}=-aM,\ \frac{\partial f_5}{\partial x_7}=0,\ \frac{\partial f_5}{\partial x_8}=0.$$

Sixth row of the **Jacobian matrix** $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_6}{\partial x_1}=0,\ \frac{\partial f_6}{\partial x_2}=0,\ \frac{\partial f_6}{\partial x_3}=0,$$

$$\frac{\partial f_6}{\partial x_4}=n_p x_7,\ \frac{\partial f_6}{\partial x_5}=\frac{a\beta-aMx_7^2}{x_5^2},\ \frac{\partial f_6}{\partial x_6}=-\gamma,\ \frac{\partial f_6}{\partial x_7}=\frac{n_p x_4+2aMx_7}{x_5},\ \frac{\partial f_6}{\partial x_8}=0.$$

Example 3: Nonlinear control and state estimation using approximate linearization

3. Approximate linearization of the electric ship propulsion system

Seventh row of the **Jacobian matrix** $\nabla_x [f(x)] \,|_{(x^*,u^*)}$

$$\frac{\partial f_7}{\partial x_1} = 0, \ \frac{\partial f_7}{\partial x_2} = 0, \ \frac{\partial f_7}{\partial x_3} = 0,$$

$$\frac{\partial f_7}{\partial x_4} = -\beta n_p x_5 + n_p x_6, \ \frac{\partial f_7}{\partial x_5} = -\frac{\beta n_p x_4 + aMx_6x_7}{x_5^2}, \ \frac{\partial f_7}{\partial x_6} = n_p x_4 - \frac{aMx_7}{x_5}, \ \frac{\partial f_7}{\partial x_7} = -\gamma - \frac{aMx_6}{x_5}, \ \frac{\partial f_7}{\partial x_8} = 0.$$

Eigth row of the **Jacobian matrix** $\nabla_x [f(x)] \,|_{(x^*,u^*)}$

$$\frac{\partial f_8}{\partial x_1} = 0, \ \frac{\partial f_8}{\partial x_2} = 0, \ \frac{\partial f_8}{\partial x_3} = 0, \ \frac{\partial f_8}{\partial x_4} = n_p, \ \frac{\partial f_8}{\partial x_5} = -\frac{aMx_7}{x_5^2}, \ \frac{\partial f_8}{\partial x_6} = 0, \ \frac{\partial f_8}{\partial x_7} = \frac{aM}{x_5}, \ \frac{\partial f_8}{\partial x_8} = 0$$

Thus, matrices A ad B of the linearized model of the electric propulsion system are given by

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{K_1}{J_p} & -\frac{B_p}{J_p} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{J_m}\frac{n_m}{n_p}K_1 & 0 & \frac{1}{J_m}\frac{n_m}{n_p}K_1 & \frac{B_m}{J_m} & \mu x_7 & 0 & \mu x_5 & 0 \\ 0 & 0 & 0 & 0 & -a & -aM & 0 & 0 \\ 0 & 0 & 0 & \tilde{n}_p x_7 & \frac{a\beta - aMx_7^2}{x_5^2} & -\gamma & \frac{\tilde{n}_p x_4 + 2aMx_7}{x_5} & 0 \\ 0 & 0 & 0 & -\beta \tilde{n}_p x_5 + n_p x_6 & -\frac{\beta \tilde{n}_p x_4 + aMx_6x_7}{x_5^2} & \tilde{n}_p x_4 - \frac{aMx_7}{x_5} & -\gamma - \frac{aMx_6}{x_5} & 0 \\ 0 & 0 & 0 & \tilde{n}_p & -\frac{aMx_7}{x_5^2} & 0 & \frac{aM}{x_5} & 0 \end{pmatrix}$$

$B = g(x)$

Example 3: Nonlinear control and state estimation using approximate linearization

4. Design of the H-infinity feedback controller

The **state vector notation** *x* is used for the model of Eq. (17)

At every time instant the control input u^* is assumed to differ from the control input u appearing above by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \quad (18)$$

The **dynamics of the system** of Eq. (16) can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad (19)$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad (20)$$

By subtracting Eq. (18) from Eq. (20) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad (21)$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$ the **tracking error dynamics** becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad (22)$$

Example 3: Nonlinear control and state estimation using approximate linearization

4. Design of the H-infinity feedback controller

The initial model of the electric ship propulsion system assumed to be in the form

$$\dot{x} = f(x, u) \quad x \in R^n, \ u \in R^m$$

Linearization of the system is performed at each iteration of the control algorithm round its present operating point

$$(x^*, u^*) = (x(t), u(t - T_s)).$$

The **linearized equivalent of the system** is described by

$$\dot{x} = Ax + Bu + Ld \quad x \in R^n, \ u \in R^m, \ d \in R^q$$

where matrices A and B are obtained from te **computation of the Jacobians**

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$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Big|_{(x^*, u^*)} \qquad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Big|_{(x^*, u^*)}$$

and vector d denotes disturbance terms due to linearization errors.

The problem of **disturbance rejection** for the linearized model that is described by

$$\dot{x} = Ax + Bu + Ld$$
$$y = Cx$$

Example 3: Nonlinear control and state estimation using approximate linearization

4. Design of the H-infinity feedback controller

where $x \in R^n$, $u \in R^m$, $d \in R^q$ and $y \in R^p$,cannot be handled efficiently if the classical LQR control scheme is applied. This because of the existence of the perturbation term d.

In the $H\infty$ control approach, a **feedback control scheme** is designed for **setpoints tracking** by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner

The disturbances' effect are incorporated in the following **quadratic cost function**

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2 \tilde{d}^T(t)\tilde{d}(t)]dt, \quad r, \rho > 0 \tag{24}$$

The coefficient r determines **the penalization of the control input** and the weight coefficient ρ determines the **reward of the disturbances**' effects. It is assumed that

Then, the **optimal feedback control** law is $u(t) = -Kx(t)$ vith $K = \frac{1}{r}B^T P$

where P **is a positive semi-definite symmetric matrix** which is obtained from the solution of a **Riccati equation** of the form

$$A^T P + PA + Q - P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P = 0 \tag{25}$$

where Q is also a positive definite symmetric matrix.

Parameter ρ in Eq. (25), is an **indication of the closed-loop system robustness**. If the values of ρ> 0 are excessively decreased with respect to r, then the solution of the **Riccati equation** is no longer a **positive definite matrix**. Consequently, there is a lower bound ρ_{min} of for which the H-infinity control problem has a solution.

Example 3: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

The **tracking error dynamics** for the **electric ship propulsion system** is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \qquad (26)$$

where in the electric ship propulsion system $L = I \in I^{8x8}$ with I being the identity matrix. The following **Lyapunov function** is considered

$$V = \frac{1}{2}e^T P e \qquad (27)$$

where $e = x - x_d$ is the **tracking error**. By differentiating with respect to time one obtains

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow$$

$$\dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}e^T A^T P e + \frac{1}{2}u^T B^T P e + \frac{1}{2}\tilde{d}^T L^T P e + \\ \frac{1}{2}e^T P A e + \frac{1}{2}e^T P B u + \frac{1}{2}e^T P L\tilde{d}$$

Example 3: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(A^TP + PA)e + (\frac{1}{2}u^TB^TPe + \frac{1}{2}e^TPBu) + \\ +(\frac{1}{2}\tilde{d}^TL^TPe + \frac{1}{2}e^TPL\tilde{d})$$

Assumption: For given positive definite matrix Q *and coefficients* r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \quad (28)$$

Moreover, the following **feedback control law** is applied to the **electric ship propulsion system**

$$u = -\frac{1}{r}B^TPe \quad (29)$$

By substituting Eq. 28 and Eq. 29 one obtains

$$\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P]e + \\ +e^TPB(-\frac{1}{r}B^TPe) + e^TPL\tilde{d} \Rightarrow$$

Example 3: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Continuing with computations one obtains

$$\dot{V} = -\frac{1}{2}e^TQe + \frac{1}{r}e^TPBB^TPe - \frac{1}{2\rho^2}e^TPLL^TPe \\ -\frac{1}{r}e^TPBB^TPe + e^TPL\tilde{d}$$

which next gives

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe + e^TPL\tilde{d}$$

or equivalently

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe + \\ +\frac{1}{2}e^TPL\tilde{d} + \frac{1}{2}\tilde{d}^TL^TPe \quad (30)$$

Lemma: The following inequality holds

$$\frac{1}{2}e^TL\tilde{d} + \frac{1}{2}\tilde{d}L^TPe - \frac{1}{2\rho^2}e^TPLL^TPe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d}$$

Example 3: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Proof : *The binomial* $(\rho a - \frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d} - \frac{1}{2\rho^2}e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (31)$$

Eq. (31) is substituted in Eq. (30) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (32)$$

Eq. (32) shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T ||\tilde{d}||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||\tilde{d}||^2 dt$$

Example 3: Nonlinear control and state estimation using approximate linearization

5. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d$$

then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded.

Moreover, V(T) is bounded and from the definition of the Lyapunov function V it becomes clear that **e(t) will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}.$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$lim_{t \to \infty} e(t) = 0.$$

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Example 3: Nonlinear control and state estimation using approximate linearization

6. State estimation with the H-infinity Kalman Filter

The **H-infinity KF** is an optimal **state estimator under model uncertainty** and perturbations and thus its use under the variable operating conditions of the electric hop propulsion system is advantageous

The H-infinity KF is addressed to linear systems and to use it in the model of the electric ship propulsion, the previously analyzed **approximate linearization**. was applied

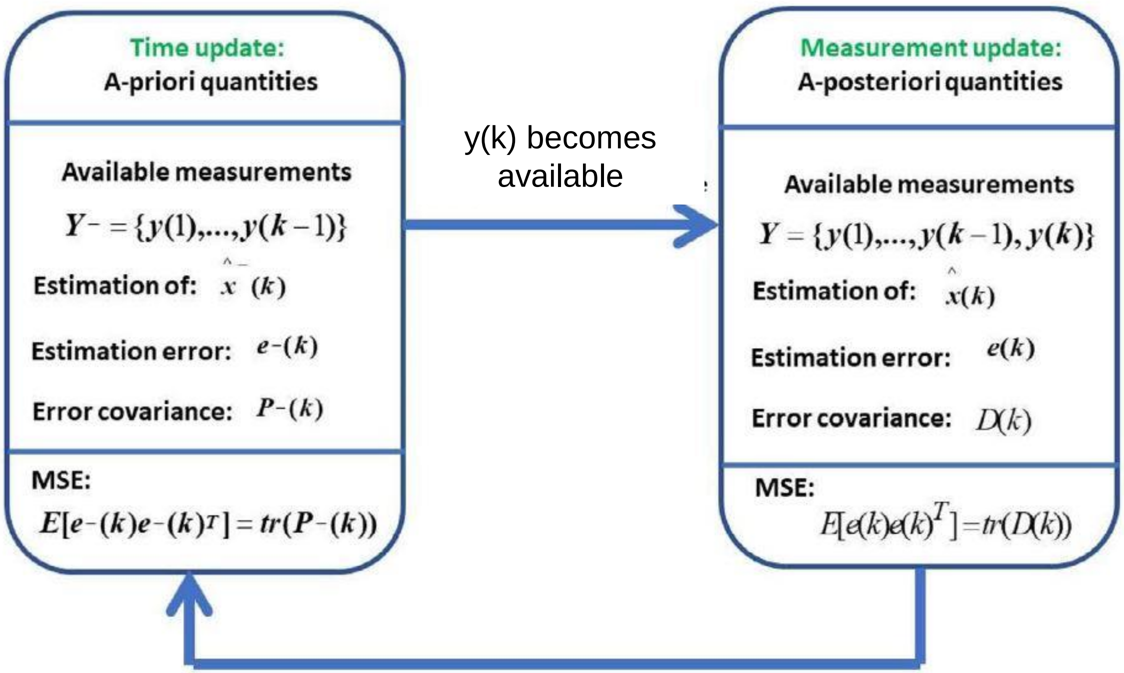


Fig. 2 Diagram of the H-infinity Kalman Filter comprising a time-update part and a measurement update part

Example 3: Nonlinear control and state estimation using approximate linearization

6. State estimation with the H-infinity Kalman Filter

● The **recursion of the H-infinity Kalman Filter**, for the **electric ship propulsion system**, can be formulated in terms of a measurement update and a time update part

Measurement update

$$\begin{aligned} D(k) &= [I - \theta W(k)P^{-}(k) + C^{T}(k)R(k)^{-1}C(k)P^{-}(k)]^{-1} \\ K(k) &= P^{-}(k)D(k)C^{T}(k)R(k)^{-1} \\ \hat{x}(k) &= \hat{x}^{-}(k) + K(k)[y(k) - C\hat{x}^{-}(k)] \end{aligned} \tag{20}$$

Time update

$$\begin{aligned} \hat{x}^{-}(k+1) &= A(k)x(k) + B(k)u(k) \\ P^{-}(k+1) &= A(k)P^{-}(k)D(k)A^{T}(k) + Q(k) \end{aligned} \tag{21}$$

where it is assumed that parameter θ is sufficiently small to assure that **matrix**

$$P^{-1}(k) - \theta W(k) + C^{T}(k)R(k)^{-1}C(k)$$

is **positive definite**

● By dynamically updating the elements of the process noise covariance matrix Q and of the measurement noise covariance matrix R the functioning of the filter under variable noise levels is ensured

Example 3: Nonlinear control and state estimation using approximate linearization

6. State estimation with the H-infinity Kalman Filter

The **H-infinity Kalman Filter exhibits advantages** against other nonlinear filters

EKF is not robust enough against linearization errors and measurement noise.
UKF methods are not of proven convergence and stability.
PF demands high computation power and has slow convergence

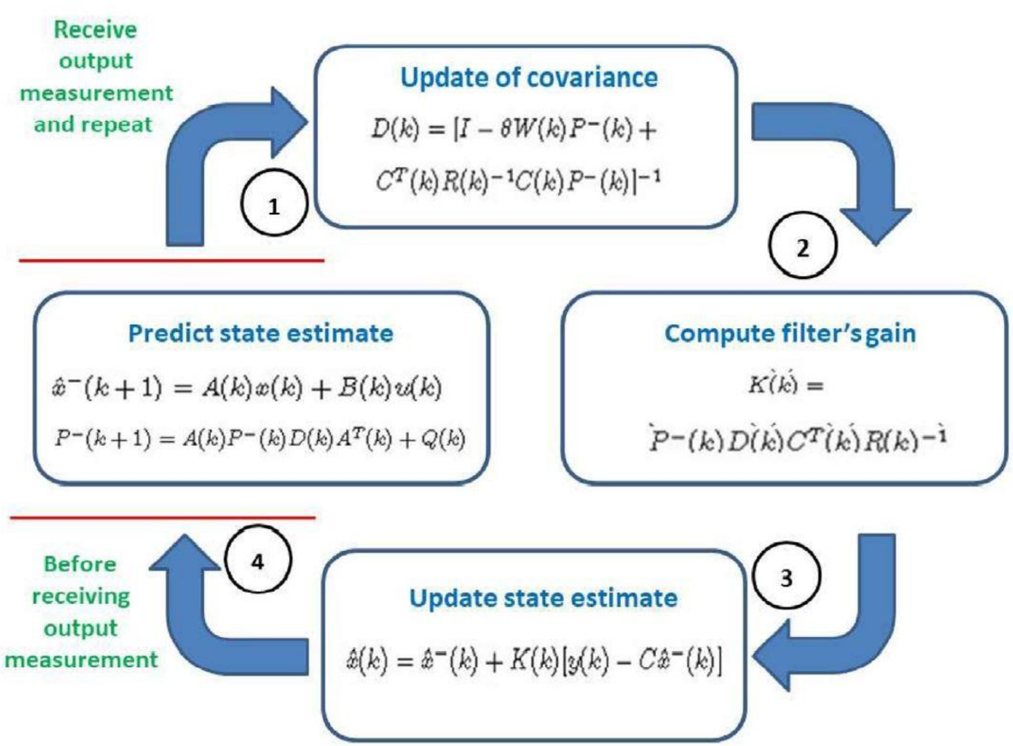


Fig. 3 The sequence of computations that constitute the H-infinity Kalman Filter.

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

- The performance of the proposed nonlinear **H-infinity control scheme** for the **electric ship propulsion system** is tested through simulation:

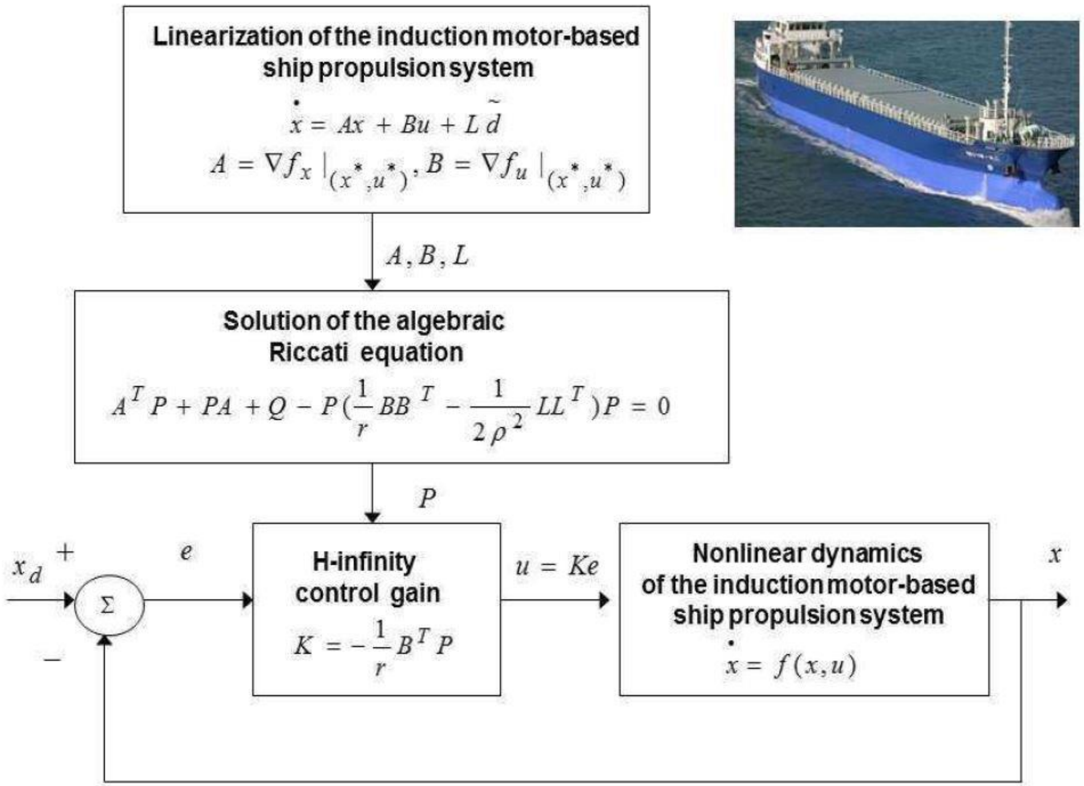


Fig.4 Diagram of the nonlinear optimal control for the electric ship propulsion system

With the use of the H-infinity control method, **fast and accurate tracking of the reference setpoints** of the state variables of the electric ship propulsion system was achieved

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Tracking performance of the electric ship propulsion system in case of **setpoint 1**:

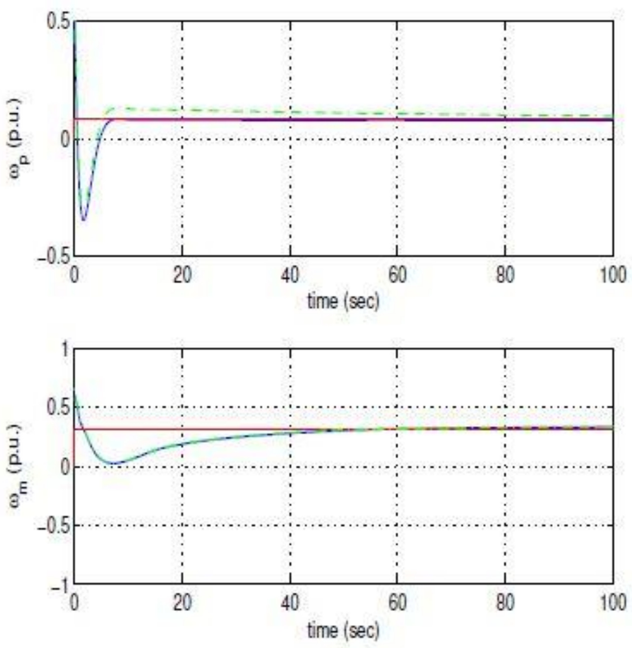


Fig. 5(a) Convergence of the rotational speed of the propeller and of the rotational speed of the induction motor to the reference setpoints.

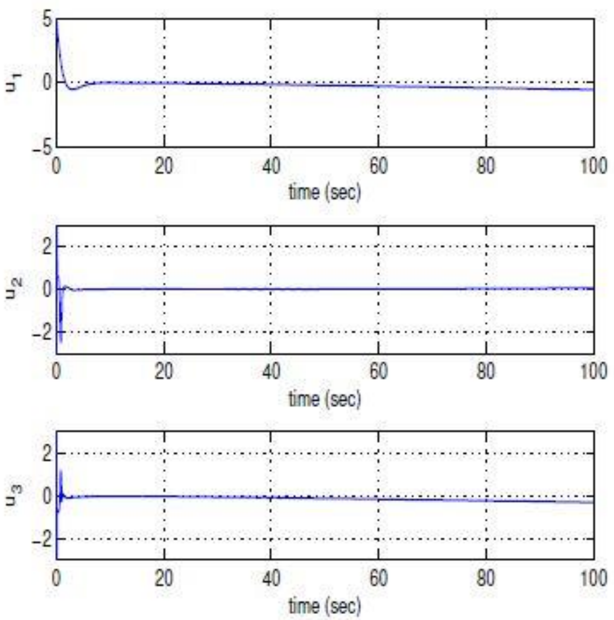


Fig. 5(b) Control inputs u_1, u_2 and u_3 applied to the propulsion system

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Tracking performance of the electric ship propulsion system in case of **setpoint 2**:

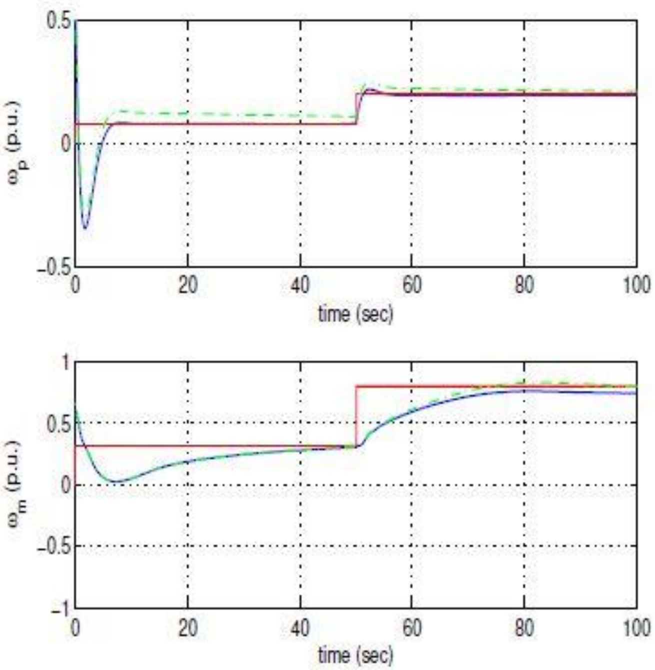


Fig. 6(a) Convergence of the rotational speed of the propeller and of the rotational speed of the induction motor to the reference setpoints.

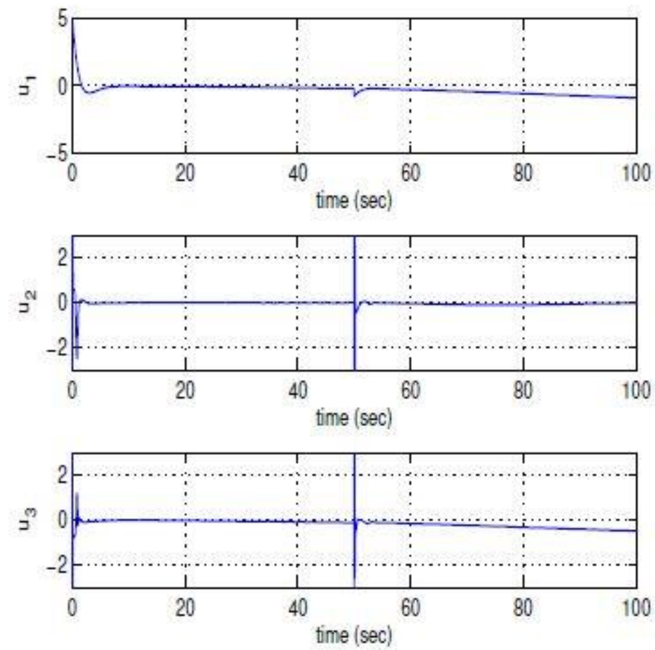


Fig. 6(b) Control inputs u_1, u_2 and u_3 applied to the propulsion system

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Tracking performance of the electric ship propulsion system in case of **setpoint 3**:

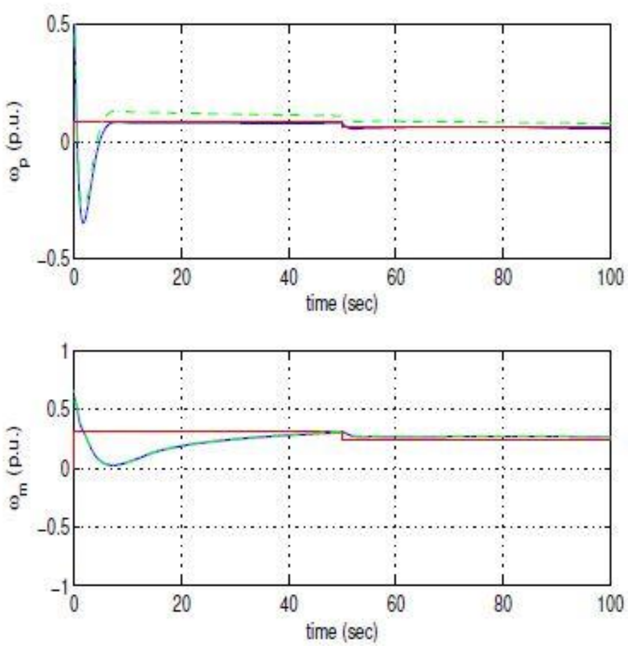


Fig. 7(a) Convergence of the rotational speed of the propeller and of the rotational speed of the induction motor to the reference setpoints.

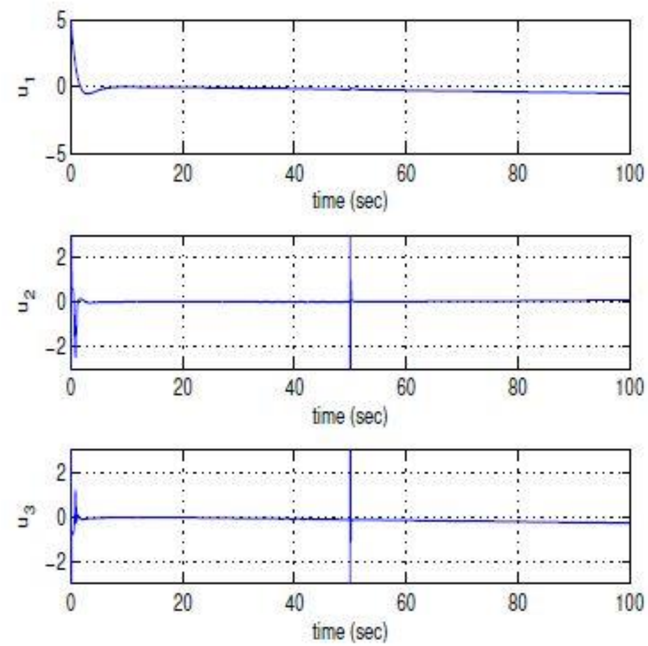


Fig. 7(b) Control inputs u_1, u_2 and u_3 applied to the propulsion system

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Tracking performance of the electric ship propulsion system in case of **setpoint 4**:

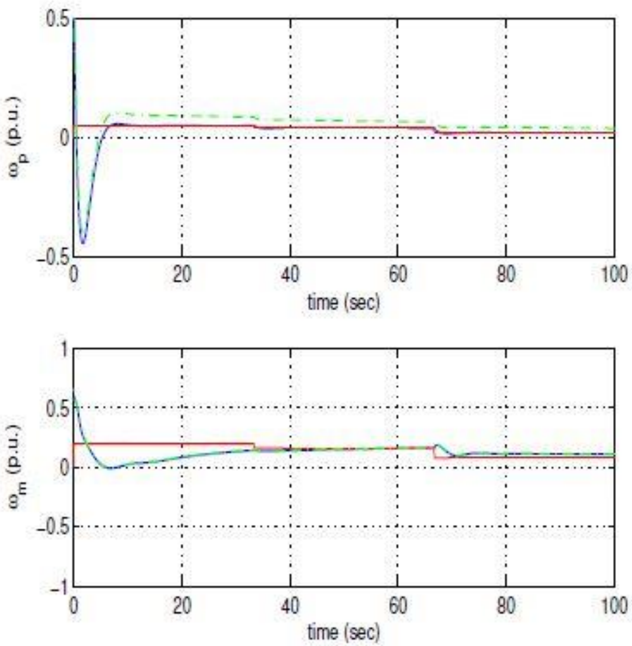


Fig. 8(a) Convergence of the rotational speed of the propeller and of the rotational speed of the induction motor to the reference setpoints.

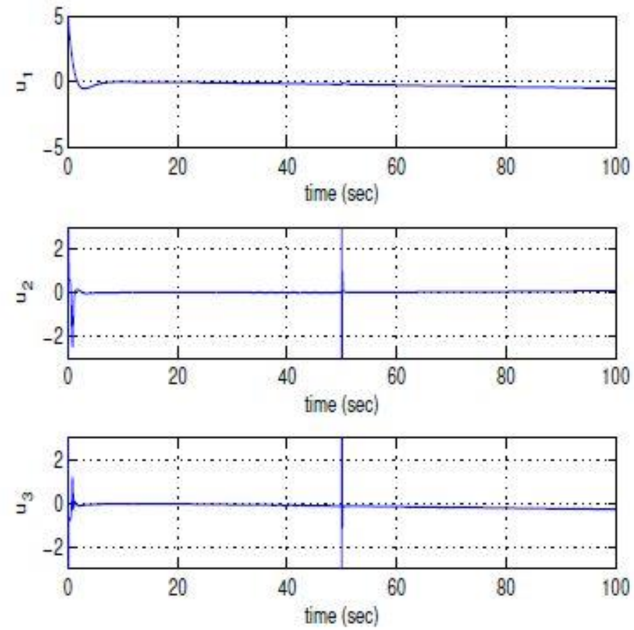


Fig. 8(b) Control inputs u_1, u_2 and u_3 applied to the propulsion system

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Tracking performance of the electric ship propulsion system in case of **setpoint 5**:

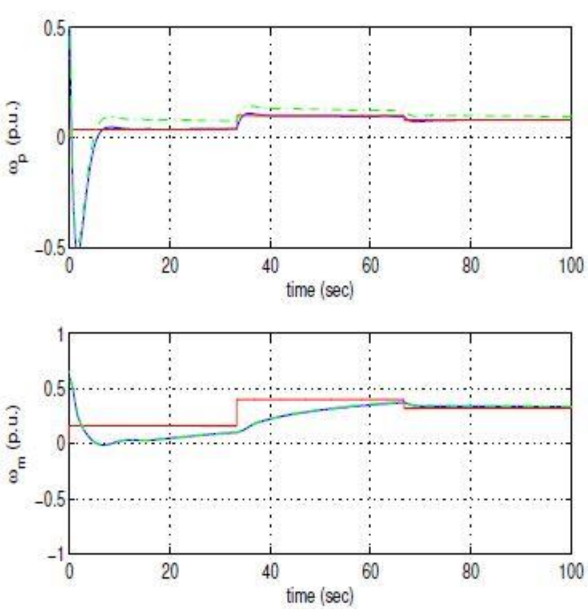


Fig. 9(a) Convergence of the rotational speed of the propeller and of the rotational speed of the induction motor to the reference setpoints.

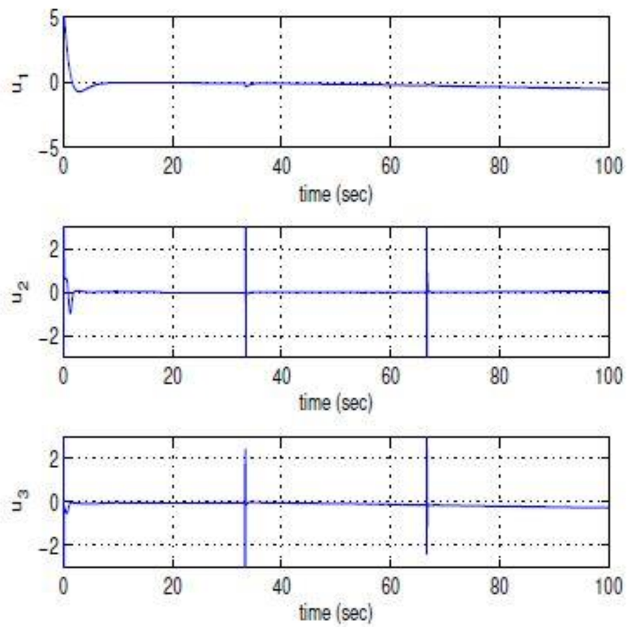


Fig. 9(b) Control inputs u_1, u_2 and u_3 applied to the propulsion system

Example 3: Nonlinear control and state estimation using approximate linearization

7. Simulation tests

Variations of the elements of the drift vector f(x) when tracking **setpoint 4 and 5**:

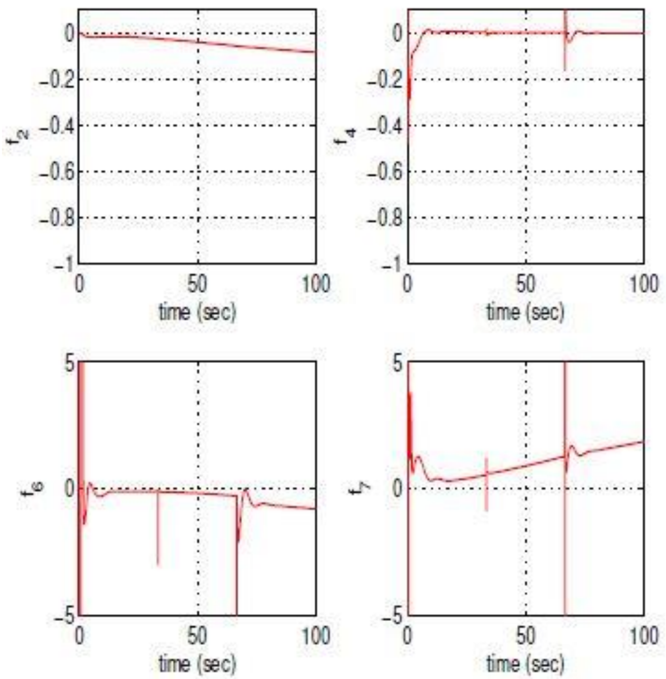


Fig. 10(a) Drift vector elements $f_i(x)$ when tracking setpoint 4.

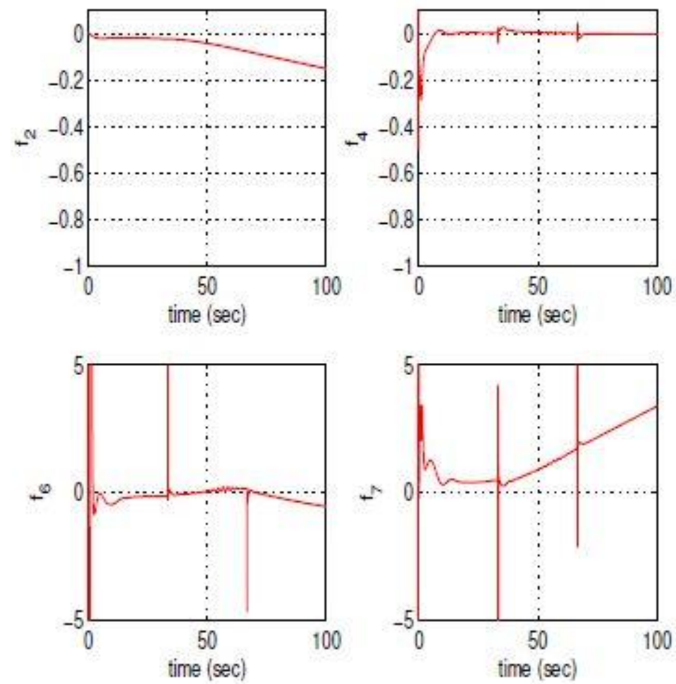


Fig. 10(b) Drift vector elements $f_i(x)$ when tracking setpoint 4.

Example 3: Nonlinear control and state estimation using approximate linearization

8. Conclusions

• **Electric propulsion schemes** are widely used in **USVs and AUVs**. Such propulsion schemes may comprise synchronous or **asynchronous (induction) motors** which finally provide rotational motion to propellers

• A **nonlinear optimal (H-infinity) control** method has been proposed for electric ship propulsion systems, comprising a **three-phase induction motor**, a **drivetrain** and a **propeller.**

• The dynamic model of the propulsion system has undergone **approximate linearization** around a **temporary operating point** that was redefined at each iteration of the control method.

• The linearization procedure relied on **Taylor series expansion** and on the computation of the associated **Jacobian matrices**.

• For the approximately linearized model of the propulsion system, an **optimal (H-infinity) feedback controller** has been designed.

• This control represents the solution to a **min-max differential game** in which the controller tries to minimize a quadratic cost function of the state vector's error whereas the model uncertainty and external perturbation terms try to maximize this cost functional.

Example 3: Nonlinear control and state estimation using approximate linearization

8. Conclusions

- The **stability properties** of the control scheme have been proven through **Lyapunov analysis**.

- First, it has been demonstrated that the control loop of the propulsion system satisfies the **H-infinity tracking performance**, which signifies elevated **robustness against parametric uncertainty** and **exogenous disturbances**.

- Moreover, conditions have been provided under which the control loop is **globally asymptotically stable.**

- To implement **state estimation-based control** through the measuring of small number of state variables, the **H-infinity Kalman Filter** has been used as a **robust state estimator.**

- The proposed **nonlinear optimal control scheme** avoids complicated state-space transformations for the propulsion system, as well as the singularity problems that can be met in global linearization-based control methods

- The proposed control method **retains the known advantages of linear optimal control,** that is fast and accurate tracking of reference setpoints under moderate variations of the control inputs

Example 4: Nonlinear control and state estimation using Lyapunov methods

1. Control of turbocharged ship diesel engines

• A **nonlinear control method** for **turbocharged Diesel engines** is developed with the use of **Differential flatness theory** and **adaptive fuzzy control**.

• It is shown that the **dynamic model of the turbocharged Diesel engine** is **differentially flat** and admits **dynamic feedback linearization**. It is also shown that this dynamic model can be written in the **linear Brunovsky canonical form** for which a **state feedback controller can be easily designed**.

• To compensate for **modeling errors** and **external disturbances** an **adaptive fuzzy control scheme** is implemented making use of the **transformed state-space description** of the diesel engine that is obtained through the application of differential flatness theory.

• Since **only the system's output is measurable** the complete state vector has to be reconstructed with the use of a **state observer**.

• It is shown that a **suitable learning law** can be defined for **neuro-fuzzy approxim**ators, which are part of the controller, so as to **preserve the closed-loop system stability**.

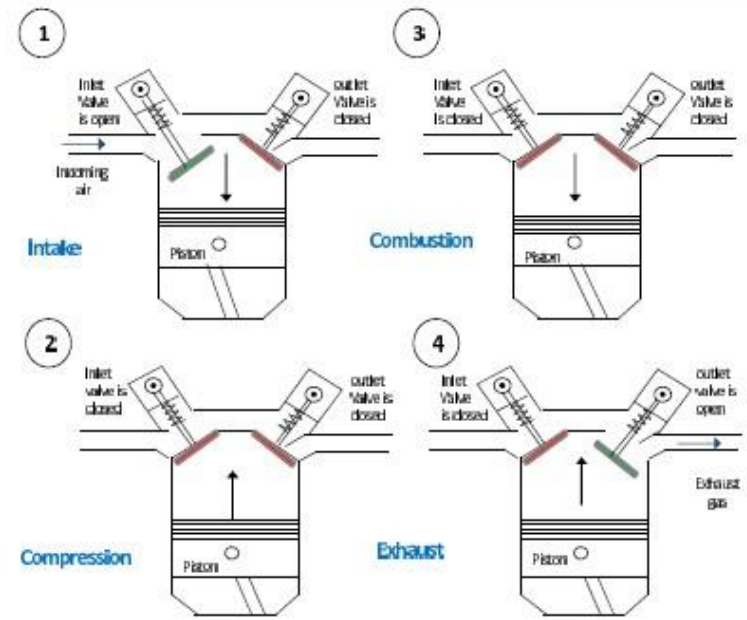
• With the use of **Lyapunov stability analysis** it is proven that the proposed observer-based adaptive fuzzy control scheme results into **H-infinity tracking performance** and finally into global stability.

Example 4: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the Diesel engine

The basic parameters of the Diesel engine are:

(i) Gas pressure in the intake manifold p_1
(ii) Gas pressure in the exhaust manifold p_2
(iii) Turbine power P_t
(iv) Compressor power P_c



Four-stroke cycle of an internal combustion diesel engine

Additional variables of importance are

W_c which is the compressor's mass flow rate

T_1 which is the intake manifold temperature

T_2 which is the exhaust manifold temperature

W_t which is the turbine mass-flow rate

W_{EGR} which is the exhaust gas recirculation flow rate

Example 4: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the Diesel engine

The **basic relations of the Diesel-engine's dynamics** are:

$$\dot{p}_1 = K_1(W_c + u_1 - K_e p_1)$$
$$\dot{p}_2 = K_2(K_e p_1 - u_1 - u_2)$$
$$\dot{P}_c = \frac{1}{\tau}(\eta_m P_t - P_c)$$

The **control inputs** to this model are:

(i) The exhaust-gas recirculation (EGR) flow rate $u_1 = W_{EGR}$

(ii) The turbine's mass flow rate $u_2 = W_t$

Moreover, it holds that: $W_c = P_c \frac{K_c}{p_1^{\mu} - 1}$ $\quad P_t = K_t(1 - p_2{}^{\mu})u_2$

The **description of the Diesel engine in state-space form** is given by:

$$\dot{x} = f(x) + g_a(x)u_1 + g_b(x)u_2$$

where:

$$f(x) = \begin{pmatrix} K_1 K_c \frac{P_c}{p_1^{\mu}} - K_1 K_e p_1 \\ K_2 K_e p_1 \\ -\frac{P_c}{\tau} \end{pmatrix} \quad g_a(x) = \begin{pmatrix} K_1 \\ -K_2 \\ 0 \end{pmatrix} \quad g_b(x) = \begin{pmatrix} 0 \\ -K_2 \\ K_o(1 - p_2{}^{-\mu}) \end{pmatrix}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

2. Dynamic model of the Diesel engine

The **output variables** of the Diesel engine model are:

$$y = \begin{pmatrix} p_1 \\ W_c \end{pmatrix} = \begin{pmatrix} p_1 \\ P_c \frac{K_c}{p_1^{\mu} - 1} \end{pmatrix}$$

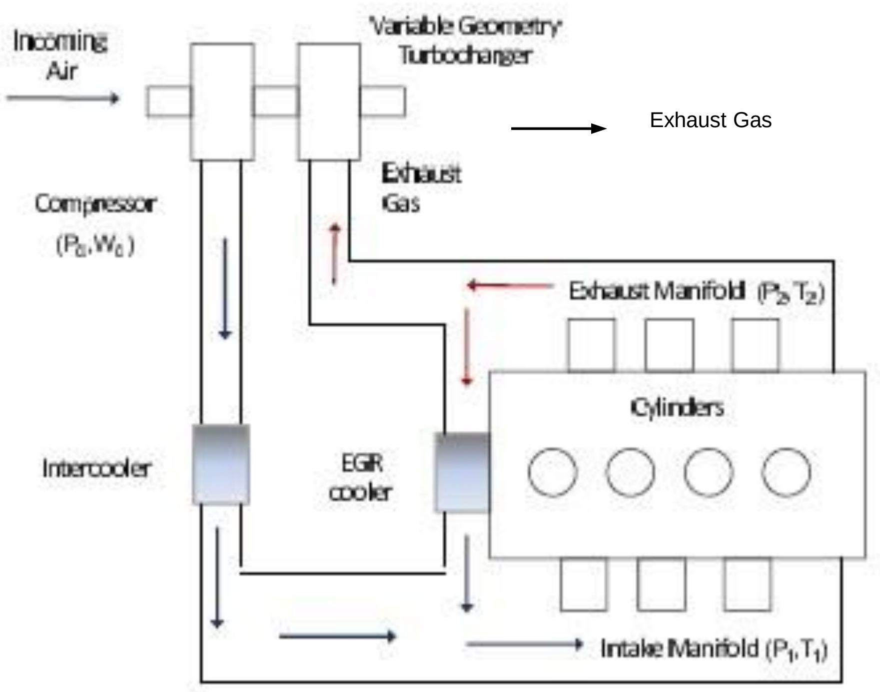


Fig.1 : Diagram of the turbocharged Diesel engine

Example 4: Nonlinear control and state estimation using Lyapunov methods

3. Lie algebra-based control

Dynamic feedback linearization is applied to the Diesel engine's model:

The **state vector of the system is extended** by considering as **additional state variables** the control inputs

The **transformed control inputs** which appear in the linearized equivalent of the system are functions of not only the initial control variables u_1, u_2 but also of their derivatives $\dot{u}_1, \dot{u}_2$.

The **extended state vector of the diesel engine** becomes:

$$\bar{x} = [\bar{x}_1, \bar{x}_2, \dot{x}_3, \dot{x}_4]^T = [p_1, p_2, \dot{P}_c, \dot{z}]^T$$

The **control inputs** to the linearized model of the Diesel engine become:

$$v_1 = \dot{u}_1 = \ddot{z},\ v_2 = u_2$$

The **control inputs which are finally exerted on the system** contain an integral action:

$$\bar{u}_1 = \int v_1 dt,\ u_2 = v_2$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

3. Lie algebra-based control

The **extended state-space description** of the Diesel engine becomes:

$$\begin{aligned}
\dot{x}_1 &= K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_g x_1 + K_1 x_4 \\
\dot{x}_2 &= K_2 K_g x_1 - K_2 x_4 - K_2 v_2 \\
\dot{x}_3 &= -\frac{x_3}{\tau} + K_o\left(1 - x_2{}^{-\mu}\right) \\
\dot{x}_4 &= v_1
\end{aligned}$$

Consequently, **in matrix form** one has:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_g x_1 + K_1 x_4 \\ K_2 K_g x_1 - K_2 x_4 \\ -\frac{x_3}{\tau} + K_o(1 - x_2^{\mu}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} v_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} v_2$$

The **system's outputs** are chosen to be:

$$\begin{aligned}
y_1 &= x_1 = p_1 \\
y_2 = P_c \frac{K_c}{p_1^{\mu}-1} &\Rightarrow y_2 = x_3 \frac{K_c}{x_1^{\mu}-1}
\end{aligned}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

3. Lie algebra-based control

Linearization of the system's dynamics is performed using the following state variables:

$$z_1^1 = h_1(x) \qquad z_2^1 = L_f h_1(x) \qquad \dot{z}_2^1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$

and equivalently:

$$z_1^2 = h_2(x) \qquad z_2^2 = L_f h_2(x) \qquad \dot{z}_2^2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

After intermediate computations one obtains:

$$L_f^2 h_1(x) = L_f z_2^1 \Rightarrow L_f^2 h_1(x) = \frac{\partial z_2^1}{\partial x_1} f_1 + \frac{\partial z_2^1}{\partial x_1} f_2 + \frac{\partial z_2^1}{\partial x_3} f_3 \Rightarrow$$

$$L_f^2 h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right) f_1 + 0 f_2 + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right) f_3 + K_1 f_4 \Rightarrow$$

$$L_f^2 h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right)\left(K_1 K_3 \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right)\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right)$$

and also

$$L_{g_a} L_f h_1(x) = L_{g_a} z_2^1 \Rightarrow L_{g_a} L_f h_1(x) = \frac{\partial z_2^1}{\partial x_1} g_{a_1} + \frac{\partial z_2^1}{\partial x_2} g_{a_2} + \frac{\partial z_2^1}{\partial x_3} g_{a_3} + \frac{\partial z_2^1}{\partial x_4} g_{a_4} \Rightarrow$$

$$L_{g_a} L_f h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right) g_{a_1} + 0 g_{a_2} + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right) g_{a_3} + K_1 g_{a_4} \Rightarrow$$

$$L_{g_a} L_f h_1(x) = K_1$$

and

$$L_{g_b} L_f h_1(x) = L_{g_b} z_2^1 \Rightarrow L_{g_b} L_f h_1(x) = \frac{\partial z_2^1}{\partial x_1} g_{b_1} + \frac{\partial z_2^1}{\partial x_2} g_{b_2} + \frac{\partial z_2^1}{\partial x_3} g_{b_3} + \frac{\partial z_2^1}{\partial x_4} g_{b_4} \Rightarrow$$

$$L_{g_b} L_f h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right) g_{b_1} + 0 g_{b_2} + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right) g_{b_3} + K_1 g_{b_4} \Rightarrow$$

$$L_{g_b} L_f h_1(x) = 0$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

3. Lie algebra-based control

In a similar manner one obtains:

$$L_f^2 h_2(x) = \frac{\partial z_2^2}{\partial x_1} f_1 + \frac{\partial z_2^2}{\partial x_2} f_2 + \frac{\partial z_2^2}{\partial x_3} f_3 + \frac{\partial z_2^2}{\partial x_4} f_4 \Rightarrow$$

$$L_f^2 h_2(x) = \{x_3 \frac{(-K_c\mu\mu-1x_1^{\mu-2})(x_1^{\mu}-1)^2-(-K_c\mu x_1^{\mu-1})2(x_1^{\mu}-1)\mu x_1^{\mu-1}}{x_1^{\mu}-1^4}(\frac{K_1K_cx_3}{x_1^{\mu}-1-K_1K_c+K_1x_4})+$$
$$x_3\frac{-K_c\mu x_1^{\mu-1}}{x_1^{\mu}-1}(\frac{K_1K_c-x_3\mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}-K_1K_c)+$$
$$+\frac{-K_c\mu x_1^{\mu-1}}{x_1^{\mu}-1^2}(-\frac{x_2}{\tau}+K_o(1-x_2^{-\mu}))\}(K_1K_c\frac{x_3}{x_1^{\mu}-1}-K_1K_ex_1+K_1x_4)+$$
$$+\frac{K_c}{x_1^{\mu}-1}K_o\mu x_2^{-\mu-1}(K_2K_ex_1-K_2x_4)+$$
$$+\{x_3\frac{-K_c\mu x_1^{\mu-1}}{x_1^{\mu}-1^2}\frac{K_1K_c}{x_1^{\mu}-1}+\frac{K_c}{x_1^{\mu}-1}(-\frac{1}{\tau})\}(-\frac{x_2}{\tau}+K_o(1-x_2^{-\mu}))+$$
$$\{x_3\frac{-K_c\mu x_1^{\mu-1}}{x_1^{\mu}-1^2}K_1\}0$$

and also

$$L_{g_a}L_f h_2(x) = \frac{\partial z_2^2}{\partial x_1} g_{a1} + \frac{\partial z_2^2}{\partial x_2} g_{a2} + \frac{\partial z_2^2}{\partial x_3} g_{a3} + \frac{\partial z_2^2}{\partial x_4} g_{a4} \Rightarrow$$
$$L_{g_a}L_f h_2(x) = \frac{\partial z_2^2}{\partial x_4} 1 \Rightarrow L_{g_a}L_f h_2(x) = x_3\frac{-K_c\mu x_1^{\mu-1}}{(x_1^{\mu}-1^2)}K_1$$

and

$$L_{g_b}L_f h_2(x) = \frac{\partial z_2^2}{\partial x_1} g_{b1} + \frac{\partial z_2^2}{\partial x_2} g_{b2} + \frac{\partial z_2^2}{\partial x_3} g_{b3} + \frac{\partial z_2^2}{\partial x_4} g_{b4} \Rightarrow$$
$$L_{g_b}L_f h_2(x) = \frac{\partial z_2^2}{\partial x_2} 1 \Rightarrow L_{g_b}L_f h_2(x) = \frac{K_c}{x_1^{\mu}-1}K_o\mu x_2^{-\mu-1}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

3. Lie algebra-based control

Thus, **after the change of coordinates** the following description of the system is obtained

$$\begin{gathered}\dot{z}_1^1 = z_2^1 \\ \dot{z}_2^1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) v_1 + L_{g_b} L_f h_1(x) v_2 \\ \dot{z}_1^2 = z_2^2 \\ \dot{z}_2^2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) v_1 + L_{g_b} L_f h_2(x) v_2\end{gathered}$$

which is also written in the state-space form

$$\begin{pmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{pmatrix} = \begin{pmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} + \begin{pmatrix} L_{g_a} L_f h_1(x) & L_{g_b} L_f h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

or also in the more compact form

$$\ddot{z} = \bar{f}_a + \bar{M}_a v$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

3. Lie algebra-based control

Moreover, by **defining the control inputs**

$$
\begin{aligned}
v_{in}^1 &= L_f^2 h_1(x) + L_{g_a} L_f h_1(x) v_1 + L_{g_b} L_f h_1(x) v_2 \\
v_{in}^2 &= L_f^2 h_2(x) + L_{g_a} L_f h_2(x) v_1 + L_{g_b} L_f h_2(x) v_2
\end{aligned}
$$

the system's description comes to the following **canonical form**:

$$
\begin{pmatrix} \dot{z}_1^1 \\ \dot{z}_2^1 \\ \dot{z}_1^2 \\ \dot{z}_2^2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^1 \\ z_1^2 \\ z_1^2 \\ z_2^2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_{in}^1 \\ v_{in}^2 \end{pmatrix}
$$

The selection of the **state feedback control law**, which assures elimination of the tracking error is:

$$
\begin{aligned}
v_{in}^1 &= \ddot{z}_{1,d}^1 - K_d^1(\dot{z}_1^1 - \dot{z}_{1,d}^1) - K_p^1(z_1^1 - z_{1,d}^1) \\
v_{in}^2 &= \ddot{z}_{1,d}^2 - K_d^2(\dot{z}_1^2 - \dot{z}_{1,d}^2) - K_p^2(z_1^2 - z_{1,d}^2)
\end{aligned}
$$

The **control input** that is finally exerted to the system is:

$$
\bar{v}_{in} = \bar{f}_a + \bar{M}_a \bar{v} \Rightarrow \bar{v} = \bar{M}_a^{-1}(\bar{v}_{in} - \bar{f}_a)
$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

4. Nonlinear control of the diesel engine using differential flatness theory

The results about **dynamic state feedback system linearization** can be obtained with the **computation of time derivatives and differential flatness theo**ry. The following differentially flat system outputs are considered

$$y_1 = p_1 = x_1$$
$$y_2 = P_c \frac{K_c}{p_1^{\mu}-1} \Rightarrow y_2 = x_3 \frac{K_c}{x_1^{\mu}-1}$$

The dynamics of the extended system are:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{pmatrix} = \begin{pmatrix} K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4 \\ K_2 K_e x_1 - K_2 x_4 \\ \frac{-x_3}{\tau} + K_o(1 - x_2^{\mu}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} v_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} v_2$$

It holds that $y_1 = x_1$ therefore:

$$x_1 = q_1(y, \dot{y})$$
$$y_2 = x_3 \frac{K_c}{x_1^{\mu}-1} \Rightarrow x_3 = \frac{y_2 (x_1^{\mu}-1)}{K_c} \Rightarrow x_3 = \frac{y_2 (y_1^{\mu}-1)}{K_c}$$

which means that variable x_3 is also a function of the flat output and its derivatives.

Example 4: Nonlinear control and state estimation using Lyapunov methods

4. Nonlinear control of the diesel engine using differential flatness theory

Moreover, from the first row of the state-space equations one obtains:

$$\dot{x}_1 = K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4 \Rightarrow$$

$$x_4 = \frac{\dot{x}_1 - K_1 K_c \frac{x_3}{x_1^{\mu}-1} + K_1 K_e x_1}{K_1} \Rightarrow x_4 = q_4(y,\dot{y})$$

which means that variable x_4 is also a function of the flat output and its derivatives.

Additionally, from the fourth row of the state-space equations one obtains:

$$\dot{x}_4 = v_1 \Rightarrow v_1 = q_5(y,\dot{y})$$

This means that the control input v_1 is also a function of the flat output and its derivatives.

Similarly, from the third row of the state-space equations one has:

$$\dot{x}_3 = -\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu}) \Rightarrow$$

$$x_2^{-\mu} = -\frac{\dot{x}_3 + \frac{x_3}{\tau}}{K_o} \Rightarrow x_2 = \left(\frac{-\dot{x}_3 + \frac{x_3}{\tau}}{K_o}\right)^{\mu}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

4. Nonlinear control of the diesel engine using differential flatness theory

while, from the second row of the state-space equations one obtains:

$$\dot{x}_2 = K_2 K_e x_1 - K_2 x_4 + v_2 \Rightarrow$$
$$v_2 = \dot{x}_2 - K_2 K_e x_1 + K_2 x_4 \Rightarrow$$
$$v_2 = q_6(y, \dot{y})$$

Therefore, **all state variables of the system and the control inputs** can be written as **functions of the flat output and its derivatives**. Therefore, the system of the diesel engine **is differentially flat** and can be subjected to dynamic feedback linearization.

Next, by **considering the flat outputs and by differentiating with respect to time** one obtains:

$$y_1 = x_1$$
$$\dot{y}_1 = \dot{x}_1 \Rightarrow \dot{y}_1 = K_1 K_c \frac{x_3}{x_1^{\mu} - 1} - K_1 K_e x_1 + K_1 x_4$$

By differentiating once more with respect to time one gets:

$$\ddot{y}_1 = \left(K_1 K_c \frac{-x_3 \mu x_1^{\mu-1}}{{x_1^{\mu} - 1}^2} - K_1 K_e\right)\left(K_1 K_c \frac{x_3}{x_1^{\mu} - 1} - K_1 K_e x_1 + K_1 x_4\right) +$$
$$+\left(K_1 K_c \frac{1}{x_1^{\mu} - 1}\right)\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right) + K_1 v_1$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

4. Nonlinear control of the diesel engine using differential flatness theory

In a similar manner one has:

$$y_2 = x_3 \frac{K_c}{x_1^{\mu}-1}$$

$$\dot{y}_2 = \frac{x_3 K_c \mu x_1^{\mu-1}}{{x_1^{\mu}-1}^2}\dot{x}_1 + \frac{K_c}{x_1^{\mu}-1}\dot{x}_3 \Rightarrow$$

$$\dot{y}_2 = \frac{x_3 K_c \mu x_1^{\mu-1}}{{x_1^{\mu}-1}^2}\dot{x}_1 + \frac{K_c}{x_1^{\mu}-1}\dot{x}_3 \Rightarrow$$

while by **differentiating once more with respect to time** one obtains:

$$\begin{aligned}\ddot{y}_2 = \{ & \tfrac{x_3 K_c \mu(\mu-1)x_1^{\mu-2}(x_1^{\mu}-1)^2 - x_3 K_c \mu x_1^{\mu-1} 2(x_1^{\mu}-1)\mu x_1^{\mu-1}}{(x_1^{\mu}-1)^4}(K_1 K_c \tfrac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4) + \\ & + \tfrac{-K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}(-\tfrac{x_3}{\tau} + K_o(1-x_2^{\mu}))\}(K_1 K_c \tfrac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4) + \tfrac{K_c}{x_1^{\mu}-1}(K_o \mu x_2^{\mu-1})(K_2 K_e x_1 - K_2 x_4 + v_2) + \\ & + \{ \tfrac{K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}(K_1 K_c \tfrac{x_3}{x_1^{\mu}-1}) - K_1 K_e x_1 + K_1 x_4 + \tfrac{K_c}{x_1^{\mu}-1}(-\tfrac{1}{\tau})\}(-\tfrac{x_3}{\tau} + K_o(1 - x_2^{-\mu})) + \\ & + \tfrac{x_3 K_c \mu x_1^{\mu-1}}{{x_1^{\mu}-1}^2} K_1 \} v_1\end{aligned}$$

Thus one arrives at a representation of the system's dynamics that is analogous to the one obtained by applying the **Lie algebra-based approach**:

Example 4: Nonlinear control and state estimation using Lyapunov methods

4. Nonlinear control of the diesel engine using differential flatness theory

$$\ddot{y}_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$
$$\ddot{y}_2 = L_f^2 h_2(x) + L_{g_a} L_f h_2(x) u_1 + L_{g_b} L_f h_2(x) u_2$$

where:

$$L_f^2 h_1(x) = \left(K_1 K_3 \frac{-x_3 \mu x_1^{\mu-1}}{x_1^{\mu}-1} - K_1 K_e\right)\left(K_1 K_3 \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \left(\frac{K_1 K_3}{x_1^{\mu}-1}\right)\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right)$$

$$L_{g_a} L_f h_1(x) = K_1 \qquad \text{and} \qquad L_{g_b} L_f h_1(x) = 0$$

and also:

$$\begin{aligned} L_f^2 h_2(x) = &\left\{\frac{x_3 K_c \mu(\mu-1) x_1^{\mu-2}(x_1^{\mu}-1)^2 - x_3 K_c \mu x_1^{\mu-1} 2(x_1^{\mu}-1)\mu x_1^{\mu-1}}{(x_1^{\mu}-1)^4}\left(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \right. \\ &+ \frac{-K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{\mu})\right)\Big\}\left(K_1 K_c \frac{x_3}{x_1^{\mu}-1} - K_1 K_e x_1 + K_1 x_4\right) + \frac{K_c}{x_1^{\mu}-1}\left(K_o \mu x_2^{\mu-1}\right)\left(K_2 K_e x_1 - K_2 x_4\right) + \\ &+ \left\{\frac{K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2}\left(K_1 K_c \frac{x_3}{x_1^{\mu}-1}\right) - K_1 K_e x_1 + K_1 x_4 + \frac{K_c}{x_1^{\mu}-1}\left(-\frac{1}{\tau}\right)\right\}\left(-\frac{x_3}{\tau} + K_o(1 - x_2^{-\mu})\right) \end{aligned}$$

$$L_{g_a} L_f h_2(x) = \frac{x_3 K_c \mu x_1^{\mu-1}}{(x_1^{\mu}-1)^2} K_1 \qquad \text{and} \qquad L_{g_b} L_f h_2(x) = \frac{K_c}{x_1^{\mu}-1}\left(K_o \mu x_2^{\mu-1}\right)$$

The **design of the state feedback controller** proceeds as in case of linearization with the use of Lie algebra-based computations

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.1. Transformation of MIMO nonlinear systems into the Brunovsky form

It is assumed now that **after defining the flat outputs of the initial MIMO nonlinear system**, and after expressing the system state variables and control inputs as functions of the flat output and of the associated derivatives, the system can be **transformed in the Brunovsky canonical form**

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\cdots \\
\dot{x}_{r_1-1} &= x_{r_1} \\
\dot{x}_{r_1} &= f_1(x) + \sum_{j=1}^{p} g_{1_j}(x) u_j + d_1 \\
\\
\dot{x}_{r_1+1} &= x_{r_1+2} \\
\dot{x}_{r_1+2} &= x_{r_1+3} \\
&\cdots \\
\dot{x}_{p-1} &= x_p \\
\dot{x}_p &= f_p(x) + \sum_{j=1}^{p} g_{p_j}(x) u_j + d_p
\end{aligned}
$$

$$
\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_{r_1-1} \\
&\cdots \\
y_p &= x_{n-r_p+1}
\end{aligned}
$$

$x = [x_1, \cdots, x_n]^T$: is the state vector

$u = [u_1, \cdots, u_p]^T$: is the inputs vector

$y = [y_1, \cdots, y_p]^T$: is the outputs vector

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Next **the following vectors and matrices** can be defined

$$f(x) = [f_1(x), \quad \dots, \quad f_n(x)]^T$$
$$g(x) = [g_1(x), \quad \dots, \quad g_n(x)]^T$$
$$\text{with } g_i(x) = [g_{1i}(x), \quad \dots, \quad g_{pi}(x)]^T$$
$$A = diag[A_1, \dots, A_p], \qquad B = diag[B_1, \dots, B_p]$$
$$C^T = diag[C_1, \dots, C_p], \qquad d = [d_1, \dots, d_p]^T$$

where matrix A has the **MIMO canonical form**, i.e. with elements

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}_{r_i \times r_i}$$

$$B_i^T = [0 \quad 0 \quad \dots \quad 0 \quad 1]_{1 \times r_i}$$

Thus, the initial nonlinear system can be written in the **state-space form**

$$\dot{x} = Ax + B[f(x) + g(x)u + \tilde{d}]$$
$$y = Cx$$

or equivalently in the state space form

$$\dot{x} = Ax + Bv + B\tilde{d}$$
$$y = Cx$$

where $v = f(x) + g(x)u$

For the case of the **MIMO diesel engine model** it is assumed that the functions $f(x)$ and $g(x)$ are unknown and have to be approximated by **neuro-fuzzy networks**

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Thus, the nonlinear system can be written in **state-space form**

$$\dot{x} = Ax + B[f(x) + g(x)u + \tilde{d}]$$
$$y = C^T x$$

which equivalently can be written as

$$\dot{x} = Ax + Bv + B\tilde{d}$$
$$y = C^T x$$

where $v = f(x) + g(x)u.$

The **reference setpoints** for the system's outputs $y_1, \cdots, y_p$

are denoted as $y_{1m}, \cdots, y_{pm}$ and the associated tracking errors are defined as

$$e_1 = y_1 - y_{1m}$$
$$e_2 = y_2 - y_{2m}$$
$$\cdots$$
$$e_p = y_p - y_{pm}$$

The **error vector of the outputs** of the transformed MIMO system is denoted as

$$E_1 = [e_1, \cdots, e_p]^T$$
$$y_m = [y_{1m}, \cdots, y_{pm}]^T$$
$$\cdots$$
$$y_m^{(r)} = [y_{1m}^{(r)}, \cdots, y_{pm}^{(r)}]^T$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.2. Control law

The **control signal of the MIMO nonlinear system** contains the **unknown nonlinear functions** f(x) and g(x) which can be approximated by

$$\boxed{\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f, \quad \hat{g}(x|\theta_g) = \Phi_g(x)\theta_g}$$

where $\Phi_f(x) = \left(\xi_f^1(x), \xi_f^2(x), \cdots \xi_f^n(x)\right)^T,$

$$\xi_f^i(x) = \left(\phi_f^{i,1}(x), \phi_f^{i,2}(x), \cdots, \phi_f^{i,N}(x)\right)$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_f{}^T = \left(\theta_f^1, \theta_f^2, \cdots \theta_f^N\right).$

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.2. Control law

Similarly, it holds $\Phi_g(x) = (\xi_g^1(x), \xi_g^2(x), \cdots \xi_g^N(x))^T$.

$$\xi_g^i(x) = (\phi_g^{i,1}(x), \phi_g^{i,2}(x), \cdots, \phi_g^{i,N}(x)).$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_g = (\theta_g^1, \theta_g^2, \cdots, \theta_g^p)^T$.

However, here each row of θ_g is vector thus giving

$$\theta_g = \begin{pmatrix} \theta_{g1}^1 & \theta_{g1}^2 & \cdots & \theta_{g1}^p \\ \theta_{g2}^1 & \theta_{g2}^2 & \cdots & \theta_{g2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{gN}^1 & \theta_{gN}^2 & \cdots & \theta_{gN}^p \end{pmatrix}$$

If the state variables of the system are available for measurement then a **state-feedback control law can be formulated** as

$$u = \hat{g}^{-1}(x|\theta_g)[-\hat{f}(x|\theta_f) + y_m^{(r)} + K_c^T e + u_c]$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.2. Estimation of the state vector

The control of the system which has been described in the canonical form becomes more complicated **when the state vector x is not directly measurable** and has to be reconstructed through a state observer. The following definitions are used

$e = x - x_m$: is the error of the state vector

$\hat{e} = \hat{x} - x_m$ is the error of the estimated state vector

$\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$ is the observation error

When an **observer is used to reconstruct the state vector**, the control law

$$u = \hat{g}^{-1}(\hat{x}|\theta_g)[-\hat{f}(\hat{x}|\theta_f) + y_m^{(r)} - K^T\hat{e} + u_c]$$

By applying the previous feedback control law one obtains the closed-loop dynamics

$$y^{(n)} = f(x) + g(x)\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(n)} = f(x) + [g(x) - \hat{g}(\hat{x}) + \hat{g}(\hat{x})]\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(n)} = [f(x) - \hat{f}(\hat{x})] + [g(x) - \hat{g}(\hat{x})]u + y_m^{(n)} - K^T\hat{e} + u_c + d$$

It holds $e = x - x_m \Rightarrow y^{(n)} = e^{(n)} + y_m^{(n)}$

and by substituting $y^{(n)}$ n the **previous tracking error dynamics** gives

Example 4: Nonlinear control and state estimation using Lyapunov methods

5. Flatness-based adaptive neurofuzzy control for MIMO nonlinear systems

5.2. Estimation of the state vector

the **new tracking error dynamics**

$$e^{(n)} + y_m^{(n)} = y_m^{(n)} - K^T\hat{e} + u_c + [f(x) - \hat{f}(\hat{x})] + \\ +[g(x) - \hat{g}(\hat{x})]u + d$$

or equivalently

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\{[f(x) - \hat{f}(\hat{x})] + \\ +[g(x) - \hat{g}(\hat{x})]u + \tilde{d}\}$$ A

$$e_1 = C^T e$$

where $e = [e^1, e^2, \cdots, e^p]^T$ with $e^i = [e_i, \dot{e}_i, \ddot{e}_i, \cdots, e_i^{r_i-1}]^T,\ i = 1, 2, \cdots, p$

and equivalently $\hat{e} = [\hat{e}^1, \hat{e}^2, \cdots, \hat{e}^p]^T$ with $\hat{e}^i = [\hat{e}_i, \dot{\hat{e}}_i, \ddot{\hat{e}}_i, \cdots, \hat{e}_i^{r_i-1}]^T,\ i = 1, 2, \cdots, p.$

A state observer is designed as:

$$\dot{\hat{e}} = A\hat{e} - BK^T\hat{e} + K_o[e_1 - C^T\hat{e}]$$ B

$$\hat{e}_1 = C^T\hat{e}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

6. Application of adaptive fuzzy control to the MIMO diesel engine model

6.1. Differential flatness of the diesel engine

By **applying differential flatness theory, and in the presence of disturbances**, the dynamic model of the Diesel engine comes to the form

$$\ddot{x}_1 = f_1(x,t) + g_1(x,t)u + d_1$$
$$\ddot{x}_3 = f_2(x,t) + g_2(x,t)u + d_2 \qquad \text{C}$$

The following **control input** is defined:

$$u = \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} \qquad \text{D}$$

where: $[u_{c_1}\ u_{c_2}]^T$ is a **robust control term** that is used for the compensation of the model's uncertainties as well as of the external disturbances

and: $K_i^T = [k_1^i, k_2^i, \cdots, k_{n-1}^i, k_n^i]$. is the feedback gain

Substituting the control input D into the system C one obtains

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

6. Application of adaptive fuzzy control to the MIMO diesel engine model

6.1. Differential flatness of the diesel engine

Moreover, using again Eq. D one obtains the **tracking error dynamics**

$$\begin{pmatrix} \ddot{e}_1 \\ \ddot{e}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{pmatrix} u - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

The **approximation error** is defined as:

$$w = \begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{pmatrix} u$$

Using matrices A,B;K, and considering that **the estimated state vector is used in the control loop** the following description of the tracking error dynamics is obtained:

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\left\{ \begin{pmatrix} f_1(x,t) - \hat{f}_1(\hat{x},t) \\ f_2(x,t) - \hat{f}_2(\hat{x},t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(\hat{x},t) \\ g_2(x,t) - \hat{g}_2(\hat{x},t) \end{pmatrix} u + \tilde{d} \right\}$$

When the **estimated state vector** is used in the loop the **approximation error is written as**

$$w = \begin{pmatrix} f_1(x,t) - \hat{f}_1(\hat{x},t) \\ f_2(x,t) - \hat{f}_2(\hat{x},t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(\hat{x},t) \\ g_2(x,t) - \hat{g}_2(\hat{x},t) \end{pmatrix} u$$

while the **tracking error dynamics becomes**

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + Bw + B\tilde{d}$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

6. Application of adaptive fuzzy control to the MIMO diesel engine model

6.2. Dynamics of the observation error

The **observation error** is defined as: $\tilde{e} = e - \hat{e} = x - \hat{x}$.

By subtracting Eq. B from Eq. A one obtains:

$$\dot{e} - \dot{\hat{e}} = A(e - \hat{e}) + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T(e - \hat{e})$$

$$e_1 - \hat{e}_1 = C^T(e - \hat{e})$$

or equivalently:

$$\dot{\tilde{e}} = A\tilde{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T\tilde{e}$$

$$\tilde{e}_1 = C^T\tilde{e}$$

which can be also written as:

$$\dot{\tilde{e}} = (A - K_o C^T)\tilde{e} + Bu_c + Bw + \tilde{d}\}$$

$$\tilde{e}_1 = C^T\tilde{e}$$

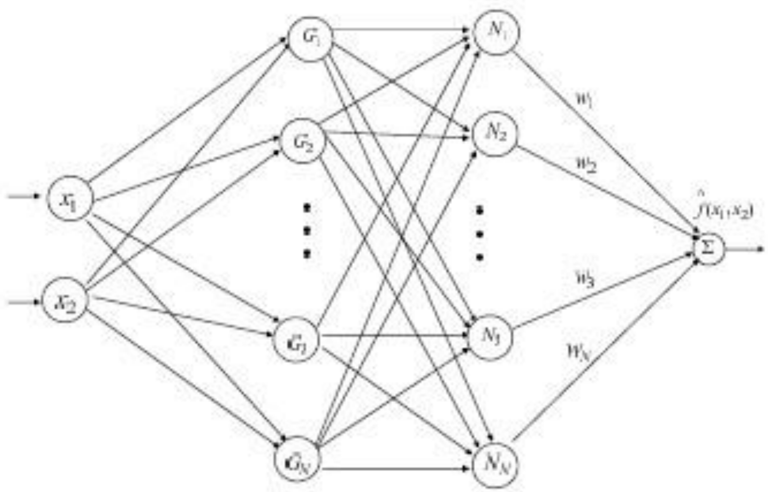
Example 4: Nonlinear control and state estimation using Lyapunov methods

6. Application of adaptive fuzzy control to the MIMO diesel engine model

6.3. Approximation of functions f(x,t) and g(x,t)

Next, the **first of the approximators** of the unknown system dynamics is defined

$$\hat{f}(\hat{x}) = \begin{pmatrix} \hat{f}_1(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_1(\hat{x}|\theta_f) & \in & R^{1\times 1} \\ \hat{f}_2(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_2(\hat{x}|\theta_f) & \in & R^{1\times 1} \end{pmatrix}$$



containing kernel functions $\phi_f^{l}(\hat{x}) = \frac{\prod_{j=1}^{n}\mu_{A_j^l}(\hat{x}_j)}{\sum_{l=1}^{N}\prod_{j=1}^{n}\mu_{A_j^l}(\hat{x}_j)}$

where $\mu_{A_j^l}(\hat{x})$ are fuzzy membership functions

appearing in the antecedent part of the *l-th* fuzzy rule

Example 4: Nonlinear control and state estimation using Lyapunov methods

6. Application of adaptive fuzzy control to the MIMO diesel engine model

6.3. Approximation of functions f(x,t) and g(x,t)

Similarly, **the second of the approximators** of the unknown system dynamics is defined

$$\hat{g}(\hat{x}) = \begin{pmatrix} \hat{g}_1(\hat{x}|\theta_g) \ \hat{x}\in R^{4\times 1} \ \hat{g}_1(\hat{x}|\theta_g) \ \in \ R^{1\times 2} \\ \hat{g}_2(\hat{x}|\theta_g) \ \hat{x}\in R^{4\times 1} \ \hat{g}_2(\hat{x}|\theta_g) \ \in \ R^{1\times 2} \end{pmatrix}$$

The **values of the weights that result in optimal approximation** are

$$\theta_f^* = arg\ min_{\theta_f\in M_{\theta_f}}[sup_{\hat{x}\in U_{\hat{x}}}(f(x) - \hat{f}(\hat{x}|\theta_f))]$$
$$\theta_g^* = arg\ min_{\theta_g\in M_{\theta_g}}[sup_{\hat{x}\in U_{\hat{x}}}(g(x) - \hat{g}(\hat{x}|\theta_g))]$$

The variation ranges for the weights are given by

$$M_{\theta_f} = \{\theta_f\in R^h : \ ||\theta_f||\leq m_{\theta_f}\}$$
$$M_{\theta_g} = \{\theta_g\in R^h : \ ||\theta_g||\leq m_{\theta_g}\}$$

The **value of the approximation error** that corresponds to the optimal values of the weights vectors is

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f^*)\right) + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

6. Application of adaptive fuzzy control to the MIMO diesel engine model

6.3. Approximation of functions f(x,t) and g(x,t)

which is next written as

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f) + \hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)\right) + \\ + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g) + \hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

which can be also written in the following form

$$w = (w_a + w_b)$$

with

$$w_a = \{[f(x,t) - \hat{f}(\hat{x}|\theta_f)] + [g(x,t) - \hat{g}(\hat{x}|\theta_g)]\} u$$

and

$$w_b = \{[\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x},\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]\} u$$

Moreover, the following **weights error vectors are defined**

$$\tilde{\theta}_f = \theta_f - \theta_f^*$$
$$\tilde{\theta}_g = \theta_g - \theta_g^*$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

The **following Lyapunov function** is considered:

$$V = \frac{1}{2}\hat{e}^T P_1 \hat{e} + \frac{1}{2}\bar{e}^T P_2 \bar{e} + \frac{1}{2\gamma_1}\bar{\theta}_f^T \bar{\theta}_f + \frac{1}{2\gamma_2} tr[\bar{\theta}_g^T \bar{\theta}_g]$$

The selection of the **Lyapunov function** is based on the following principle of indirect adaptive control

$$\hat{e} : \lim_{t \to \infty} \hat{x}(t) = x_d(t)$$
$$\bar{e} : \lim_{t \to \infty} \hat{x}(t) = x(t).$$

this results into

$$\lim_{t \to \infty} x(t) = x_d(t)$$

By deriving the **Lyapunov function** with respect to time one obtains:

$$\dot{V} = \frac{1}{2}\dot{\hat{e}}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \dot{\hat{e}} + \frac{1}{2}\dot{\bar{e}}^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 \dot{\bar{e}} + \\ + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g] \Rightarrow$$

$$\dot{V} = \frac{1}{2}\{(A - BK^T)\hat{e} + K_o C^T \bar{e}\}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1\{(A - BK^T)\hat{e} + K_o C^T \bar{e}\} + \\ + \frac{1}{2}\{(A - K_o C^T)\bar{e} + Bu_c + B\bar{d} + Bw\}^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P_2\{(A - K_o C^T)\bar{e} + Bu_c + B\bar{d} + Bw\} + \\ + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g] \Rightarrow$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

The equation is rewritten as:

$$\dot{V} = \frac{1}{2}\{\hat{e}^T(A-BK^T)^T + \tilde{e}^T C K_o^T\} P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1\{(A-BK^T)\hat{e} + K_o C^T \tilde{e}\} +$$
$$+\frac{1}{2}\{\tilde{e}^T(A-K_oC^T)^T + u_c^T B^T + w^T B^T + \tilde{d}^T B^T\} P_2 \tilde{e} +$$
$$\frac{1}{2}\tilde{e}^T P_2\{(A-K_oC^T)\tilde{e} + Bu_c + Bw + B\tilde{d}\} + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow$$

which finally takes the form:

$$\dot{V} = \frac{1}{2}\hat{e}^T(A-BK^T)^T P_1 \hat{e} + \frac{1}{2}\tilde{e}^T C K_o^T P_1 \hat{e} +$$
$$+\frac{1}{2}\hat{e}^T P_1 (A-BK^T)\hat{e} + \frac{1}{2}\hat{e}^T P_1 K_o C^T \tilde{e} +$$
$$+\frac{1}{2}\tilde{e}^T(A-K_oC^T)^T P_2 \tilde{e} + \frac{1}{2}(u_c^T + w^T + \tilde{d}^T) B^T P_2 \tilde{e} +$$
$$+\frac{1}{2}\tilde{e}^T P_2 (A-K_oC^T)\tilde{e} + \frac{1}{2}\tilde{e}^T P_2 B(u_c + w + \tilde{d}) +$$
$$+\frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Assumption 1: For given positive definite matrices Q_1 and Q_2 there exist positive definite matrices P_1 and P_2, which are the solution of the following **Riccati equations**

$$(A-BK^T)^T P_1 + P_1(A-BK^T) + Q_1 = 0$$

$$(A-K_oC^T)^T P_2 + P_2(A-K_oC^T) -$$
$$-P_2 B\left(\frac{2}{r} - \frac{1}{\rho^2}\right) B^T P_2 + Q_2 = 0$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

By substituting the conditions from the previous Riccati equations into the derivative of the Lyapunov function one gets:

$$\begin{aligned}\dot{V} = \tfrac{1}{2}\hat{e}^T\{(A-BK^T)^T P_1 + P_1(A-BK^T)\}\hat{e} + \tilde{e}^T C K_o^T P_1 \hat{e} + \\ + \tfrac{1}{2}\tilde{e}^T\{(A-K_o C^T)^T P_2 + P_2(A-K_o C^T)\}\tilde{e} + \\ + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \tfrac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]\end{aligned}$$

or:

$$\begin{aligned}\dot{V} = -\tfrac{1}{2}\hat{e}^T Q_1 \hat{e} + \tilde{e}^T C K_o^T P_1 \hat{e} - \tfrac{1}{2}\tilde{e}^T\{Q_2 - P_2 B(\tfrac{2}{r} - \tfrac{1}{\rho^2})B^T P_2\}\tilde{e} + \\ + \tilde{e}^T P_2 B(u_c + w + \tilde{d}) + \tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \tfrac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]\end{aligned}$$

The **supervisory control term** u_c consists of two terms:

$$u_a = -\frac{1}{r} B^T P_2 \tilde{e}$$

$$u_b = -[(P_2 B)^T (P_2 B)]^{-1} (P_2 B)^T C K_o^T P_1 \hat{e}$$

u_a is an H_∞ control used for the compensation of the approximation error w and the additive disturbance $\tilde{d}$ (the control term u_a has been chosen so as to satisfy the condition

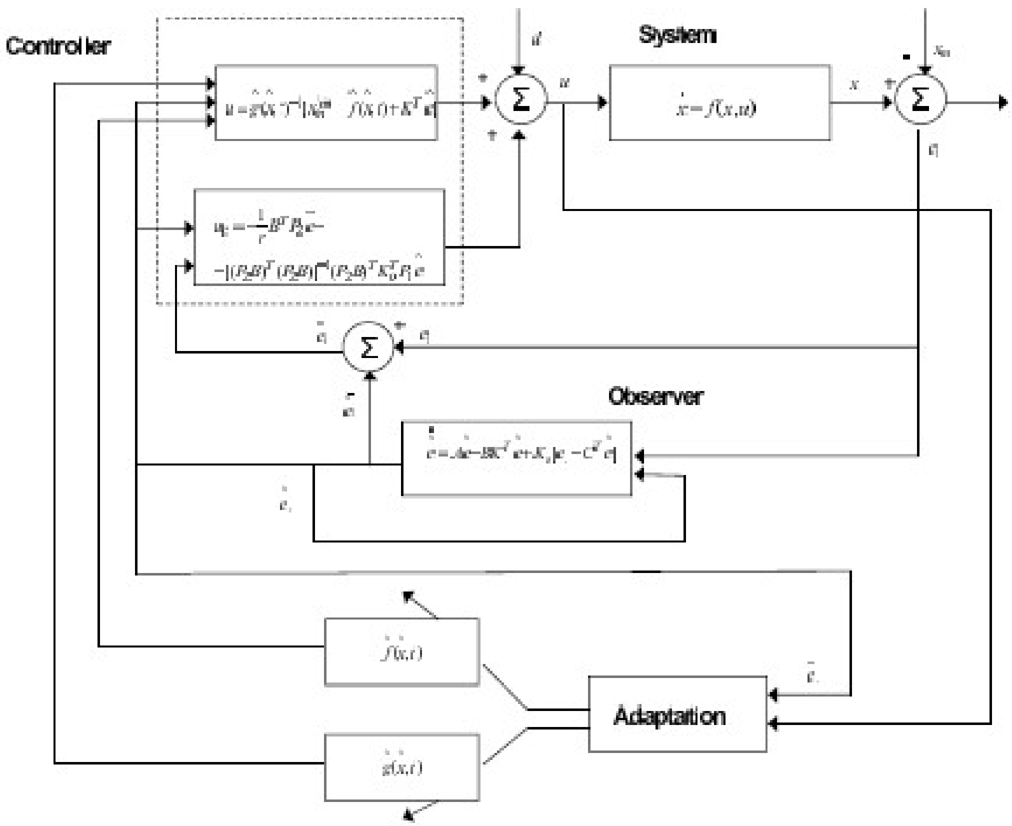
$$\tilde{e} P_2 B u_a = -\tfrac{1}{r} P_2 B B^T P_2 \tilde{e}.$$

u_b is a control used for the compensation of the observation error (the control term u_b has been chosen so as to satisfy the condition $\tilde{e}^T P_2 B u_b = -\tilde{e}^T C K_o^T P_1 \hat{e}$.

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

The control scheme is depicted in the following diagram



Substituting the supervisory control term in the derivative of the Lyapunov function gives:

$$\dot{V} = -\frac{1}{2}\hat{e}^TQ_1\hat{e} + \bar{e}^TCK_o^TP_1\hat{e} - \frac{1}{2}\bar{e}^TQ_2\bar{e} + \frac{1}{r}\bar{e}^TP_2BB^TP_2\bar{e} - \frac{1}{2\rho^2}\bar{e}^TP_2BB^TP_2\bar{e} +$$
$$+\bar{e}^TP_2Bu_a + \bar{e}^TP_2Bu_b + \bar{e}^TP_2B(w+\tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

or equivalently

$$\dot V = -\frac{1}{2}\hat e^T Q_1 \hat e - \frac{1}{2}\bar e^T Q_2 \bar e - \frac{1}{2\rho^2}\bar e^T P_2 B B^T P_2 \bar e + \\ + \bar e^T P_2 B(w + \bar d) + \frac{1}{\gamma_1}\dot{\bar\theta}_f^T \bar\theta_f + \frac{1}{\gamma_2} tr[\dot{\bar\theta}_g^T \bar\theta_g]$$

Besides, about the adaptation of the weights of the neurofuzzy network it holds

$$\dot{\bar\theta}_f = \dot\theta_f - \dot\theta_f^* = \dot\theta_f \qquad \dot{\bar\theta}_g = \dot\theta_g - \dot\theta_g^* = \dot\theta_g.$$

and also

$$\dot\theta_f = -\gamma_1 \Phi(\hat x)^T B^T P_2 \bar e$$
$$\dot\theta_g = -\gamma_2 \Phi(\hat x)^T B^T P_2 \bar e u^T$$

By substituting the above relations in the derivative of the Lyapunov function one obtains

$$\dot V = -\frac{1}{2}\hat e^T Q_1 \hat e - \frac{1}{2}\bar e^T Q_2 \bar e - \frac{1}{2\rho^2}\bar e^T P_2 B B^T P_2 \bar e + B^T P_2 \bar e(w + d) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\bar e^T P_2 B \Phi(\hat x)(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[u \bar e^T P_2 B \Phi(\hat x)(\theta_g - \theta_g^*)]$$

or

$$\dot V = -\frac{1}{2}\hat e^T Q_1 \hat e - \frac{1}{2}\bar e^T Q_2 \bar e - \frac{1}{2\rho^2}\bar e^T P_2 B B^T P_2 \bar e + B^T P_2 \bar e(w + \bar d) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\bar e^T P_2 B \Phi(\hat x)(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[u \bar e^T P_2 B(\hat g(\hat x|\theta_g) - \hat g(\hat x|\theta_g^*))]$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

Taking into account that $u \in R^{2\times 1}$ and $\bar{e}^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1\times 2}$

one gets

$$\begin{aligned}\dot{V} = &-\tfrac{1}{2}\hat{e}^T Q_1 \hat{e} - \tfrac{1}{2}\bar{e}^T Q_2 \bar{e} - \tfrac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \tilde{d}) + \\ &+ \tfrac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ &+ \tfrac{1}{\gamma_2}(-\gamma_2) tr[\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u]\end{aligned}$$

Since $\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \in R^{1\times 1}$

it holds

$$\begin{aligned}&tr(\bar{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)u) = \\ &= \bar{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u\end{aligned}$$

Therefore, one finally obtains

$$\begin{aligned}\dot{V} = &-\tfrac{1}{2}\hat{e}^T Q_1 \hat{e} - \tfrac{1}{2}\bar{e}^T Q_2 \bar{e} - \tfrac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \tilde{d}) + \\ &+ \tfrac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ &+ \tfrac{1}{\gamma_2}(-\gamma_2)\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u\end{aligned}$$

Next, the following **approximation error** is defined

$$w_a = [\hat{f}(\hat{x}|\theta_f^*) - \hat{f}(\hat{x}|\theta_f)] + [\hat{g}(\hat{x}|\theta_g^*) - \hat{g}(\hat{x}|\theta_g)]u$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

Thus, one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + B^T P_2 \bar{e}(w + \tilde{d}) + \bar{e}^T P_2 B w_a$$

Denoting the **aggregate approximation e**rror and disturbances vector as

$$w_1 = w + \tilde{d} + w_a$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \bar{e}^T P_2 B w_1$$

which in turn is written as

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e}$$

Lemma: The following inequality holds

$$\frac{1}{2}\bar{e}^T P_2 B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \\ \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

Proof: The binomial $(\rho a - \frac{1}{\rho} b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab \geq 0 \Rightarrow$$
$$\frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow$$
$$\frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2$$

By substituting $a = w_1$ and $b = \bar{e}^T P_2 B$ one gets

$$\frac{1}{2} w_1^T B^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 B w_1 - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Moreover, by substituting the above inequality into the derivative of the Lyapunov function one gets

$$\dot{V} \leq -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{2}\rho^2 w_1^T w_1$$

which is also written as $\dot{V} \leq -\frac{1}{2}E^T Q E + \frac{1}{2}\rho^2 w_1^T w_1$

with

$$E = \begin{pmatrix} \hat{e} \\ \bar{e} \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} = diag[Q_1, Q_2]$$

Example 4: Nonlinear control and state estimation using Lyapunov methods

7. Lyapunov stability analysis

Hence, the H_∞ performance criterion is derived. For sufficiently small ρ the inequality will be true and the H_∞ tracking criterion will be satisfied. In that case, the integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||E||^2 dt + \frac{1}{2}\rho^2 \int_0^T ||w_1||^2 dt \Rightarrow$$
$$2V(T) - 2V(0) \leq -\int_0^T ||E||_Q^2 dt + \rho^2 \int_0^T ||w_1||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||E||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||w_1||^2 dt$$

It is assumed that there exists a positive constant $M_w > 0$ such that

$$\int_0^\infty ||w_1||^2 dt \leq M_w$$

Therefore for the integral $\int_0^T ||E||_Q^2 dt$ one gets

$$\int_0^\infty ||E||_Q^2 dt \leq 2V(0) + \rho^2 M_w$$

Thus, the integral $\int_0^\infty ||E||_Q^2 dt$ is bounded and **according to Barbalat's Lemma**

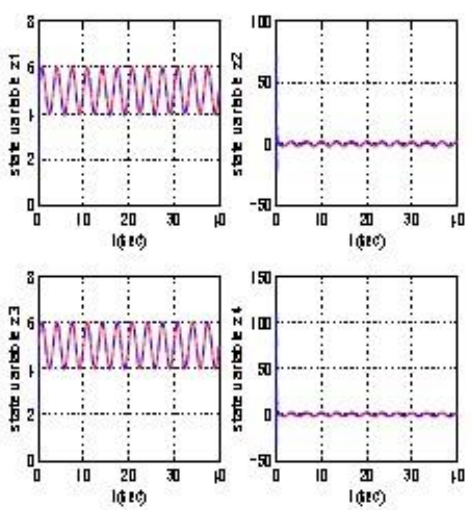
$$\lim_{t\to\infty} e(t) = 0$$

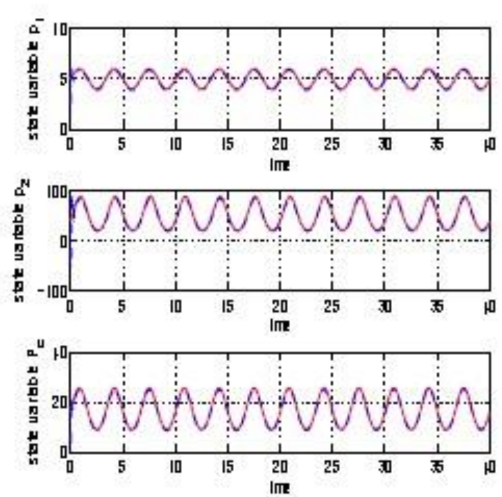
Example 4: Nonlinear control and state estimation using Lyapunov methods

8. Simulation tests

The performance of the proposed observer-based adaptive fuzzy MIMO controller was tested in the **MIMO nonlinear model of the turbocharged Diesel engine**

The fuzzy rule base used for the approximation of the unknown dynamics of the diesel engine comprised **81 rules**



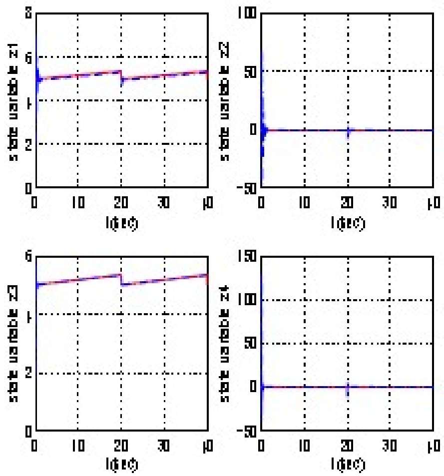


(a) Tracking of set-point 1 by the state variables z of the transformed model

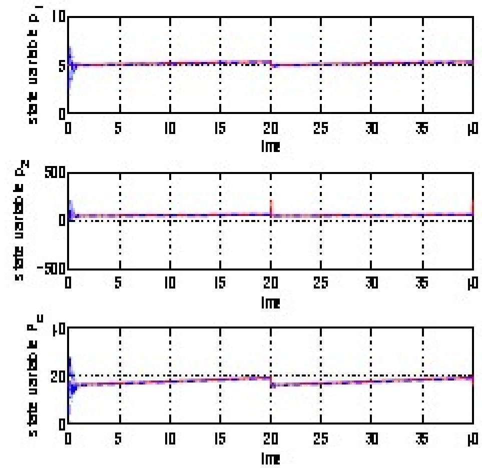
(b) Tracking of set-point 1 by the state variables x of the initial model

Example 4: Nonlinear control and state estimation using Lyapunov methods

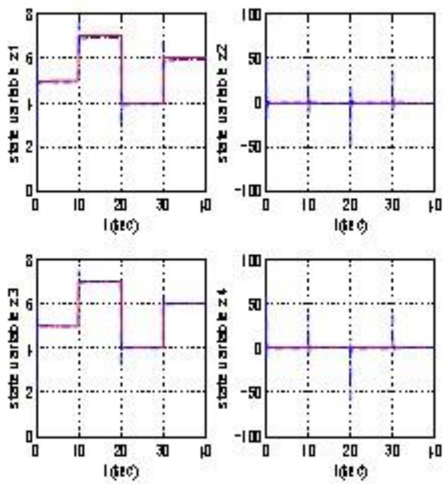
8. Simulation tests



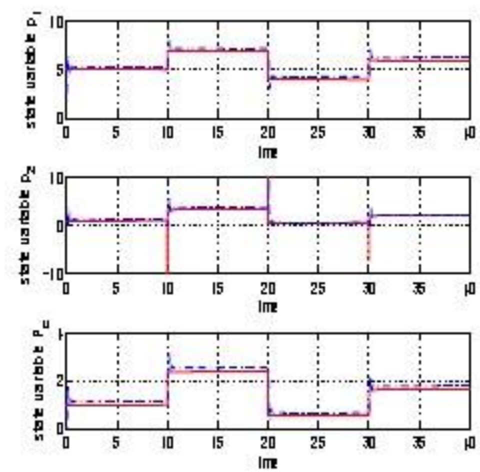
(a) Tracking of set-point 2 by the state variables z of the transformed model



(b) Tracking of set-point 2 by the state variables x of the initial model



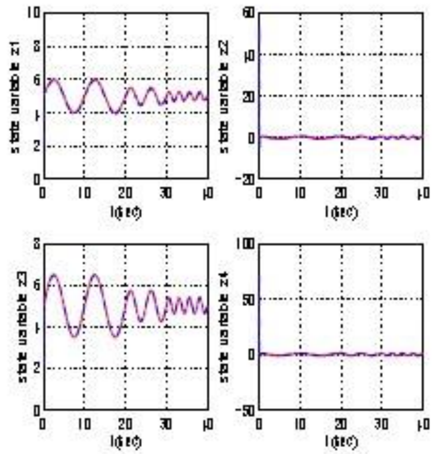
(a) Tracking of set-point 3 by the state variables z of the transformed model

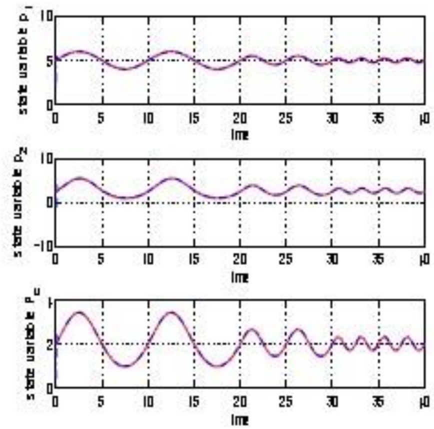


(b) Tracking of set-point 3 by the state variables x of the initial model

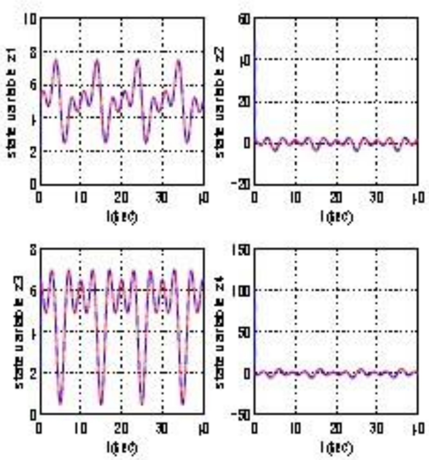
Example 4: Nonlinear control and state estimation using Lyapunov methods

8. Simulation tests

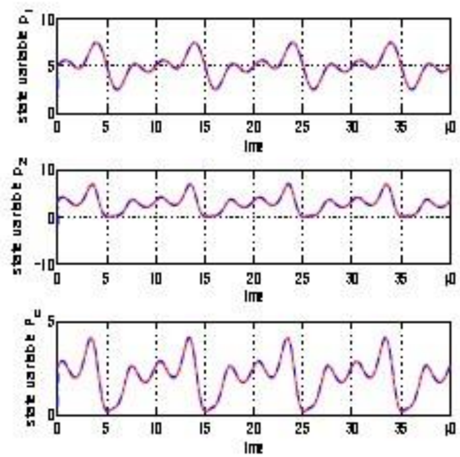




(a) Tracking of set-point 4 by the state variables z of the transformed model



(a) Tracking of set-point 4 by the state variables z of the transformed model



(b) Tracking of set-point 5 by the state variables x of the initial model

Example 4: Nonlinear control and state estimation using Lyapunov methods

8. Simulation tests

The simulation tests confirmed the **disturbance rejection capability of the control loop**. **No prior knowledge of the diesel engine's dynamics was required**.

It can be observed that the proposed adaptive fuzzy control scheme achieved fast and accurate tracking of all these setpoints..

Table I: RMSE of Diesel engine's state variables

parameter	p_1	p_2	P_c
$RMSE_a$	0.0001	0.0004	0.0002
$RMSE_b$	0.0202	0.0204	0.0055
$RMSE_c$	0.0079	0.0411	0.0087
$RMSE_d$	0.0001	0.0009	0.0005
$RMSE_e$	0.0001	0.0215	0.0128

The RMSE (root mean square error) of the examined control loop is also calculated (assuming the same parameters of the controller) in the case of tracking of the previous setpoints 1 to 5.

The results are summarized in Table I. From the simulation diagrams it can be confirmed that the transient characteristics of the control scheme are also quite satisfactory

Example 4: Nonlinear control and state estimation using Lyapunov methods

9. Conclusions

• It has been shown that the **extended state-space model of the turbocharged diesel engine** admits **dynamic feedback linearization** and that by applying differential flatness properties it can be transformed into the **MIMO canonical (Brunovsky) form**..

• The nonlinear terms which appear in the transformed control inputs contained unknown parameters and had to **be approximated with the use of neuro-fuzzy networks**.

• Moreover, since only the system's output is measurable the complete **state vector** had to be **reconstructed with the use of a state observer**.

• It has been shown that a suitable **learning law** can be defined for the aforementioned **neuro-fuzzy approximators** so as to preserve the closed-loop system stability.

• With the use of **Lyapunov stability analysis** it has also been proven that the proposed observer-based adaptive fuzzy control scheme results in H_∞ tracking performance, while global stability has been also proven

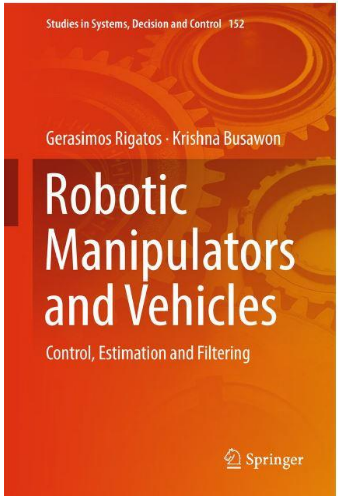
• For the design of the observer-based adaptive fuzzy controller **one had to solve two Riccati equations**, where the first one was associated with the controller and the second one was associated with the observer

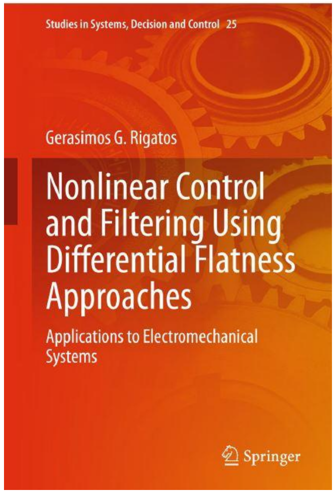
V. Final conclusions

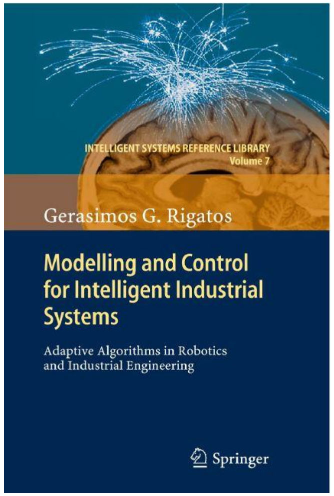
● Methods of nonlinear control and state estimation for optimized propulsion in USVs and AUVs have been developed

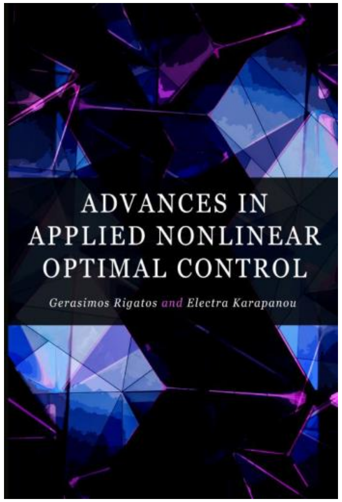
● The main approaches for nonlinear control have been: (i) **control with global linearization** method (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods (adaptive control)** in case that the model of the propulsion system of the USVs and AUVs is unknown

● The main approaches for nonlinear state estimation are: (i) nonlinear state estimation with methods of global linearization (ii) nonlinear state estimation with methods of approximate (asymptotic) linearization









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