Lecture on

New approaches to gradient-based optimization for modelling and control of nonlinear systems of uncertain dynamics:

applications to robotics and electric power generation

Gerasimos Rigatos

Unit of Industrial Automation
Industrial Systems Institute
26504, Rion Patras, Greece

email: grigat@ieee.org

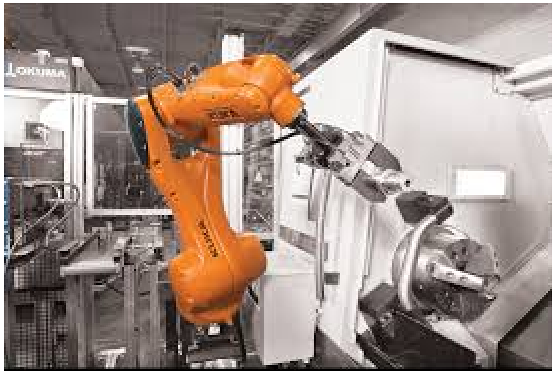
1. Outline

- The functioning of **nonlinear dynamical systems** in real conditions is characterized by **model uncertainty**, **parametric changes** and **external perturbations**.

- Control schemes must perform **simultaneously identification** and **stabilization** of such uncertain dynamics.

- This is a **dual optimization problem** since **modelling errors** and **deviation of the system's state vector elements from the associated setpoints** have to be **minimized** in real-time.

- To achieve these objectives an initial **transformation** (diffeomorphism) of the system's dynamic model to an **equivalent linearized form**, is proposed.



- The **transformed control inputs** consist of **unknown nonlinear functions** which are **identified** with the use of nonlinear regressors.

- **Learning in such networks** is performed through **gradient algorithms** in which the **adaptation rate** (step for the search of an optimum) is defined by conditions for the **minimization of an aggregate energy function** (Lyapunov function).

1. Outline

- In each iteration of the control algorithm, the **estimates of the nonlinear functions** that constitute the system's dynamics are **fed into a state feedback controller**.

- It has been proven that this control approach assures the **minimization of the aforementioned energy function** and thus the nonlinear system becomes **a globally asymptotically stable one**.

- The proposed method can be applied to **all dynamical systems** which satisfy the **differential flatness property**.

- This is the **widest class of nonlinear dynamical systems** to which one can apply **optimization and control with gradient methods**, while assuring the convergence of the optimization procedure and the stability of the control loop.

- The efficiency of the proposed **optimization-based modelling and control approac**h has been confirmed in several test cases, concerning complex nonlinear dynamical systems

- In particular, **the method has been applied to several electromechanical systems**, including robotic systems and electric power generation systems

2. Differential flatness of MIMO nonlinear systems

- **Differential flatness theory** has been developed as a **global linearization control method** by M. Fliess (Ecole Polytechnique, France) and co-researchers (Lévine, Rouchon, Mounier, Rudolph, Petit, Martin, Zhu, Sira-Ramirez et. al)

- A dynamical system can be written in the ODE form $S_i(w,\dot{w},\ddot{w},...,w^{(i)}),\ i=1,2,...,q$

  where $w^{(i)}$ stands for the i-th derivative of either a state vector element or of a control input

- The system is said to be **differentially flat** with respect to the **flat output**

$$y_i=\phi(w,\dot{w},\ddot{w},...,w^{(a)}),\ i=1,...,m \quad \text{where} \quad y=(y_1,y_2,...,y_m)$$

if the following two conditions are satisfied

(i) There does not exist any differential relation of the form

$$R(y,\dot{y},\ddot{y},...,y^{(\beta)})=0$$

which means that **the flat output and its derivatives are linearly independent**

(ii) All system variables are **functions of the flat output and its derivatives**

$$w^{(i)}=\psi(y,\dot{y},\ddot{y},...,y^{(\gamma_i)})$$

2. Differential flatness of MIMO nonlinear systems

The proposed optimization-based control method is based on the **transformation** of the nonlinear system's model into the **linear canonical form**, and this transformation is succeeded by exploiting the system's differential flatness properties

• **All single input nonlinear systems** are differentially flat and can be transformed into the linear canonical form

One has to define also which are the **MIMO nonlinear systems** which are differentially flat.

• Differential flatness holds for **MIMO nonlinear systems** that admit **static feedback linearization.**and which can be transformed into the linear canonical form through a change of variables (diffeomorphism) and feedback of the state vector.

• Differential flatness holds for **MIMO nonlinear models** that admit **dynamic feedback linearization**, This **is the case of specific underactuated robotic models.** In the latter case the state vector of the system is extended by considering as additional flat outputs some of the control inputs and their derivatives

• Finally, a more rare case is the so-called **Liouvillian systems**. These are systems for which differential flatness properties hold for part of their state vector (constituting a flat subsystem) while the non-flat state variables can be obtained by integration of the elements of the flat subsystem.

3. State-space modelling of MIMO nonlinear systems

3.1. Transformation of MIMO nonlinear systems into the Brunovsky form

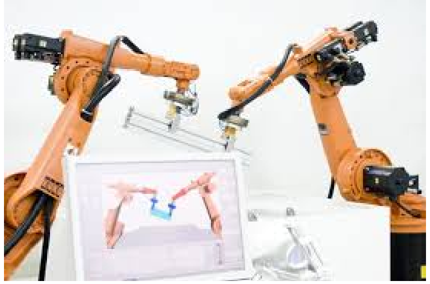
The **initial MIMO nonlinear system** is taken to be in the generic form:

$$\dot{x} = f(x, u)$$

It is assumed now that **after defining the flat outputs of the initial MIMO nonlinear system**, and after expressing the system state variables and control inputs as functions of the flat output and of the associated derivatives, the system can be **transformed in the Brunovsky canonical form**

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\cdots \\
\dot{x}_{r_1-1} &= x_{r_1} \\
\dot{x}_{r_1} &= f_1(x) + \sum_{j=1}^{p} g_{1_j}(x) u_j + d_1 \\
\\
\dot{x}_{r_1+1} &= x_{r_1+2} \\
\dot{x}_{r_1+2} &= x_{r_1+3} \\
&\cdots \\
\dot{x}_{p-1} &= x_p \\
\dot{x}_p &= f_p(x) + \sum_{j=1}^{p} g_{p_j}(x) u_j + d_p
\end{aligned}$$

$$\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_{r_1-1} \\
&\cdots \\
y_p &= x_{n-r_p+1}
\end{aligned}$$



$x = [x_1, \cdots, x_n]^T$: is the state vector

$u = [u_1, \cdots, u_p]^T$: is the inputs vector

$y = [y_1, \cdots, y_p]^T$: is the outputs vector

3. State-space modelling of MIMO nonlinear systems

3.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Next **the following vectors and matrices** can be defined

$$f(x)=[f_1(x), \quad ..., \quad f_n(x)]^T$$
$$g(x)=[g_1(x), \quad ..., \quad g_n(x)]^T$$

with $g_i(x)=[g_{1i}(x), \quad ..., \quad g_{pi}(x)]^T$

$$A=diag[A_1,...,A_p], \ B=diag[B_1,...,B_p]$$
$$C^T=diag[C_1,...,C_p], \ d=[d_1,...,d_p]^T$$

where matrix A has the **MIMO canonical form**, i.e. with elements

$$A_i=\begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 \\ 0 & 0 & 0 & ... & 0 \end{bmatrix}_{r_i \times r_i}$$

$$B_i^T=[0 \quad 0 \quad ... \quad 0 \quad 1]_{1\times r_i} \qquad C_i=[1 \quad 0 \quad ... \quad 0 \quad 0]_{1\times r_i}$$

Thus, the initial nonlinear system can be written in the **state-space form**

$$\dot{x}=Ax+B[f(x)+g(x)u+\tilde{d}]$$
$$y=Cx$$

or equivalently in the state space form

$$\dot{x}=Ax+Bv+B\tilde{d}$$
$$y=Cx$$

where $v=f(x)+g(x)u$

For the generic case of the **MIMO nonlinear system** it is assumed that the functions $f(x)$ and $g(x)$ are unknown and have to be approximated by **nonlinear regressors** (e.g. neuro-fuzzy networks)

3. State-space modelling of MIMO nonlinear systems

3.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Thus, the nonlinear system can be written in **state-space form**

$$\dot{x} = Ax + B[f(x) + g(x)u + \tilde{d}]$$
$$y = C^T x$$

which equivalently can be written as

$$\dot{x} = Ax + Bv + B\tilde{d}$$
$$y = C^T x$$

where $v = f(x) + g(x)u.$

The **reference setpoints** for the system's outputs $y_1, \cdots, y_p$:

are denoted as $y_{1m}, \cdots, y_{pm}$ and the associated tracking errors are defined as

$$e_1 = y_1 - y_{1m}$$
$$e_2 = y_2 - y_{2m}$$
$$\cdots$$
$$e_p = y_p - y_{pm}$$

The **error vector of the outputs** of the transformed MIMO system is denoted as

$$E_1 = [e_1, \cdots, e_p]^T$$
$$y_m = [y_{1m}, \cdots, y_{pm}]^T$$
$$\cdots$$
$$y_m^{(r)} = [y_{1m}^{(r)}, \cdots, y_{pm}^{(r)}]^T$$

3. State-space modelling of MIMO nonlinear systems

3.2. Control law under measurable state vector

The **control signal** $v = f(x) + g(x)u$, **of the MIMO nonlinear system** contains the **unknown nonlinear functions** *f(x)* and *g(x)* which can be approximated by

$$\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f, \quad \hat{g}(x|\theta_g) = \Phi_g(x)\theta_g$$

where $\Phi_f(x) = \left(\xi_f^1(x), \xi_f^2(x), \cdots \xi_f^n(x)\right)^T,$

$$\xi_f^i(x) = \left(\phi_f^{i,1}(x), \phi_f^{i,2}(x), \cdots, \phi_f^{i,N}(x)\right)$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_f^T = \left(\theta_f^1, \theta_f^2, \cdots \theta_f^N\right)$.

3. State-space modelling of MIMO nonlinear systems

3.2. Control law under measurable state vector

Similarly, it holds $\Phi_g(x) = (\xi_g^1(x), \xi_g^2(x), \cdots \xi_g^N(x))^T$

$$\xi_g^i(x) = (\phi_g^{i,1}(x), \phi_g^{i,2}(x), \cdots, \phi_g^{i,N}(x)).$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_g = (\theta_g^1, \theta_g^2, \cdots, \theta_g^p)^T$.

However, here each row of θ_g is vector thus giving

$$\theta_g = \begin{pmatrix} \theta_{g1}^1 & \theta_{g1}^2 & \cdots & \theta_{g1}^p \\ \theta_{g2}^1 & \theta_{g2}^2 & \cdots & \theta_{g2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{gN}^1 & \theta_{gN}^2 & \cdots & \theta_{gN}^p \end{pmatrix}$$

If the state variables of the system are available for measurement then a **state-feedback control law can be formulated** as

$$u = \hat{g}^{-1}(x|\theta_g)[-\hat{f}(x|\theta_f) + y_m^{(r)} + K_c^T e + u_c]$$

3. State-space modelling of MIMO nonlinear systems

3.2. Control law under non-measurable state vector

The control of the system $\dot{x} = f(x,u)$ becomes more complicated **when the state vector x is not directly measurable** and has to be reconstructed through a state observer. The following definitions are used

$e = x - x_m$: is the error of the state vector

$\hat{e} = \hat{x} - x_m$ is the error of the estimated state vector

$\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$ is the observation error

When an **observer is used to reconstruct the state vector**, the control law

$$u = \hat{g}^{-1}(\hat{x}|\theta_g)[-\hat{f}(\hat{x}|\theta_f) + y_m^{(r)} - K^T\hat{e} + u_c]$$

By applying the previous feedback control law one obtains the closed-loop dynamics

$$y^{(r)} = f(x) + g(x)\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(r)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(r)} = f(x) + [g(x) - \hat{g}(\hat{x}) + \hat{g}(\hat{x})]\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(r)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(r)} = [f(x) - \hat{f}(\hat{x})] + [g(x) - \hat{g}(\hat{x})]u + y_m^{(r)} - K^T\hat{e} + u_c + d$$

It holds $e = x - x_m \Rightarrow y^{(r)} = e^{(r)} + y_m^{(r)}$

and by substituting $y^{(r)}$ in the **previous feedback control loop dynamics** gives

3. State-space modelling of MIMO nonlinear systems

3.2. Control law under non-measurable state vector

the **tracking error dynamics**

$$e^{(n)} + y_m^{(n)} = y_m^{(n)} - K^T\hat{e} + u_c + [f(x) - \hat{f}(\hat{x})] + \\ +[g(x) - \hat{g}(\hat{x})]u + d$$

or equivalently

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\{[f(x) - \hat{f}(\hat{x})] + \\ +[g(x) - \hat{g}(\hat{x})]u + d\}$$ (A)

$$e_1 = C^T e$$

where $e = [e^1, e^2, \cdots, e^p]^T$ with $e^i = [e_i, \dot{e}_i, \ddot{e}_i, \cdots, e_i^{n_i - 1}]^T,\ i = 1, 2, \cdots, p$

and equivalently $\hat{e} = [\hat{e}^1, \hat{e}^2, \cdots, \hat{e}^p]^T$ with $\hat{e}^i = [\hat{e}_i, \dot{\hat{e}}_i, \ddot{\hat{e}}_i, \cdots, \hat{e}_i^{n_i - 1}]^T,\ i = 1, 2, \cdots, p.$

A **state observer** is designed as:

$$\dot{\hat{e}} = A\hat{e} - BK^T\hat{e} + K_o[e_1 - C^T\hat{e}]$$ (B)

$$\hat{e}_1 = C^T\hat{e}$$

4. An application example of optimization-based control

4.1. Dynamics of the tracking error

Without loss of generality consider a two-input MIMO system:

By **applying differential flatness theory, and in the presence of disturbances**, the dynamic model of the system comes to the form

$$\ddot{x}_1 = f_1(x,t) + g_1(x,t)u + d_1$$
$$\ddot{x}_3 = f_2(x,t) + g_2(x,t)u + d_2 \qquad (C)$$

The following **control input** is defined:

$$u = \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} \qquad (D)$$

where: $[u_{c_1} \; u_{c_2}]^T$ is a **robust control term** that is used for the compensation of the model's uncertainties as well as of the external disturbances

and: $K_i^T = [k_1^i, k_2^i, \cdots, k_{n-1}^i, k_n^i]$ is the feedback gain

Substituting the control input (D) into the system (C) one obtains

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot \left\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \right\} + \begin{pmatrix} d_1 \\ d_2 \end{pmatrix}$$

4. An application example of optimization-based control

4.1. Dynamics of the tracking error

Moreover, using again Eq. (D) one obtains the **tracking error dynamics**

$$\begin{pmatrix}\ddot{e}_1\\ \ddot{e}_3\end{pmatrix} = \begin{pmatrix} f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u - \begin{pmatrix}K_1^T\\ K_2^T\end{pmatrix}e + \begin{pmatrix}u_{c_1}\\ u_{c_2}\end{pmatrix} + \begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

The **approximation error** is defined as:

$$w = \begin{pmatrix} f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u$$

Using matrices A,B;K, and considering that **the estimated state vector is used in the control loop** the following description of the tracking error dynamics is obtained:

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\left\{\begin{pmatrix} f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u + \tilde{d}\right\}$$

When the **estimated state vector** is used in the loop the **approximation error is written as**

$$w = \begin{pmatrix} f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u$$

while the **tracking error dynamics becomes**

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + Bw + B\tilde{d}$$

4. An application example of optimization-based control

4.2. Dynamics of the observation error

The **observation error** is defined as: $\bar{e} = e - \hat{e} = x - \hat{x}$.

By subtracting Eq. (B) from Eq. (A) one obtains:

$$\dot{e} - \dot{\hat{e}} = A(e - \hat{e}) + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + \\ +[g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T(e - \hat{e})$$

$$e_1 - \hat{e}_1 = C^T(e - \hat{e})$$

or equivalently:

$$\dot{\bar{e}} = A\bar{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T\bar{e}$$

$$\bar{e}_1 = C^T\bar{e}$$

which can be also written as:

$$\dot{\bar{e}} = (A - K_o C^T)\bar{e} + Bu_c + Bw + \tilde{d}$$

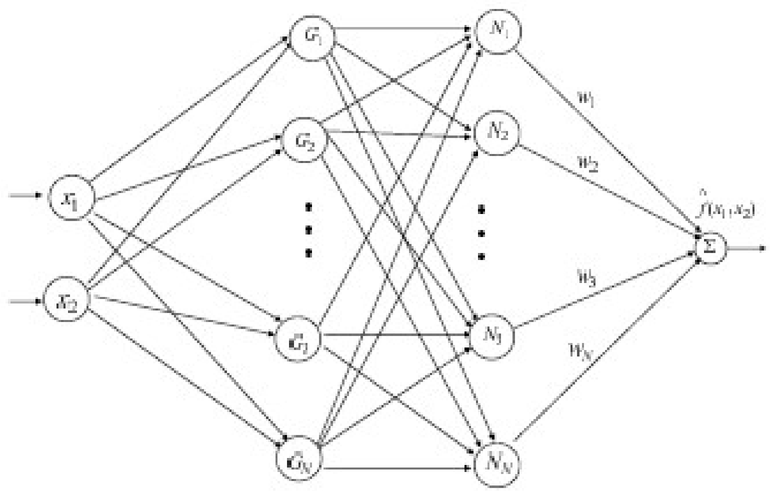
$$\bar{e}_1 = C^T\bar{e}$$

4. An application example of optimization-based control

4.3. Approximation of the unknown system dynamics

Next, **the first of the approximators** of the unknown system dynamics is defined

$$\hat{f}(\hat{x}) = \begin{pmatrix} \hat{f}_1(\hat{x}|\theta_f) \ \hat{x}\in R^{4\times 1} \ \hat{f}_1(\hat{x}|\theta_f) & \in & R^{1\times 1} \\ \hat{f}_2(\hat{x}|\theta_f) \ \hat{x}\in R^{4\times 1} \ \hat{f}_2(\hat{x}|\theta_f) & \in & R^{1\times 1} \end{pmatrix}$$



containing kernel functions $\phi_f^{i,j}(\hat{x}) = \frac{\prod_{j=1}^{n}\mu_{A_j^i}(\hat{x}_j)}{\sum_{i=1}^{N}\prod_{j=1}^{n}\mu_{A_j^i}(\hat{x}_j)}$

where $\mu_{A_j^i}(\hat{x})$ are fuzzy membership functions

appearing in the antecedent part of the *l-th* fuzzy rule

4. An application example of optimization-based control

4.3. Approximation of the unknown system dynamics

Similarly, **the second of the approximators** of the unknown system dynamics is defined

$$\hat{g}(\hat{x}) = \begin{pmatrix} \hat{g}_1(\hat{x}|\theta_g) & \hat{x}\in R^{4\times 1} & \hat{g}_1(\hat{x}|\theta_g) & \in & R^{1\times 2} \\ \hat{g}_2(\hat{x}|\theta_g) & \hat{x}\in R^{4\times 1} & \hat{g}_2(\hat{x}|\theta_g) & \in & R^{1\times 2} \end{pmatrix}$$

The **values of the weights that result in optimal approximation** are

$$\theta_f^* = arg\ min_{\theta_f\in M_{\theta_f}}[sup_{\hat{x}\in U_{\hat{x}}}(f(x) - \hat{f}(\hat{x}|\theta_f))]$$
$$\theta_g^* = arg\ min_{\theta_g\in M_{\theta_g}}[sup_{\hat{x}\in U_{\hat{x}}}(g(x) - \hat{g}(\hat{x}|\theta_g))]$$

The variation ranges for the weights are given by

$$M_{\theta_f} = \{\theta_f\in R^h:\ ||\theta_f||\leq m_{\theta_f}\}$$
$$M_{\theta_g} = \{\theta_g\in R^h:\ ||\theta_g||\leq m_{\theta_g}\}$$

The **value of the approximation error** that corresponds to the optimal values of the weights vectors is

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f^*)\right) + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

4. An application example of optimization-based control

4.3. Approximation of the unknown system dynamics

which is next written as

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f) + \hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)\right) + \\ + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g) + \hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

which can be also written in the following form

$$w = \left(w_a + w_b\right)$$

with

$$w_a = \{[f(x,t) - \hat{f}(\hat{x}|\theta_f)] + [g(x,t) - \hat{g}(\hat{x}|\theta_g)]\} u$$

and

$$w_b = \{[\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x},\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]\} u$$

Moreover, the following **weights error vectors are defined**

$$\tilde{\theta}_f = \theta_f - \theta_f^* \\ \tilde{\theta}_g = \theta_g - \theta_g^*$$

and these denote the distance of the **weights vectors** from the values that provide optimal model estimation

It will be shown that these **weights are updated through a gradient method**

5. Convergence proof for the optimization method

The **following Lyapunov (energy) function** is considered:

$$V = \frac{1}{2}\hat{e}^T P_1 \hat{e} + \frac{1}{2}\bar{e}^T P_2 \bar{e} + \frac{1}{2\gamma_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_2} tr[\tilde{\theta}_g^T \tilde{\theta}_g]$$

The selection of the **Lyapunov function** is based on the following principle of indirect adaptive control

$$\hat{e}: \lim_{t\to\infty} \hat{x}(t) = x_d(t)$$
$$\bar{e}: \lim_{t\to\infty} \hat{x}(t) = x(t).$$

this results into

$$\lim_{t\to\infty} x(t) = x_d(t)$$

By deriving the **Lyapunov function** with respect to time one obtains:

$$\dot{V} = \frac{1}{2}\dot{\hat{e}}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \dot{\hat{e}} + \frac{1}{2}\dot{\bar{e}}^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 \dot{\bar{e}} + \\ + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow$$

$$\dot{V} = \frac{1}{2}\{(A - BK^T)\hat{e} + K_o C^T \bar{e}\}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \{(A - BK^T)\hat{e} + K_o C^T \bar{e}\} + \\ + \frac{1}{2}\{(A - K_o C^T)\bar{e} + B u_c + B\tilde{d} + B w\}^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P_2 \{(A - K_o C^T)\bar{e} + B u_c + B\tilde{d} + B w\} + \\ + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow$$

5. Convergence proof for the optimization method

The previous equation is rewritten as:

$$\dot{V} = \frac{1}{2}\{\hat{e}^T(A-BK^T)^T + \bar{e}^T C K_o^T\} P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1\{(A-BK^T)\hat{e} + K_o C^T \bar{e}\} + \\ + \frac{1}{2}\{\bar{e}^T(A-K_oC^T)^T + u_c^T B^T + w^T B^T + \bar{d}^T B^T\} P_2 \bar{e} + \\ \frac{1}{2}\bar{e}^T P_2\{(A-K_oC^T)\bar{e} + Bu_c + Bw + B\bar{d}\} + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g] \Rightarrow$$

which finally takes the form:

$$\dot{V} = \frac{1}{2}\hat{e}^T(A-BK^T)^T P_1 \hat{e} + \frac{1}{2}\bar{e}^T C K_o^T P_1 \hat{e} + \\ + \frac{1}{2}\hat{e}^T P_1 (A-BK^T)\hat{e} + \frac{1}{2}\hat{e}^T P_1 K_o C^T \bar{e} + \\ + \frac{1}{2}\bar{e}^T (A-K_oC^T)^T P_2 \bar{e} + \frac{1}{2}(u_c^T + w^T + \bar{d}^T) B^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P_2 (A-K_oC^T)\bar{e} + \frac{1}{2}\bar{e}^T P_2 B(u_c + w + \bar{d}) + \\ + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Assumption 1: For given positive definite matrices Q1 and Q2 there exist positive definite matrices P1 and P2, which are the solution of the following **Riccati equations**

$$(A-BK^T)^T P_1 + P_1(A-BK^T) + Q_1 = 0$$

$$(A-K_oC^T)^T P_2 + P_2(A-K_oC^T) - \\ - P_2 B(\frac{2}{r} - \frac{1}{\rho^2}) B^T P_2 + Q_2 = 0$$

5. Convergence proof for the optimization method

By substituting the relations described by the previous **Riccati equations** into the derivative of the Lyapunov function one gets:

$$\dot{V} = \frac{1}{2}\hat{e}^T\{(A-BK^T)^T P_1 + P_1(A-BK^T)\}\hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} + \\ + \frac{1}{2}\bar{e}^T\{(A-K_oC^T)^T P_2 + P_2(A-K_oC^T)\}\bar{e} + \\ + \bar{e}^T P_2 B(u_c + w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

or:

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2}\bar{e}^T\{Q_2 - P_2 B(\frac{2}{r} - \frac{1}{\rho^2})B^T P_2\}\bar{e} + \\ + \bar{e}^T P_2 B(u_c + w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

The **supervisory control term** u_c consists of two terms u_a and u_b

The first term u_a is

$$u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$$

where assuming that the measurable elements of vector $\tilde{e}$ are $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$

5. Convergence proof for the optimization method

The term Δu_a is such that

$$-\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a = -\frac{1}{r}\begin{pmatrix} p_{11}\tilde{e}_1 + p_{13}\tilde{e}_3 + \cdots + p_{1k}\tilde{e}_k \\ p_{13}\tilde{e}_1 + p_{33}\tilde{e}_3 + \cdots + p_{3k}\tilde{e}_k \\ \cdots \cdots\cdots \\ p_{1k}\tilde{e}_1 + p_{3k}\tilde{e}_3 + \cdots + p_{kk}\tilde{e}_k \end{pmatrix}$$

u_a is an H_∞ control used for the **compensation of the approximation error** w and the additive disturbance $\tilde{d}$ (the control term u_a has been chosen so as to satisfy the condition

The previous relation finally stands for a product between the measurable state vector elements $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$ and the elements of matrix P_2 which is obtained from the solution of the previous Riccati equation.

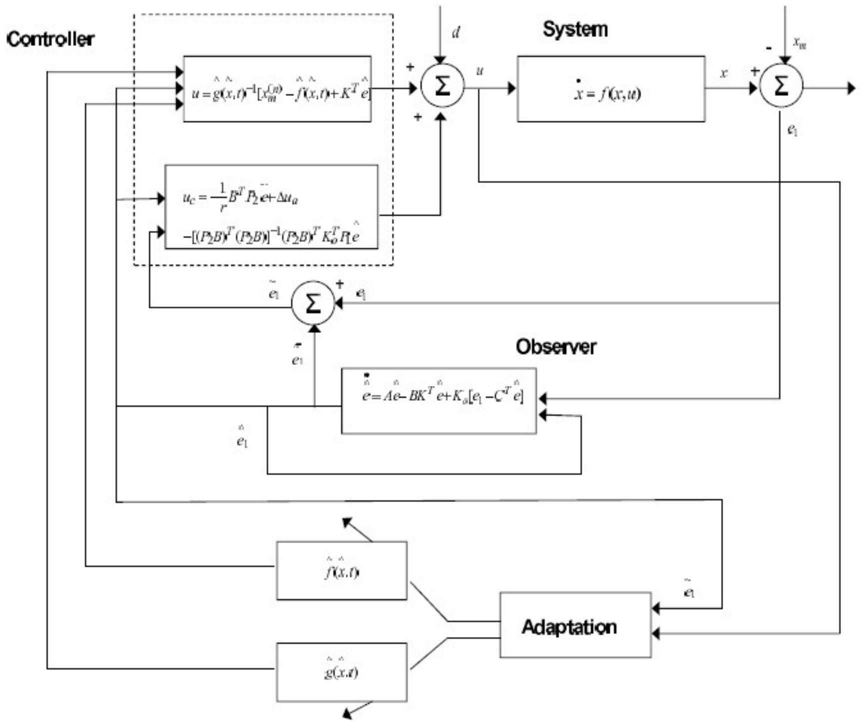
The control term u_b is given by

$$u_b = -[(P_2B)^T(P_2B)]^{-1}(P_2B)^T C K_o^T P_1 \hat{e}$$

u_b is a control used for the **compensation of the observation error** (the control term u_b has been chosen so as to satisfy the condition $\tilde{e}^T P_2 B u_b = -\tilde{e}^T C K_o^T P_1 \hat{e}$.

5. Convergence proof for the optimization method

The **optimization-based control scheme** is depicted in the following diagram



By substituting the supervisory control term in the **derivative of the Lyapunov function** one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{r}\bar{e}^T P_2 B B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + $$
$$+ \bar{e}^T P_2 B u_a + \bar{e}^T P_2 B u_b + \bar{e}^T P_2 B (w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

.

5. Convergence proof for the optimization method

or equivalently

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \tilde{e}^T P_2 B(w + \tilde{d} + \Delta u_a) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Besides, about the **adaptation of the weights** of the neurofuzzy approximator it holds

$$\dot{\tilde{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f \qquad \dot{\tilde{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g.$$

A **gradient-based update** is applied to the approximator's weights

$$\dot{\theta}_f = -\gamma_1 \Phi(\hat{x})^T B^T P_2 \bar{e}$$
$$\dot{\theta}_g = -\gamma_2 \Phi(\hat{x})^T B^T P_2 \bar{e} u^T$$

Gradient-based optimization

The **gradient update scheme** is defined in a manner that assures that **the first derivative of the Lyapunov function will remain negative**, and thus the Lyapunov function will be monotonously decreasing.

By substituting the above relations in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(-\gamma_2) tr[u\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]$$

5. Convergence proof for the optimization method

To continue with the **convergence proof for the proposed optimization method** it is taken into account that

$$u \in R^{2\times 1} \text{ and } \tilde{e}^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1\times 2}$$

one gets

$$\begin{array}{l} \dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ \frac{1}{\gamma_2}(-\gamma_2) tr[\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u] \end{array}$$

Since

$$\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \in R^{1\times 1}$$

it holds

$$\begin{array}{l} tr(\tilde{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)u) = \\ = \tilde{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u \end{array}$$

Therefore, one finally obtains

$$\begin{array}{l} \dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ \frac{1}{\gamma_2}(-\gamma_2)\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \end{array}$$

Next, the following **approximation error** is defined

$$w_a = [\hat{f}(\hat{x}|\theta_f^*) - \hat{f}(\hat{x}|\theta_f)] + [\hat{g}(\hat{x}|\theta_g^*) - \hat{g}(\hat{x}|\theta_g)]u$$

5. Convergence proof for the optimization method

Thus, one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + \\ + B^T P_2 \tilde{e}(w + \tilde{d} + \Delta u_a) + \tilde{e}^T P_2 B w_\alpha$$

Denoting the **aggregate approximation error** and disturbances vector as

$$w_1 = w + \tilde{d} + w_\alpha + \Delta u_a$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \bar{e}^T P_2 B w_1$$

which in turn is written as

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e}$$

Lemma: The following inequality holds

$$\frac{1}{2}\bar{e}^T P_2 B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \\ \leq \frac{1}{2}\rho^2 w_1^T w_1$$

5. Convergence proof for the optimization method

Proof:

The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow$$
$$\frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow$$
$$\frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$

By substituting $a = w_1$ and $b = \bar{e}^T P_2 B$ one gets

$$\frac{1}{2}w_1^T B^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 B w_1 - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Moreover, by substituting the above inequality into the **derivative of the Lyapunov function** one gets

$$\dot{V} \leq -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{2}\rho^2 w_1^T w_1$$

which is also written as $\dot{V} \leq -\frac{1}{2}E^T Q E + \frac{1}{2}\rho^2 w_1^T w_1$

with $E = \begin{pmatrix} \hat{e} \\ \bar{e} \end{pmatrix}, \quad Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} = diag[Q_1, Q_2]$

5. Convergence proof for the optimization method

Hence, the H_∞ performance criterion is derived. **For sufficiently small ρ the inequality will be true** and the H_∞ tracking criterion will be satisfied. In that case, the integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)\,dt \leq -\frac{1}{2}\int_0^T ||E||^2\,dt + \frac{1}{2}\rho^2\int_0^T ||w_1||^2\,dt \Rightarrow$$
$$2V(T) - 2V(0) \leq -\int_0^T ||E||_Q^2\,dt + \rho^2\int_0^T ||w_1||^2\,dt \Rightarrow$$
$$2V(T) + \int_0^T ||E||_Q^2\,dt \leq 2V(0) + \rho^2\int_0^T ||w_1||^2\,dt$$

It is assumed that there exists a positive constant $M_w > 0$ such that

$$\int_0^\infty ||w_1||^2\,dt \leq M_w$$

Therefore for the integral $\int_0^T ||E||_Q^2\,dt$ one gets

$$\int_0^\infty ||E||_Q^2\,dt \leq 2V(0) + \rho^2 M_w$$

Thus, the integral $\int_0^\infty ||E||_Q^2\,dt$ is bounded and **according to Barbalat's Lemma**

$$\lim_{t\to\infty} e(t) = 0$$

and thus **global asymptotic stability** is also shown for the control loop.

6. Case studies on robotic and electric power systems

6.1 The model of multi-DOF robotic manipulators

The model of the **robot's dynamics** is a MIMO nonlinear one:

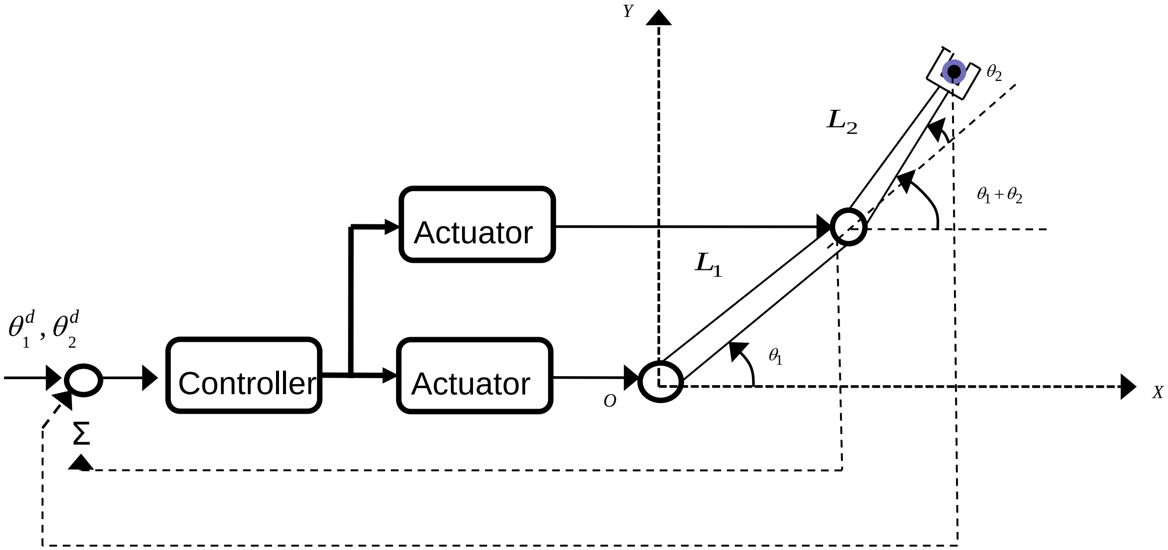
$$M(\theta)\ddot{\theta}+h(\theta,\dot{\theta})+G(\theta)=T \qquad \theta=\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}\in R^{2\times 1} \quad \dot{\theta}=\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}\in R^{2\times 1} \quad \ddot{\theta}=\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}\in R^{2\times 1}$$

$M=\begin{bmatrix}M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix}\in R^{2\times 2}$ is the inertia matrix

$h=\begin{bmatrix}h_1\\ h_2\end{bmatrix}\in R^{2\times 1}$ is the Coriolis and centrifugal forces matrix

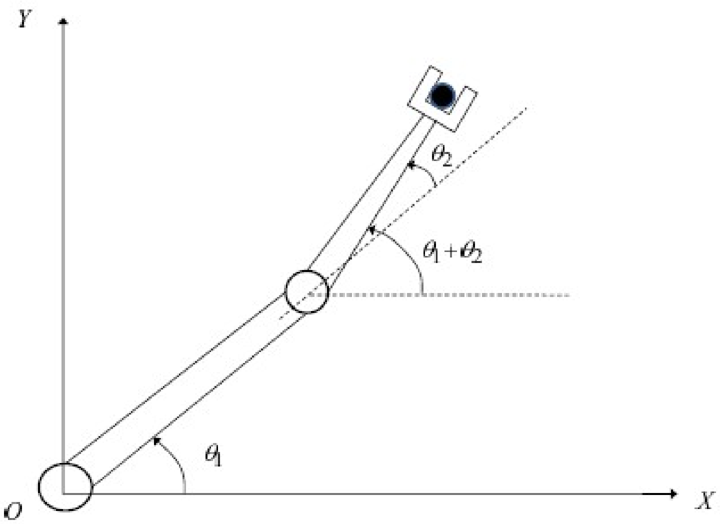
$G=\begin{bmatrix}G_1\\ G_2\end{bmatrix}\in R^{2\times 1}$ is the gravitational terms matrix

$T=\begin{bmatrix}T_1\\ T_2\end{bmatrix}\in R^{2\times 1}$ is the torques control input matrix



6. Case studies on robotic and electric power systems

6.1 The model of multi-DOF robotic manipulators

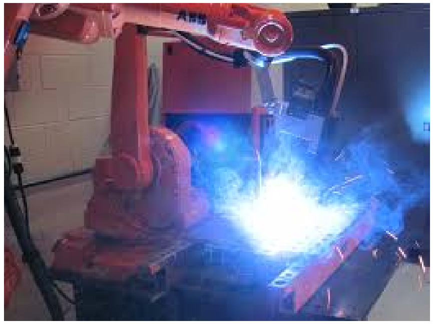


$$f_1(x) = -N_{11}F_1(\theta,\dot{\theta}) - N_{12}F_2(\theta,\dot{\theta}) - $$
$$- N_{11}G_1(\theta) - N_{12}G_2(\theta) \in R^{1\times 1}$$

$$g_1(x) = [N_{11} \quad N_{12}] \in R^{1\times 2}$$

$$f_2(x) = -N_{21}F_2(\theta,\dot{\theta}) - N_{22}F_2(\theta,\dot{\theta}) - $$
$$- N_{21}G_1(\theta) - N_{22}G_2(\theta) \in R^{1\times 1}$$

$$g_2(x) = [N_{21} \quad N_{22}] \in R^{1\times 2}$$



Defining flat outputs y_1 and y_2 for which holds

$$y = [\theta_1 \quad \theta_2] = [x_1 \quad x_3]$$

$$x_1 = \theta_1 \quad x_2 = \dot{\theta}_1 \quad x_3 = \theta_2 \quad x_4 = \dot{\theta}_2$$

$$\begin{aligned} \ddot{x}_1 &= f_1(x) + g_{11}(x)u_1 + g_{12}(x)u_2 \\ \ddot{x}_3 &= f_2(x) + g_{21}(x)u_1 + g_{22}(x)u_2 \end{aligned}$$

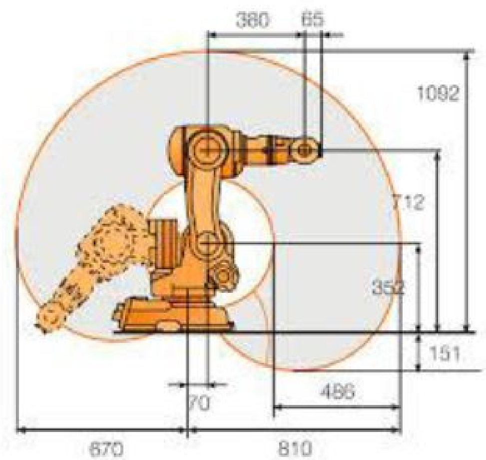
the following **Brunovsky (canonical form) of the robotic system** is finally obtained

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\begin{aligned} v_1 &= f_1(x) + g_{11}(x)u_1 + g_{12}(x)u_2 \\ v_2 &= f_2(x) + g_{21}(x)u_1 + g_{22}(x)u_2 \end{aligned}$$

6. Case studies on robotic and electric power systems

6.1 The model of multi-DOF robotic manipulators



The **optimization problem** has **multiple objectives**:

1) **Minimize the modelling error** of the system's dynamics

2) **Minimize the estimation error** for the system's state vector

3) **Minimize the tracking error** from the reference setpoints

For the **differentially flat MIMO model of the multi-DOF robotic manipulator** one gets the equivalent state-space model

$$\ddot{x}_1 = f_1(x,t) + g_1(x,t)u + \tilde{d}_1$$

$$\ddot{x}_3 = f_2(x,t) + g_2(x,t)u + \tilde{d}_2$$

• For the multi-DOF robot control scheme differential flatness properties hold and one can apply the control scheme analyzed in Sections 3 and 4.

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

The **dynamic model of the distributed power generation units** is assumed to be that of synchronous generators. The modelling approach is also applicable to PMSGs (permanent magnet synchronous generators) which are a special case of synchronous electric machines.

$$\dot{\delta} = \omega$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \frac{\omega_0}{2J}(P_m - P_e) \quad (1)$$

δ : turn angle of the rotor

ω : turn speed of the rotor

ω_0 : synchronous speed

J : moment of inertia of the rotor

P_e : active electrical power of the machine

P_m : mechanical power of the machine

D : damping coefficient

T_e : electromagnetic torque

The **generator's electrical dynamics** is:

$$\dot{E}_q' = \frac{1}{T_{d_o}}(E_f - E_q) \quad (2)$$

E_q' is the quadrature-axis transient voltage (a variable related to the magnetic flux)

E_q is quadrature axis voltage of the generator

T_{d_o} is the direct axis open-circuit transient time constant

E_f is the equivalent voltage in the excitation coil

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

The **synchronous generator's model** is complemented by a set of algebraic equations:

$$E_q = \frac{x_{d\Sigma}}{x'_{d\Sigma}} E'_q - (x_d - x'_d)\frac{V_s}{x'_{d\Sigma}} cos(\Delta\delta)$$

$$I_q = \frac{V_s}{x'_{d\Sigma}} sin(\Delta\delta)$$

$$I_d = \frac{E'_q}{x'_{d\Sigma}} - \frac{V_s}{x'_{d\Sigma}} cos(\Delta\delta)$$

$$P_e = \frac{V_s E'_q}{x'_{d\Sigma}} sin(\Delta\delta)$$

$$Q_e = \frac{V_s E'_q}{x'_{d\Sigma}} cos(\Delta\delta) - \frac{V_s^2}{x_{d\Sigma}}$$

$$V_t = \sqrt{(E'_q - X'_d I_d)^2 + (X'_d I_q)^2}$$

3

where: $x_{d\Sigma} = x_d + x_T + x_L$ $\quad x'_{d\Sigma} = x'_d + x_T + x_L$

x_d : direct-axis synchronous reactance

x_T : reactance of the transformer

x'_d : direct-axis transient reactance

x_L : transmission line reactance

I_d and I_q : direct and quadrature axis currents

V_s : infinite bus voltage

Q_e : reactive power of the generator

V_t : terminal voltage of the generator

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

From Eq. (1) and Eq. (2) one obtains the **dynamic model of the synchronous generator**:

$$\dot{\delta} = \omega - \omega_0$$

$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E_q'}{x_{d\Sigma}'} sin(\Delta\delta)$$

$$\dot{E}_q' = -\frac{1}{T_d'} E_q' + \frac{1}{T_{d_o}} \frac{x_d - x_d'}{x_{d\Sigma}'} V_s cos(\Delta\delta) + \frac{1}{T_{d_o}} E_f$$

Moreover, the generator can be written in a **state-space form**:

$$\dot{x} = f(x) + g(x)u$$

where the state vector is $x = (\Delta\delta \quad \Delta\omega \quad E_q')^T$ and

$$f(x) = \begin{pmatrix} \omega - \omega_0 \\ -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E_q'}{x_{d\Sigma}'} sin(\Delta\delta) \\ -\frac{1}{T_d'} E_q' + \frac{1}{T_{do}} \frac{x_d - x_d'}{x_{d\Sigma}'} V_s cos(\Delta\delta) \end{pmatrix}$$

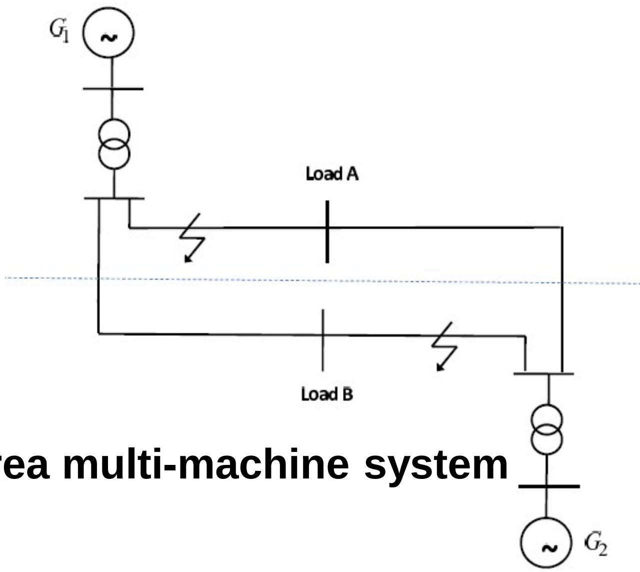
$$g(x) = \left(0 \quad 0 \quad \frac{1}{T_{do}}\right)^T$$

while the system's output is $y = h(x) = \delta - \delta_0$

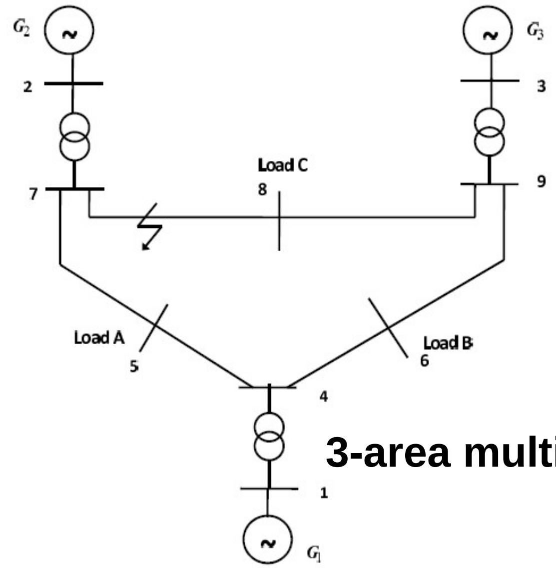
6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

The **interconnection between distributed power generators** results into a **multi-area multi-machine power system** model



2-area multi-machine system



3-area multi-machine system

The dynamic model of a power system that comprises **n-interconnected power generators** is

$$\dot{\delta}_i = \omega_i - \omega_0$$

$$\dot{\omega}_i = -\frac{D_i}{2J_i}(\omega_i - \omega_0) + \omega_0 \frac{P_{m_i}}{2J_i} -$$

$$-\omega_0 \frac{1}{2J_i}[G_{ii}{E'_{qi}}^2 + E'_{qi}\sum_{j=1,j\neq i}^{n} E'_{qj} G_{ij} sin(\delta_i - \delta_j - \alpha_{ij})]$$

$$\dot{E}'_{q_i} = -\frac{1}{T'_{d_i}} E'_{q_i} + \frac{1}{T_{d_{oi}}} \frac{x_{d_i} - x'_{d_i}}{x'_{d_{\Sigma i}}} V_{s_i} cos(\Delta\delta_i) + \frac{1}{T_{d_{oi}}} E_{f_i}$$

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

The **active power** associated with the i-th power generator is given by:

$$P_{e_i} = G_{ii}{E'_{qi}}^2 + E'_{qi}\sum_{j=1,j\neq i}^{n} E'_{qj} G_{ij} sin(\delta_i - \delta_j - \alpha_{ij})$$

The state vector of the distributed power system is given by $x = [x^1, x^2, \cdots, x^n]^T$

where $x^i = [x_1^i, x_2^i, x_3^i]^T$ with $x_1^i = \Delta\delta_i$ $x_2^i = \Delta\omega_i$ and $x_3^i = E'_{qi}$ $i = 1, 2, \cdots, n$

Next, **differential flatness** is proven for the model of the **stand-alone synchronous generator**.

In state-space form one has:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{D}{2J}x_2 + \omega_0\frac{P_m}{2J} - \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}x_3 sin(x_1)$$

$$\dot{x}_3 = -\frac{1}{T'_d}x_3 + \frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s cos(x_1) + \frac{1}{T_{do}}u$$

The **flat output** is taken to be $y = x_1$

It holds that $x_1 = y$ $x_2 = \dot{y}$ and for $x_1 \neq \pm n\pi$,

$$x_3 = \frac{\omega_0\frac{P_m}{2J} - \ddot{y} - \frac{D}{2J}\dot{y}}{\frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}sin(y)}, \text{ or } x_3 = f_a(y, \dot{y}, \ddot{y})$$

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

while for the **generator's control input** one has

$$u = T_{do}[\dot{x}_3 + \frac{1}{T_d'}x_3\frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(x_1)], \text{ or}$$
$$u = f_b(y, \dot{y}, \ddot{y})$$

Consequently, **all state variables** and the **control input** of the synchronous generator are written as **differential functions** of the flat output and thus the differential flatness of the model is confirmed.

By defining the **new state variables** $y_1 = y,\ y_2 = \dot{y},\ y_3 = \ddot{y}$

the generator's model is transformed into the **canonical (Brunovsky) form**:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v$$

with $v = f_c(y, \dot{y}, \ddot{y}) + g_c(y, \dot{y}, \ddot{y})u$ where

$$f_c(y, \dot{y}, \ddot{y}) = (\frac{D}{2J})^2\dot{y} - \omega_0\frac{D}{2J}\frac{P_m}{2J} + \omega_0\frac{D}{(2J)^2}\frac{V_s}{x_{d\Sigma}'}x_3 sin(\dot{y}) +$$
$$+\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}\frac{1}{T_d'}x_3 sin(y) - \frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}\frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(y) sin(y) -$$
$$-\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}x_3 cos(y)\dot{y}$$

and

$$g_c(y, \dot{y}, \ddot{y}) = -\frac{\omega_0}{2J}\frac{1}{T_{do}}\frac{V_s}{x_{d\Sigma}'}sin(y)$$

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

Differential flatness can be also proven for the **model of the n-interconnected power generators**

The **flat output** is taken to be the vector of the turn angles of the n-power generators

$$y = [y_1^1, y_1^2, \cdots, y_1^n] \text{ or } y = \Delta\delta^1, \Delta\delta^2, \cdots, \Delta\delta^n$$

For the n-machines power generation system it holds

$$x_1^1 = y^1,\ x_1^2 = y^2,\ x_1^3 = y^3,\ \cdots,\ x_1^n = y^n$$

$$x_2^1 = \Delta\omega^1 = \dot{y}^1,\ x_2^2 = \Delta\omega^2 = \dot{y}^2,\ x_2^3 = \Delta\omega^3 = \dot{y}^3,\ \cdots,\ x_2^n = \Delta\omega^n = \dot{y}^n$$

Moreover, it holds

$$\dot{x}_2^i = -\frac{D_i}{2J_i}x_2^i + \frac{\omega_0}{2J_i}P_{mi} - \\ -\frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2 + x_3^i\textstyle\sum_{j=1,j\neq i}^n [x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] \quad (4)$$

or using the flat outputs notation

$$\ddot{y}^i = -\frac{D_i}{2J_i}\dot{y}^i + \frac{\omega_0}{2J_i}P_{mi} - \\ -\frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2 + x_3^i\textstyle\sum_{j=1,j\neq i}^n [x_3^j G_{ij} sin(y^i - y^j - \alpha_{ij})]$$

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

The **external mechanical torque** P_{mi} is considered to be a piecewise constant variable

From Eq. (4) and for one $i=1,2,\cdots,n$ has a system of n equations which can be solved with respect to the variables $x_3^i, i=1,2,\cdots,n$

Actually, all variables x_3^i, can be expressed as **differential functions of the flat outputs**

$$y^i,\ i=1,2,\cdots,n$$

and thus one has $x_3^i = f_{x_3}(y^1, y^2, \cdots, y^n)$

Moreover, from

$$\dot{E}_{q_i} = -\frac{1}{T_{d_i}}E'_{q_i} + \frac{1}{T_{d_{oi}}}\frac{x_{d_i}-x'_{d_i}}{x_{d_{\Sigma_i}}}V_{s_i}cos(\Delta\delta_i) + \frac{1}{T_{d_{oi}}}E_{f_i}$$

one can demonstrate that the control inputs $u_i = E_{f_i}$ can be expressed as **differential functions of the flat outputs** $y^i,\ i=1,2,\cdots,n$

Consequently, all state variables and the control inputs of the distributed power system can be expressed as differential functions of the flat outputs, and **the system is a differentially flat one.**

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

Next, the **external mechanical torque** P_{mi} is considered to be time-varying

The effect of this torque is viewed as a **disturbance** to each power generator

In such a case for a model of *n=2* interconnected generators one obtains the **input-output linearized dynamics**

$$\dot{z}_3^i = a^i(x) + {b_1}^i g_1 u_1 + {b_2}^i g_2 u_2 + \tilde{d}^i \qquad \text{where} \quad z_3^i = \ddot{\delta}^i = \dot{\omega}^i$$

and

$$a^i = (\frac{D_i}{2J_i})^2 x_2^i + \frac{D_i\omega_0}{(2J_i)^2}[G_{ii}{x_3^i}^2 + x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] -$$
$$-\frac{\omega_0}{2J_i}[G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i}-x'_{d_i}}{x'_{d\Sigma_i}}V_{s_i}cos(x_1^i))] -$$
$$-\frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i}-x'_{d_i}}{x'_{d\Sigma_i}}V_{s_i}cos(x_1^i)) -$$
$$-\frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij})x_2^i \frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij})x_2^j$$

and

$$b_1^i = -\frac{\omega_0}{2J_i}[2G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})]\frac{1}{T_{doi}}$$
$$b_2^i = -\frac{\omega_0}{2J_i}G_{i2} sin(x_1^i - x_1^2 - \alpha_{i2})\frac{1}{T_{do2}}$$

while

$$\tilde{d}^i = -\frac{D_i\omega_0}{2{J_i}^2}P_m^i + \frac{\omega_0}{2J_i}\dot{P}_m^i$$

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

For the **two interconnected generators** (*i=1,2*) one has the linearized dynamics

$$\dot{z}_1^i = z_2^i$$
$$\dot{z}_2^i = z_3^i$$
$$\dot{z}_3^i = a^i(x) + {b_1}^i g_1 u_1 + {b_2}^i g_2 u_2 + \tilde{d}^i$$

It is used that

$$\dot{z}_3^1 = a^1(x) + {b_1}^1 g_1 u_1 + {b_2}^1 g_2 u_2 + \tilde{d}^1$$
$$\dot{z}_3^2 = a^2(x) + {b_1}^2 g_1 u_1 + {b_2}^2 g_2 u_2 + \tilde{d}^2$$

or in matrix form

$$\dot{z}_3 = f_a(x) + Mu + \tilde{d}$$

where

$$z_3 = [z_3^1, z_3^2]^T, \ u = [u_1, u_2]^T \text{ and } \tilde{d} = [\tilde{d}_1, \tilde{d}_2]^T$$

and

$$f_a(x) = \begin{pmatrix} a^1(x) \\ a^2(x) \end{pmatrix}, \quad M = \begin{pmatrix} {b_1}^1 g_1 & {b_2}^1 g_2 \\ {b_1}^2 g_1 & {b_2}^2 g_2 \end{pmatrix}$$

Setting, $v = f_a(x) + Mu + \tilde{d}$ one obtains

$$\begin{pmatrix} \dot{z}_1^i \\ \dot{z}_2^i \\ \dot{z}_3^i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^i \\ z_2^i \\ z_3^i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (v^i + \tilde{d}^i)$$

6. Case studies on robotic and electric power systems

6.2 The model of distributed power generators

For **the model of the 2-area distributed power generation system** it holds that

$$x_{1,1}^{(3)} = f_1(x,t) + g_1(x,t)u + d_1$$
$$x_{1,2}^{(3)} = f_2(x,t) + g_2(x,t)u + d_2$$

By denoting

$$x_1 = x_{1,1},\ \ x_2 = \dot{x}_{1,1},\ \ x_3 = \ddot{x}_{1,1}$$

$$x_2 = x_{2,1},\ \ x_5 = \dot{x}_{2,1},\ \ x_6 = \ddot{x}_{2,1}$$

the **Brunovsky (canonical form) of the distributed power system** is obtained

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where

$$v_1 = f_1(x) + g_{11}(x)u_1 + g_{12}(x)u_2$$
$$v_2 = f_2(x) + g_{21}(x)u_1 + g_{22}(x)u_2$$

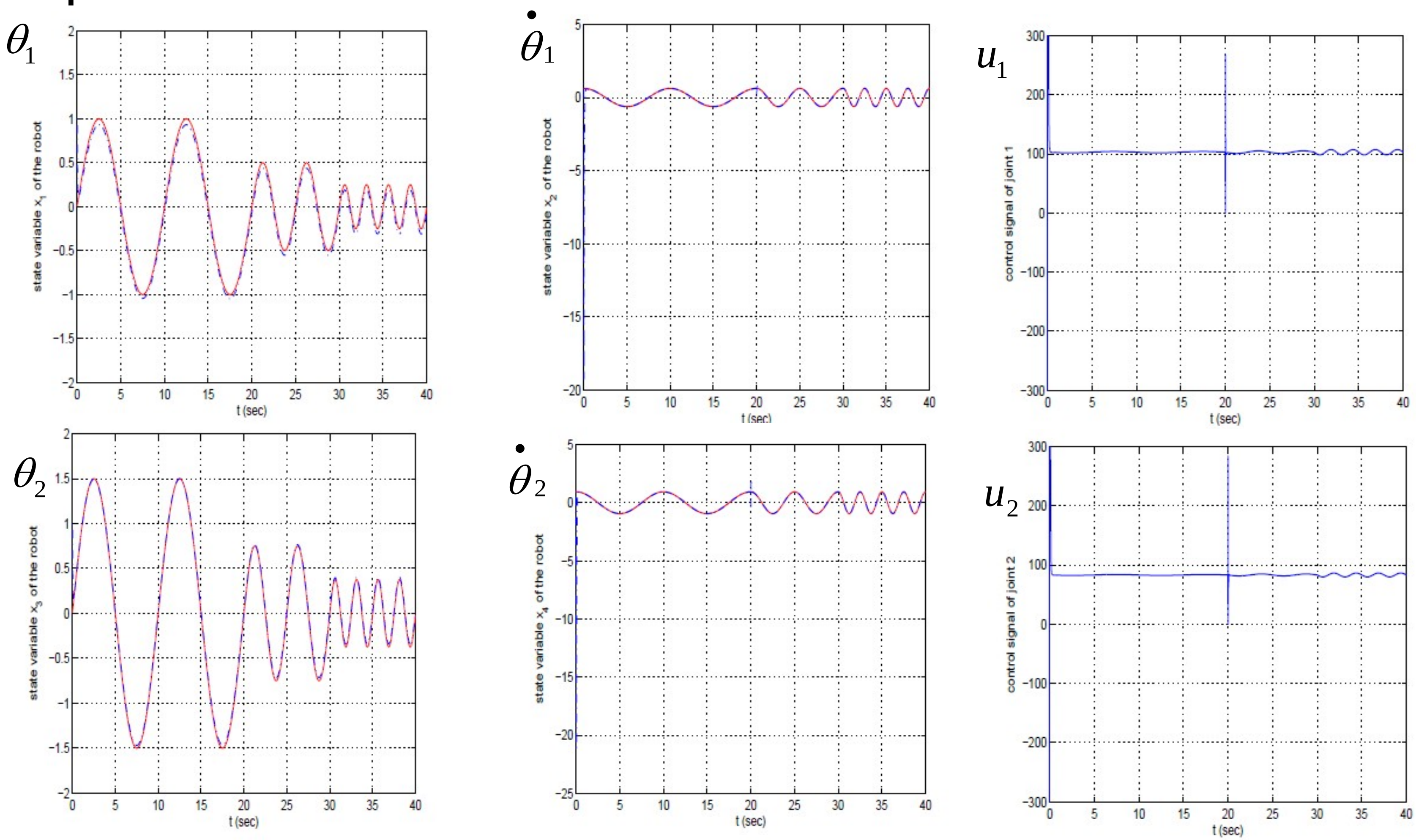
- For the 2-area distributed power system differential flatness properties hold and one can apply the control scheme analyzed in Sections 3 and 4.

7. Simulation tests

7.1 Optimization-based modelling and control of a multi-DOF robotic manipulator

The dynamic model of the robot was taken to be completely unknown, while the state vector could be partially measured

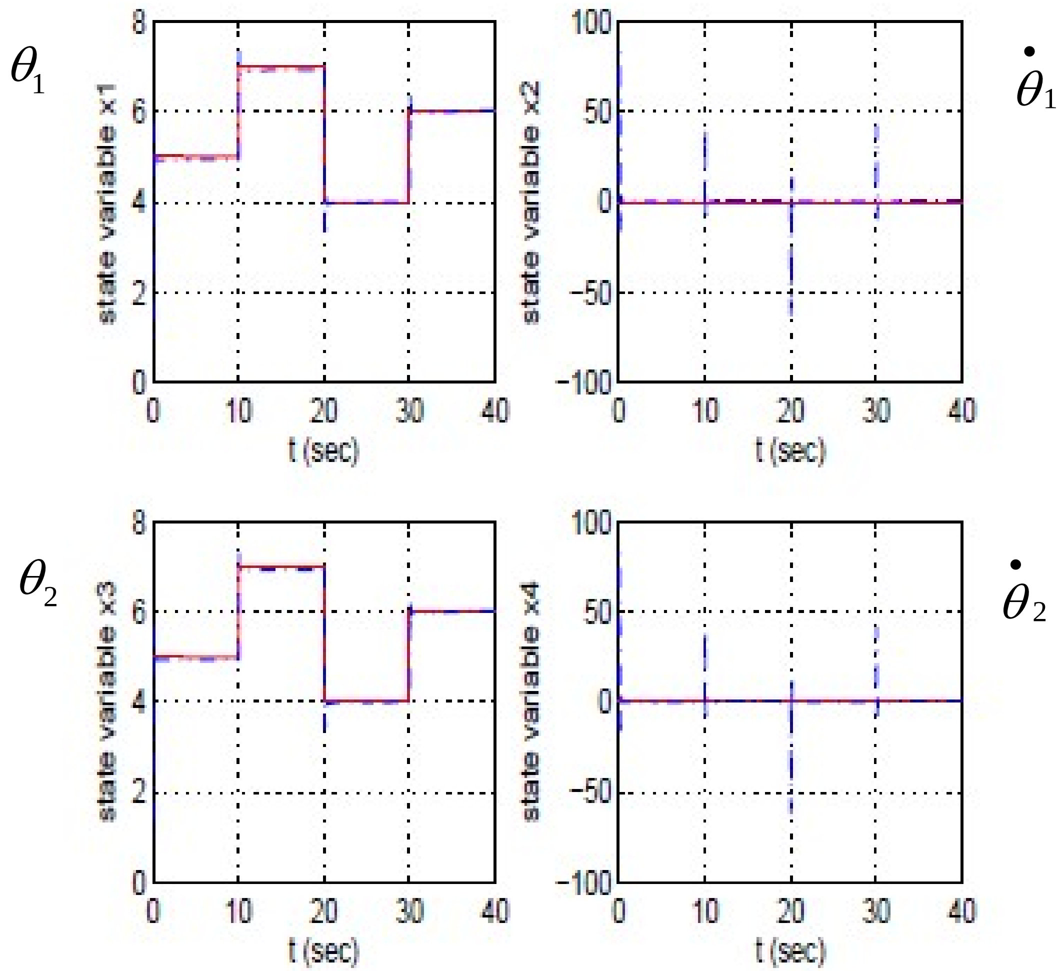
setpoint 1



7. Simulation tests

7.1 Optimization-based modelling and control of a multi-DOF robotic manipulator

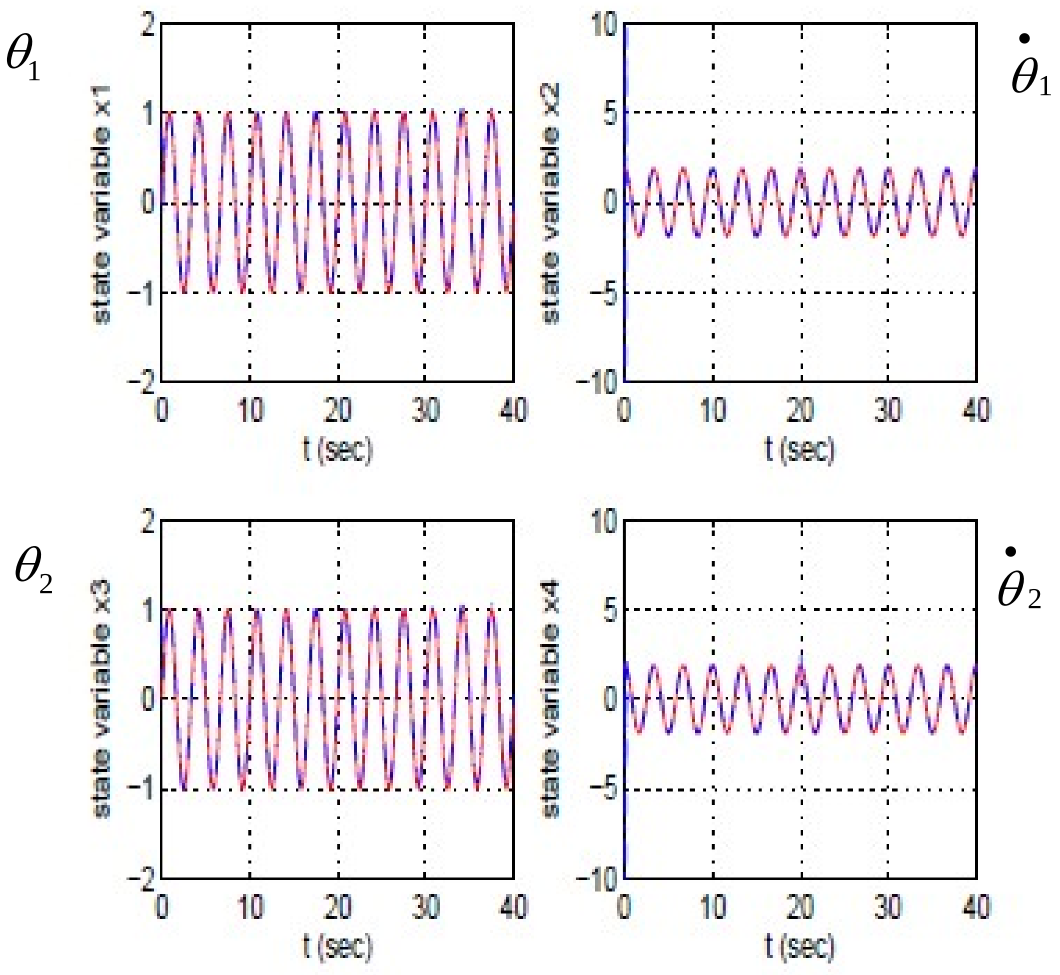
setpoint 2



7. Simulation tests

7.1 Optimization-based modelling and control of a multi-DOF robotic manipulator

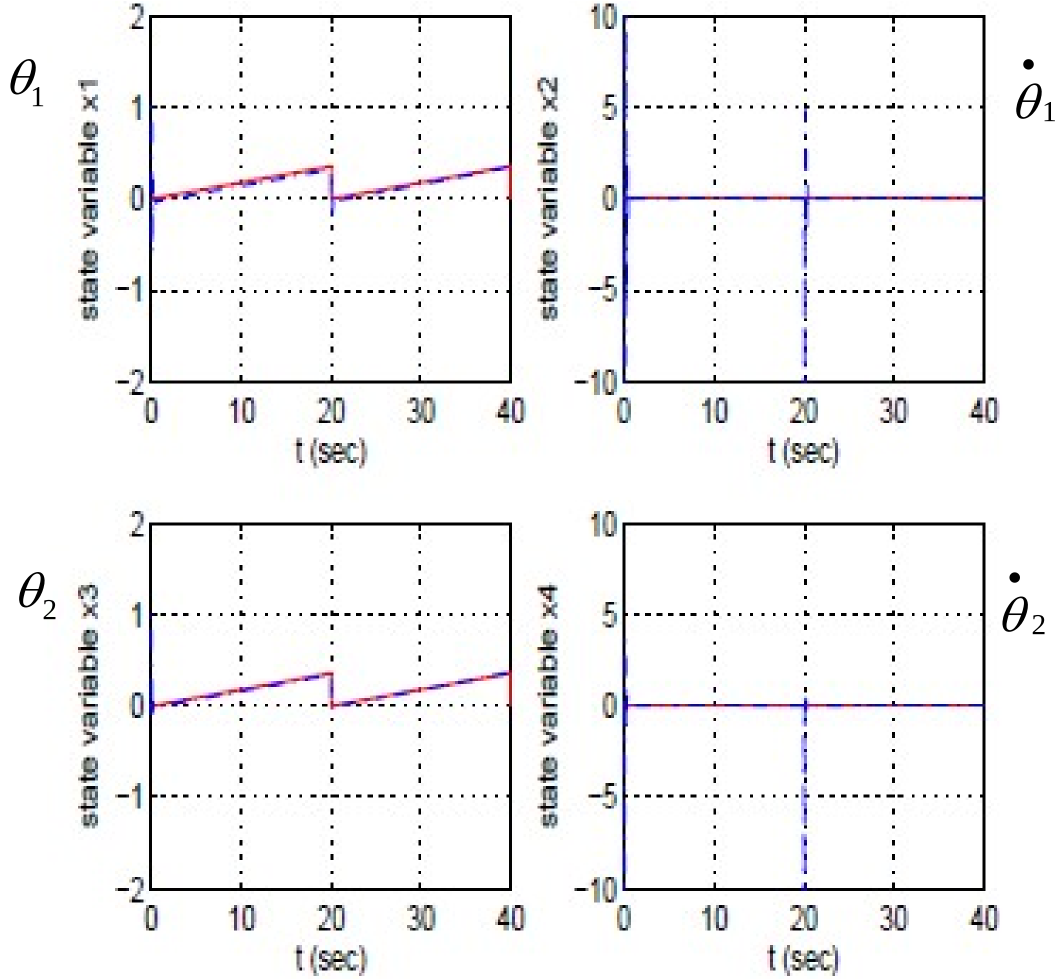
setpoint 3

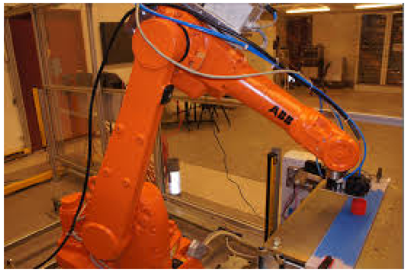


7. Simulation tests

7.1 Optimization-based modelling and control of a multi-DOF robotic manipulator

setpoint 4





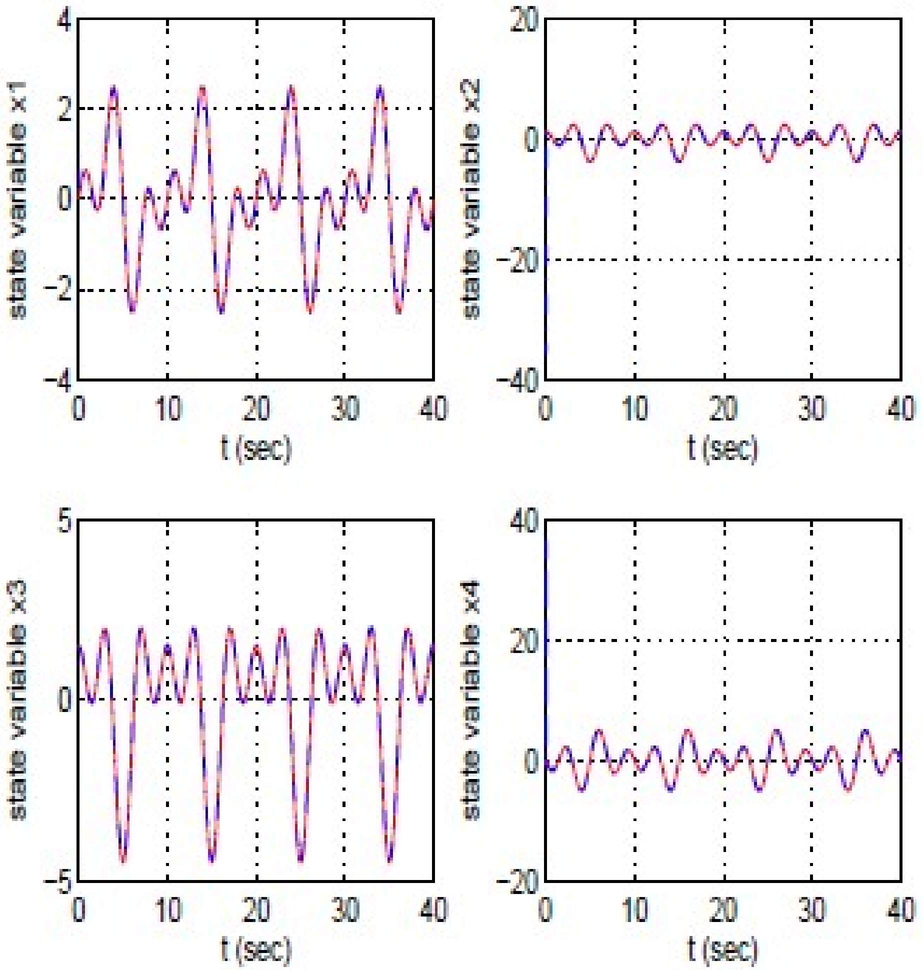
7. Simulation tests

7.1 Optimization-based modelling and control of a multi-DOF robotic manipulator

setpoint 5

θ_1 $\dot{\theta}_1$

θ_2 $\dot{\theta}_2$



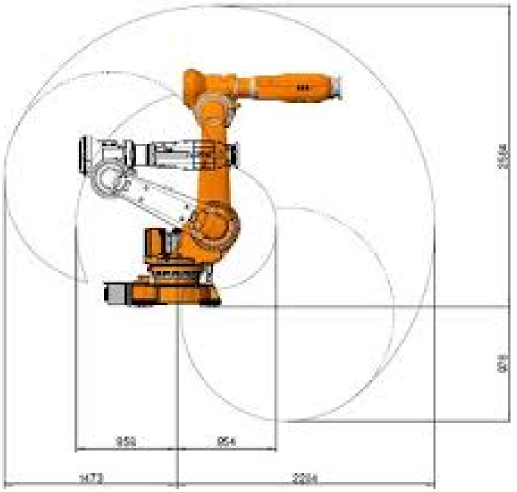


Table II: RMSE of joints' angles

parameter	θ_1	θ_2
$RMSE_a$	0.0471	0.0449
$RMSE_b$	0.0418	0.0427
$RMSE_c$	0.0495	0.0288
$RMSE_d$	0.0449	0.0472

7. Simulation tests

7.1 Optimization-based modelling and control of a multi-DOF robotic manipulator

video

robotic manipulator in a pick and place task

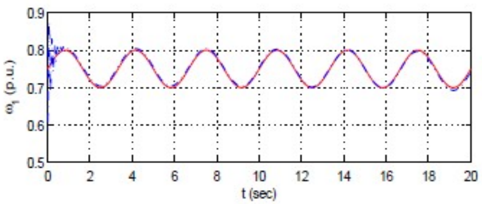
7. Simulation tests

7.2 Optimization-based modelling and control of distributed power generators

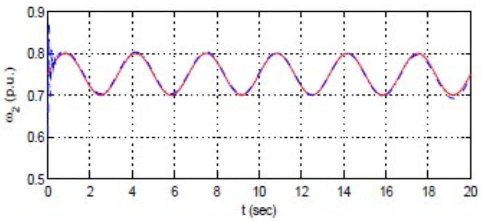
The dynamic model of the distributed power generators was taken to be completely unknown, while the state vector could be partially measured

setpoint 1

ω_1

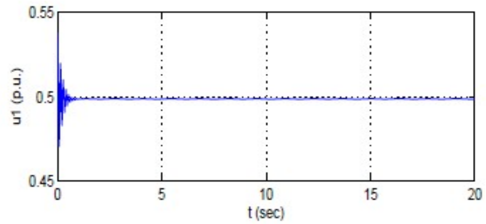


ω_2

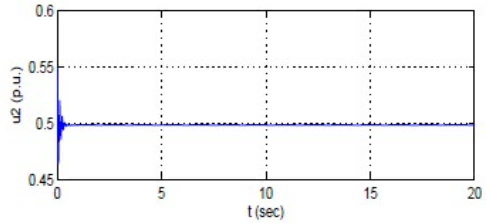


time

u_1



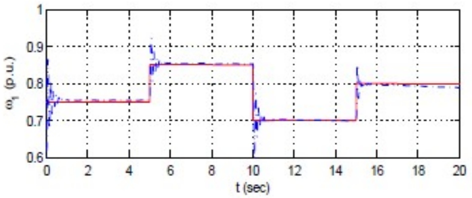
u_2



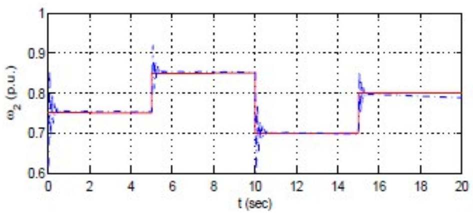
time

setpoint 2

ω_1

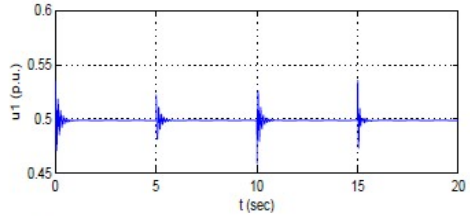


ω_2

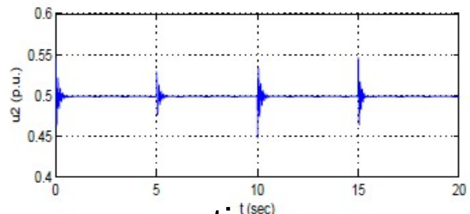


time

u_1



u_2



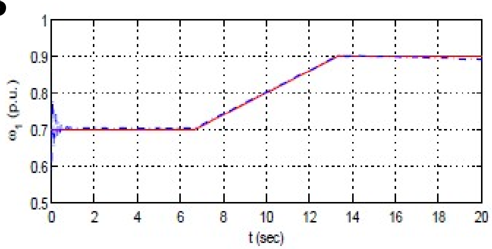
time

7. Simulation tests

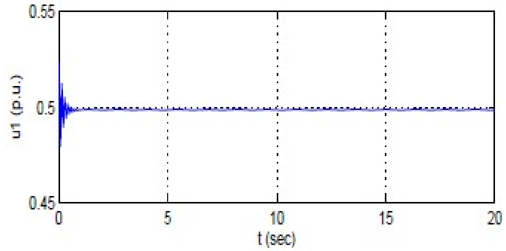
7.2 Optimization-based modelling and control of distributed power generators

setpoint 3

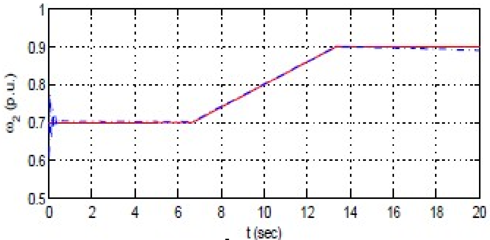
ω_1



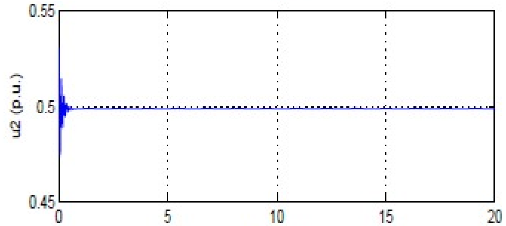
u_1



ω_2



u_2

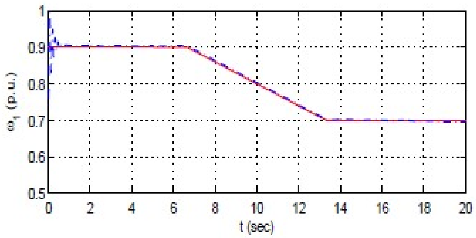


time

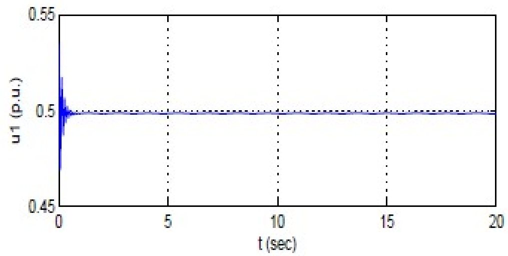
time

setpoint 4

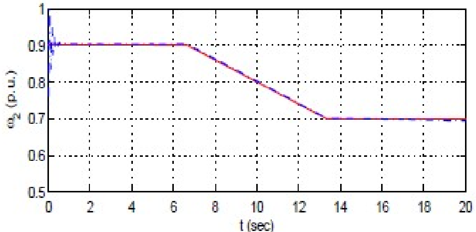
ω_1



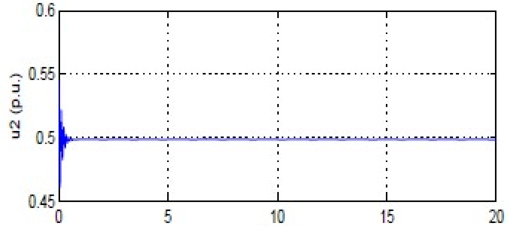
u_1



ω_2



u_2



time

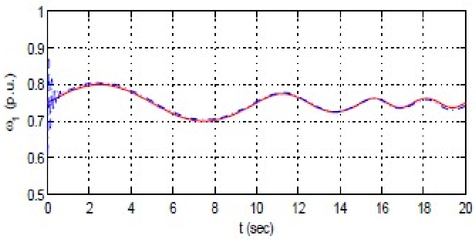
time

7. Simulation tests

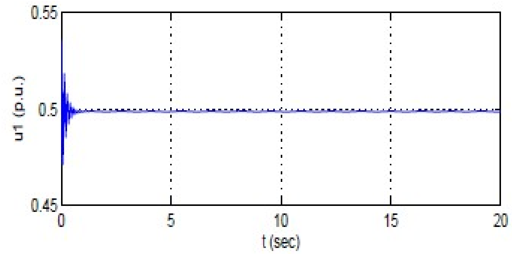
7.2 Optimization-based modelling and control of distributed power generators

setpoint 5

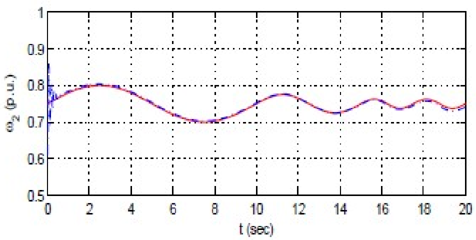
ω_1



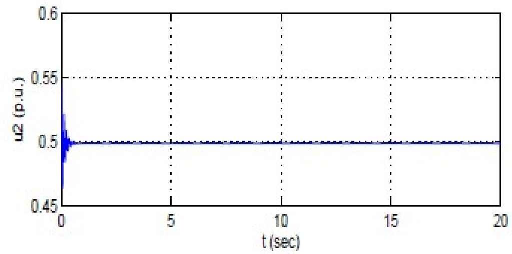
u_1



ω_2



u_2

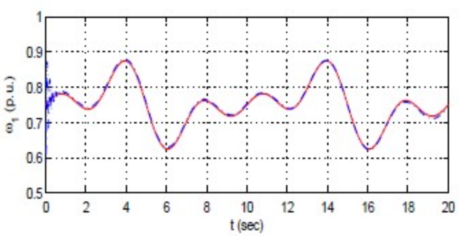


time

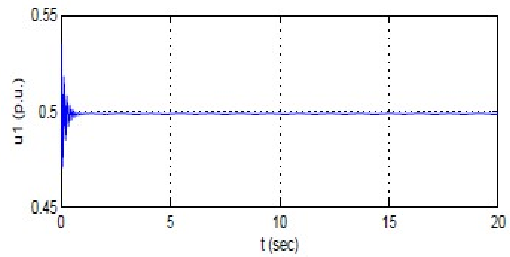
time

setpoint 6

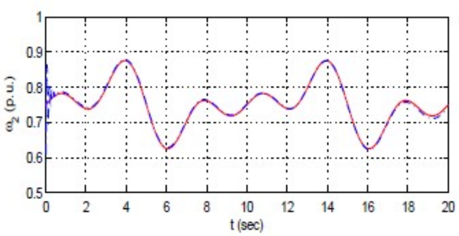
ω_1



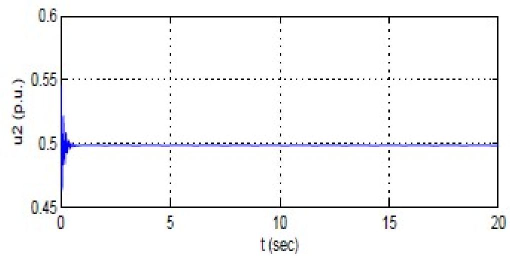
u_1



ω_2



u_2



time

time

7. Simulation tests

7.2 Optimization-based modelling and control of distributed power generators

Table I: RMSE of the power generator's state variables

parameter	ω_1	$\dot{\omega}_1$	ω_2	$\dot{\omega}_2$
$RMSE_1$	0.0035	0.0002	0.0034	0.0002
$RMSE_2$	0.0123	0.0545	0.0118	0.0602
$RMSE_3$	0.0035	0.0020	0.0035	0.0020
$RMSE_4$	0.0031	0.0020	0.0026	0.0020
$RMSE_5$	0.0034	0.0003	0.0033	0.0002
$RMSE_6$	0.0035	0.0003	0.0033	0.0002

The **tracking accuracy** of the control method was remarkable despite the fact that

(i) the **dynamic model** of the systems was **completely unknown**,

(ii) **only output feedback** was used in the implementation of the control scheme.

It has been also confirmed that the transient characteristics of the control scheme are quite satisfactory

The proposed **optimization-based modelling and control method** is of generic use and can be applied to a **wide class of nonlinear dynamical** systems of unknown model

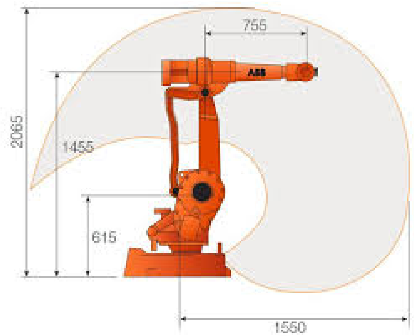
7. Simulation tests

7.2 Optimization-based modelling and control of distributed power generators

video

synchronization between the distributed power generators

8. Conclusions



• A **gradient-based method of assured convergence and stability** has been developed. The method is suitable for **modelling** and optimization-based control in a wide class of nonlinear systems

• By exploiting the **differential flatness** properties of the **MIMO nonlinear model of the dynamical systems** this was transformed into the **linear canonical (Brunovsky) form.** For the latter description the design of a feedback controller was possible.

• Moreover, to cope with **unknown nonlinear terms** appearing in the new control inputs of the transformed state-space description of the systems, the use of nonlinear regressors (neurofuzzy approximators) has been proposed..

• These estimators were online trained to **identify the unknown dynamics of the system** and the associated learning procedure was determined by the requirement the **first derivative of the control loop's Lyapunov function to be a negative one**.

• The computation of the control input required the **solution of two algebraic Riccati equation**.

• Through **Lyapunov stability analysis** it was proven that the closed loop satisfies the **H-infinity tracking performance criterion**, while also an **asymptotic stability condition** has been formulated.

8. Conclusions

• Deliverables from related research projects are:

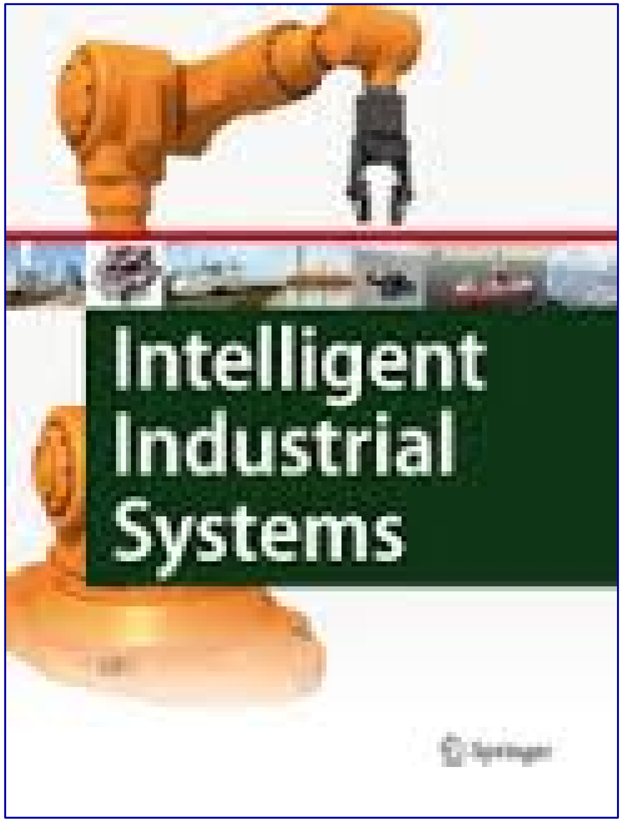
[1] G. Rigatos, **Modelling and control for intelligent industrial systems: adaptive algorithms In Robotics and Industrial Engineering**, Springer, 2011

[2] G. Rigatos, **Advanced models of Neural Networks: Nonlinear Dynamics and Stochasticity in Biological Neurons**, Springer, 2013

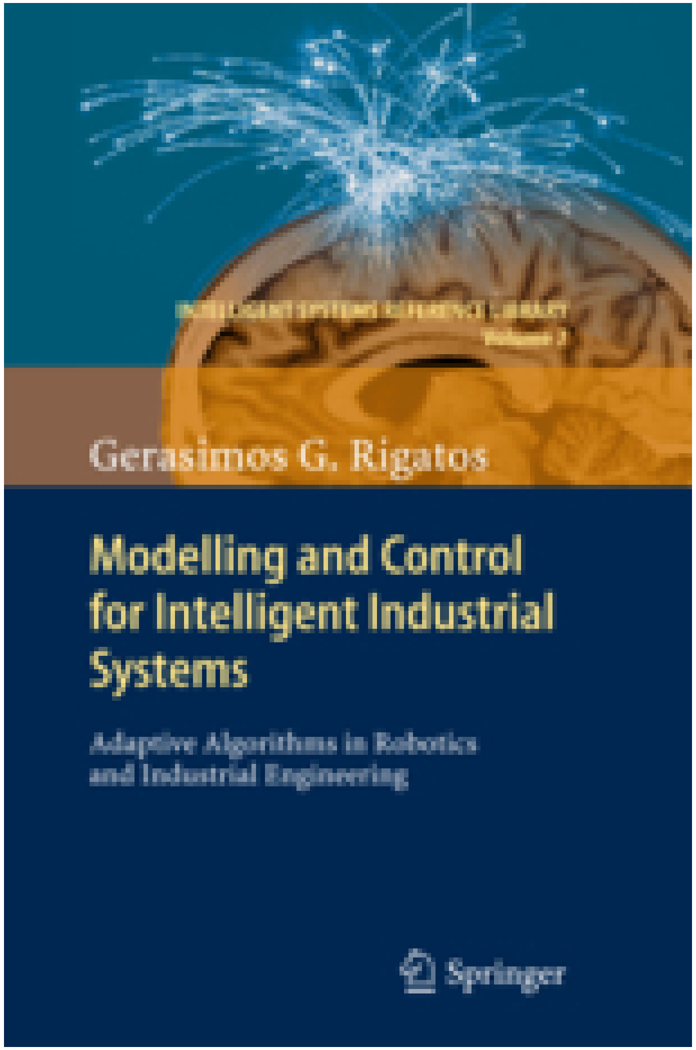
[3] G. Rigatos, **Nonlinear control and filtering using differential flatness approaches: applications to electromechanical systems**, Springer 2015.

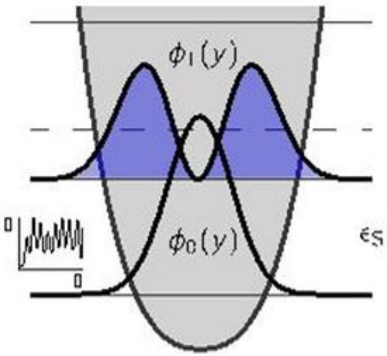
[4] G. Rigatos, **Intelligent renewable energy systems: Modelling and Control**, Springer, 2017

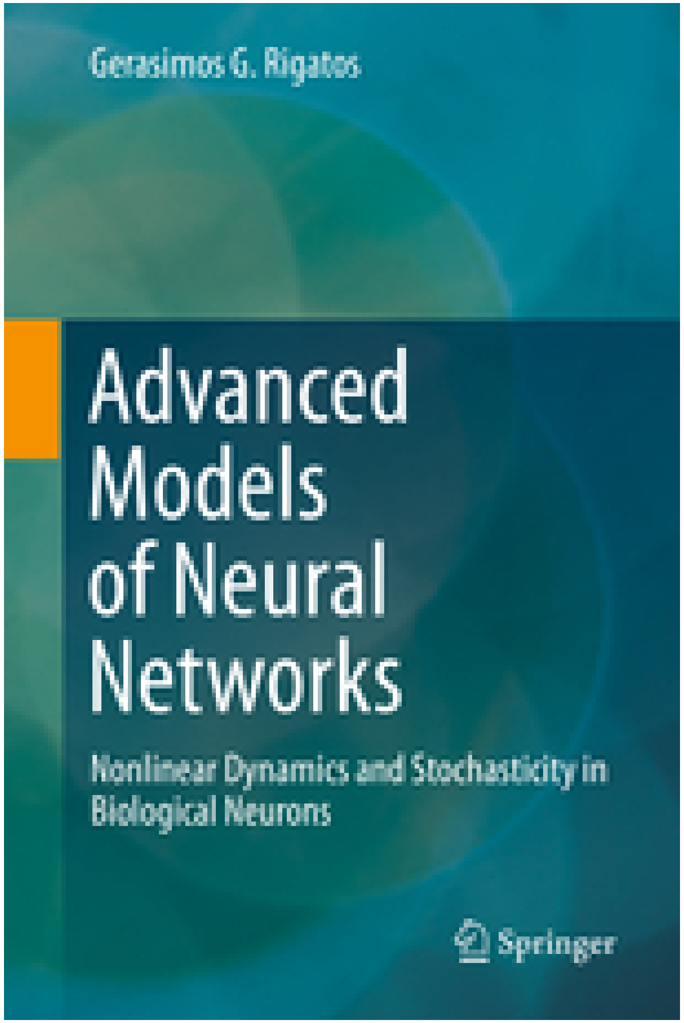
[5] G. Rigatos, **Journal of Intelligent Industrial Systems**, Springer, 2015

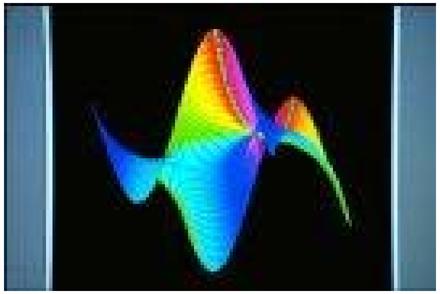


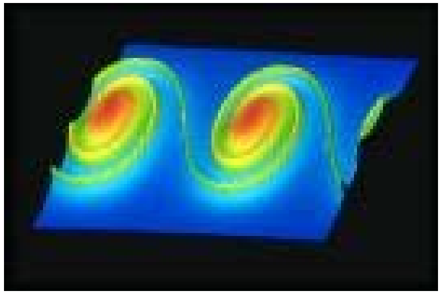
Thank you for your attention

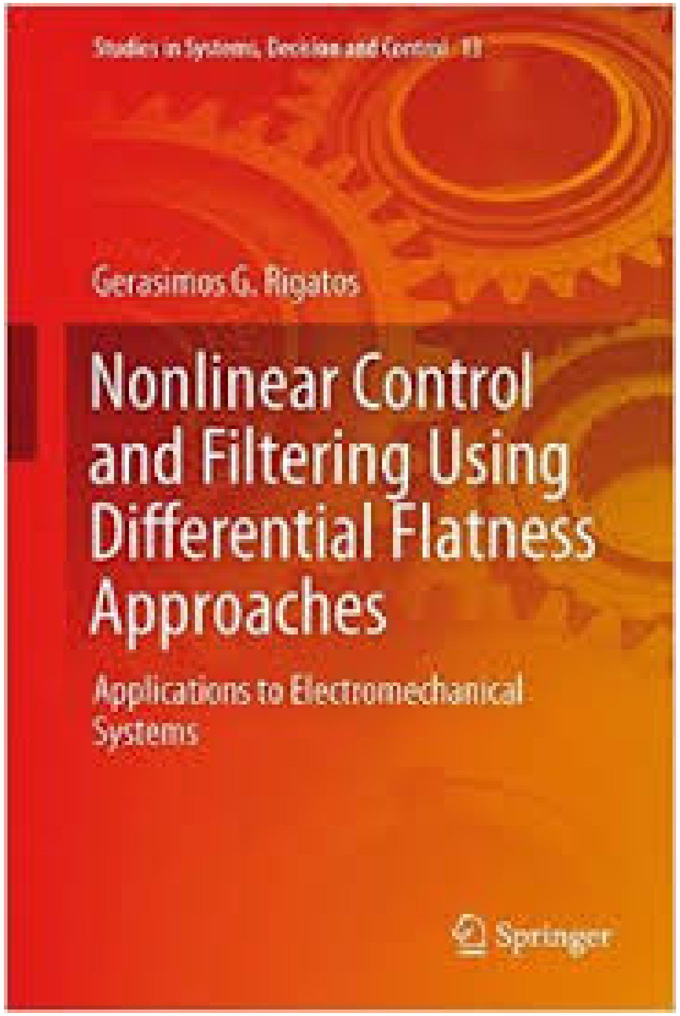

Gerasimos G. Rigatos
Modelling and Control for Intelligent Industrial Systems
Adaptive Algorithms in Robotics and Industrial Engineering
Springer


$\phi_1(y)$
$\phi_0(y)$


Gerasimos G. Rigatos
Advanced Models of Neural Networks
Nonlinear Dynamics and Stochasticity in Biological Neurons
Springer



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