Nonlinear control and state estimation for electric power systems

Gerasimos Rigatos
Electrical and Computer Eng., Ph.D.
Research Director

Unit of Industrial Automation
Industrial Systems Institute
26504, Rion Patras, Greece

email: grigat@ieee.org

1 . Outline

● The reliable functioning of electric power systems relies on the solution of the associated nonlinear control and state estimation problems

● The main approaches followed towards the solution of nonlinear control problem are as follows: (i) **control with global linearization** methods (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods** (adaptive control methods) when the dynamic model of the electric power systems is unknown

● The main approaches followed towards the solution of the nonlinear state estimation problems are as follows: (i) state estimation with methods global linearization (ii) state estimation with methods of approximate (asymptotic) linearization

● Factors of major importance for the control loop of electric power systems are as follows (i) global stability conditions for the related nonlinear control scheme (ii) global stability conditions for the related nonlinear state estimation scheme (iii) global asymptotic stability for the joint control and state estimation scheme

2 . Nonlinear control and state estimation with global linearization

● To this end the differential flatness control theory is used

● The method can be applied to all nonlinear systems which are subject to an input-output linearization and actually such systems posses the property of differential flatness

● The state-space description for the dynamic model of the electric power systems is transformed into a more compact form that is input-output linearized. This is achieved after defining the system's flat outputs

● A system is differentially flat if the following two conditions hold: (i) all state variables and control inputs of the system can be expressed as differential functions of its flat outputs (ii) the flat outputs of the system and their time-derivatives are differentially independent, which means that they are not connected through a relation having the form of an ordinary differential equation

● With the applications of change of variables (diffeomorphisms) that rely on the differential flatness property (i), the state-space description of the electric power system is written into the linear canonical form. For the latter state-space description it is possible to solve both the control and the state estimation problem for the electric power system.

3 . Nonlinear control and state estimation with approximate linearization

- To this end the theory of optimal H-infinity control and the theory of optimal H-infinity state estimation are used

- The nonlinear state-space description of the electric power system undergoes approximate linearization around a temporary operating point which is updated at each iteration of the control and state estimation algorithm

- The linearization relies on first order Taylor series expansion around the temporary operating point and makes use of the computation of the associated Jacobian matrices

- The linearization error which is due to the truncation error of higher-order terms in the Taylor series expansion is considered to be a perturbation that is finally compensated by the robustness of the control algorithm

- For the linearized description of the state-space model an optimal H-infinity controller is designed. For the selection of the controller's feedback gains an algebraic Riccati equation has to be solved at each time step of the control algorithm

- Through Lyapunov stability analysis, the global stability properties of the control method are proven

- For the implementation of the optimal control method through the processing of measurements from a small number of sensors in the electric power system, the H-infinity Kalman Filter is used as a robust state estimator

4 . Nonlinear control and state estimation with Lyapunov methods

● By initially proving the differential flatness properties for the electric power system and by defining its flat outputs a transformation of Its state-space description into an equivalent input-output linearized form is achieved.

● The unknown dynamics of the electric power systems is incorporated into the transformed control inputs of the system, which now appear in its equivalent input-output linearized state-space description

● The control problem for the electric power systems of unknown dynamics in now turned into a problem of indirect adaptive control. The computation of the control inputs of the system is performed simultaneously with the identification of the nonlinear functions which constitute its unknown dynamics.

● The estimation of the unknown dynamics of the electric power system is performed through the adaptation of neurofuzzy approximators. The definition of the learning parameters takes place through gradient algorithms of proven convergence, as demonstrated by Lyapunov stability analysis

● The Lyapunov stability method is the tool for selecting both the gains of the stabilizing feedback controller and the learning rate of the estimator of the unknown system's dynamics

● Equivalently through Lyapunov stability analysis the feedback gains of the state estimators of the electric power system are chosen. Such observers are included in the control loop so as to enable feedback control through the processing of a small number of sensor measurements

Example 1: Nonlinear control and state estimation using global linearization

5.1. Outline

• **Decentralized control for parallel inverters** connected to the power grid is developed using **differential flatness theory** and **the Derivative-free nonlinear Kalman Filter**.

• The problem is of elevated difficulty comparing to the control of stand-alone inverters because in this case in the dynamics of each inverter one has also to **compensate for interaction terms** which are due to the coupling with other inverters.

• T**he model of inverters, is differentially flat** and thus the multiple inverters model can be transformed into a set of local inverter models which are **decoupled and linearized**.

• For each local inverter the design of a **state feedback controller** becomes possible, e.g. using pole placement methods. Such a controller processes measurements not only coming from the individual inverter but also coming from other inverters connected to the grid.

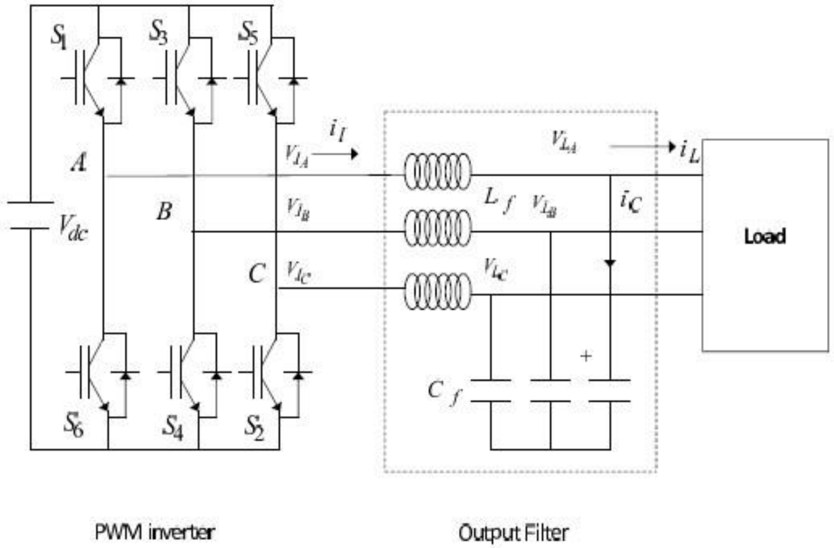
• Moreover, to **estimate the non-measurable state variables** of each local inverter, the **Derivative-free nonlinear Kalman Filter** is used. This consists of the Kalman Filter recursion applied to the local linearized model of the inverter and of an inverse transformation that is based on differential flatness theory, which enables to compute estimates of the state variables of the initial nonlinear model of the inverter.

• Furthermore, by **redesigning the aforementioned filter as a disturbance observer** it becomes also possible to estimate and compensate for disturbance terms that affect each local inverter.

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

Voltage inverters (DC to AC converters) are usually connected to their output to **a LC or a LCL filter**



By applying **Kirchhoff's voltage and current laws** one obtains

$$\frac{d}{dt}i_I = \frac{1}{L_f}V_I - \frac{1}{L_f}V_L$$
$$\frac{d}{dt}V_L = \frac{1}{C_f}i_I - \frac{1}{C_f}i_L \qquad \text{A}$$

For the representation of the voltage and current variables, denoted as $X = \{I, V\}$ in the ab static reference frame one has

$$X_{ab} = X_a e^{j0} + X_b e^{\frac{j2\pi}{3}} + X_c e^{\frac{j4\pi}{3}}$$

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

Using the **Park transformation** this is also written as a complex variable in the form

$$X_{ab} = X_a + jX_b$$

Next, the voltage and current variables are represented in the **rotating *dq* reference** frame

$$X_{dq} = X_{ab}e^{-j\theta} \Rightarrow X_{ab} = X_{dq}e^{j\theta}$$
$$\text{where } \theta(t) = \int_0^t \omega(t)dt + \theta_0$$

By differentiating with respect to time one obtains the following description

$$\dot{X}ab = \frac{d}{dt}X_{dq} + j\omega X_{dq}$$

Thus, one has for the current and voltage variables respectively,

$$\dot{i}_{I,ab} = \frac{d}{dt}i_{I,dq} + (j\omega)i_{I,dq}$$
$$\dot{V}_{L,ab} = \frac{d}{dt}V_{L,dq} + (j\omega)V_{L,dq}$$

B

By substituting Eq. B into Eq. A one obtains

$$\frac{d}{dt}i_{I,dq} + j\omega i_{I,dq} = \frac{1}{L_f}V_{I,dq} - \frac{1}{L_f}V_{L,dq}$$
$$\frac{d}{dt}V_{L,dq} + j\omega V_{L,dq} = \frac{1}{C_f}i_{I,dq} - \frac{1}{C_f}i_{L,dq}$$

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

Thus one arrives at a **description of the inverter's dynamics in the *dq* reference frame**

$$\begin{aligned}
\frac{d}{dt}V_{L,d} &= \omega V_{L,q} + \frac{1}{C_f}i_{I,d} - \frac{1}{C_f}i_{L,d} \\
\frac{d}{dt}V_{L,q} &= -\omega V_{L,d} + \frac{1}{C_f}i_{I,q} - \frac{1}{C_f}i_{L,q} \\
\frac{d}{dt}i_{I,d} &= \omega i_{I,q} + \frac{1}{L_f}V_{I,d} - \frac{1}{L_f}V_{L,d} \\
\frac{d}{dt}i_{I,q} &= -\omega i_{I,d} + \frac{1}{L_f}V_{I,q} - \frac{1}{L_f}V_{L,q}
\end{aligned}$$

The **state vector of the system** is taken to be

$$\tilde{X} = [V_{L_d}, V_{L_q}, i_{I,d}, i_{I,q}]^T$$

The **active and the reactive power of the inverter** are used next

$$p_f = V_{L_d}i_{L_d} + V_{L_q}i_{L_q} \qquad \text{(C)}$$

$$q_f = V_{L_q}i_{L_d} - V_{L_d}i_{L_q} - \omega C_f(V_{L_d}^2 + V_{L_q}^2) + \omega L_f(i_{I,d}^2 + i_{I,q}^2) \qquad \text{(D)}$$

By solving Eq. C and Eq. D with respect to the **load currents** one obtains

$$i_{L_d} = \frac{p_f V_{L_d} + q_f V_{L_q}}{V_{L_d}^2 + V_{L_q}^2} + \omega C_f V_{L_q} - \frac{\omega L_f V_{L_q}(i_{I_d}^2 + i_{I_q}^2)}{(V_{L_d}^2 + V_{L_q}^2)}$$

$$i_{L_q} = \frac{p_f V_{L_q} - q_f V_{L_d}}{V_{L_d}^2 + V_{L_q}^2} - \omega C_f V_{L_d} + \frac{\omega L_f V_{L_d}(i_{I_d}^2 + i_{I_q}^2)}{(V_{L_d}^2 + V_{L_q}^2)}$$

Example 1: Nonlinear control and state estimation using global linearization

5.2. Dynamics of the inverter

and by using the state variables notation $x_1 = V_{L_d},\ x_2 = V_{L_q},\ x_3 = i_{L_d}$ and $x_4 = i_{L_q}$

one finally obtains the **state-space description of the inverter's dynamics**

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \omega x_2 + \frac{1}{C_f}x_3 - \frac{1}{C_f}\frac{p_f x_1 + q_f x_2}{x_1^2+x_2^2} + \omega C_f x_2 - \frac{\omega L_f x_2(x_3^2+x_4^2)}{(x_1^2+x_2^2)} \\ -\omega x_1 + \frac{1}{C_f}x_4 - \frac{1}{C_f}\frac{p_f x_2 - q_f x_1}{x_1^2+x_2^2} - \omega C_f x_1 + \frac{\omega L_f x_1(x_3^2+x_4^2)}{(x_1^2+x_2^2)} \\ \omega x_4 - \frac{1}{L_f}x_1 \\ -\omega x_3 - \frac{1}{L_f}x_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} V_{L_d} \\ V_{L_q} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

thus, the inverter's model is written in the **nonlinear state-space form**

$$\dot{x} = f(x) + G(x)u$$
$$y = h(x)$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

- **Differential flatness theory** has been developed as a **global linearization control method** by M. Fliess (Ecole Polytechnique, France) and co-researchers (Lévine, Rouchon, Mounier, Rudolph, Petit, Martin, Zhu, Sira-Ramirez et. al)

- A dynamical system can be written in the ODE form $S_i(w,\dot{w},\ddot{w},\ldots,w^{(i)}),\ \ i=1,2,\ldots,q$
  where $w^{(i)}$ stands for the i-th derivative of either a state vector element or of a control input

- The system is said to be **differentially flat** with respect to the **flat output**

$$y_i=\phi(w,\dot{w},\ddot{w},\ldots,w^{(a)}),\ i=1,\ldots,m \quad \text{where} \quad y=(y_1,y_2,\ldots,y_m)$$

if the following two conditions are satisfied

(i) There does not exist any differential relation of the form

$$R(y,\dot{y},\ddot{y},\ldots,y^{(\beta)})=0$$

which means that **the flat output and its derivatives are linearly independent**

(ii) All system variables are **functions of the flat output and its derivatives**

$$w^{(i)}=\psi(y,\dot{y},\ddot{y},\ldots,y^{(\gamma_i)})$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

The **flat output of the inverter** is taken to be the vector

$$y=[y_1,y_2]=[V_{L_d},V_{L_q}]$$

The first row of the state-space equations is

$$\dot{x}_1=\omega x_2+\frac{1}{C_f}x_3-\frac{1}{C_f}\frac{p_f x_1+q_f x_2}{x_1^2+x_2^2}-\omega x_2+\frac{1}{C_f}\frac{\omega L_f x_2(x_3^2+x_4^2)}{x_1^2+x_2^2} \qquad \text{E}$$

The second row of the state-space equations is

$$\dot{x}_2=\omega x_1+\frac{1}{C_f}x_4-\frac{1}{C_f}\frac{p_f x_2+q_f x_1}{x_1^2+x_2^2}+\omega x_1+\frac{1}{C_f}\frac{\omega L_f x_1(x_3^2+x_4^2)}{x_1^2+x_2^2} \qquad \text{F}$$

These equations are rewritten as follows

$$\frac{1}{C_f}\frac{\omega L_f x_2(x_3^2+x_4^2)}{x_1^2+x_2^2}=\dot{x}_1-\omega x_2-\frac{1}{C_f}x_3+\frac{1}{C_f}\frac{p_f x_1+q_f x_2}{(x_1^2+x_2^2)}+\omega x_2$$

$$\frac{1}{C_f}\frac{\omega L_f x_1(x_3^2+x_4^2)}{x_1^2+x_2^2}=\dot{x}_2+\omega x_1-\frac{1}{C_f}x_4+\frac{1}{C_f}\frac{p_f x_2-q_f x_1}{(x_1^2+x_2^2)}-\omega x_1$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

By dividing the above two equations one gets

$$-\frac{x_2}{x_1} = \frac{\dot{x}_1 - \omega x_2 - \frac{1}{C_f}x_3 + \frac{1}{C_f}\frac{p_f x_1 + q_f x_2}{(x_1^2+x_2^2)} + \omega x_2}{\dot{x}_2 + \omega x_1 - \frac{1}{C_f}x_4 + \frac{1}{C_f}\frac{p_f x_2 - q_f x_1}{(x_1^2+x_2^2)} - \omega x_1}$$

while using in the notation the elements of the flat output vector this gives

$$-\frac{y_2}{y_1}\dot{y}_2 - \omega y_2 + \frac{1}{C_f}\left(\frac{y_2}{y_1}\right)\frac{p_f y_2 - q_f y_1}{(y_1^2+y_2^2)} + \omega y_2 =$$
$$= \dot{y}_1 - \omega y_2 - \frac{1}{C_f}x_3 + \frac{1}{C_f}\frac{p_f y_1 + q_f y_2}{(y_1^2+y_2^2)} + \omega y_2$$

By solving the above equation with respect to x_3 gives

$$x_3 = -\frac{y_2}{y_1}x_4 + C_f\{\frac{y_2}{y_1}\dot{y}_2 + \omega y_2 + \frac{1}{C_f}\frac{(y_2}{y_1)}\frac{p_f y_2 - q_f y_1}{(y_1^2+y_2^2)} - \omega y_2$$
$$+\dot{y}_1 - \omega y_2 + \frac{1}{C_f}\frac{p_f y_1 + q_f y_2}{(y_1^2+y_2^2)} + \omega y_2\}$$

which is also written as $x_3 = -(\frac{y_2}{y_1})x_4 + f_a(y_1, \dot{y}_1, y_2, \dot{y}_2)$ G

Next Eq. G is substituted into Eq. E which gives.

$$\dot{x}_2 = -\omega x_1 + \frac{1}{C_f}x_4 - \frac{1}{C_f}\frac{p_f x_2 - q_f x_1}{(x_1^2+x_2^2)} + \omega x_1 -$$
$$-\frac{1}{C_f}\frac{\omega L_f x_1\{[-\frac{(y_2}{y_1})x_4 + f_a(y_1, \dot{y}_1, y_2, doty_2)]^2 + x_4^2\}}{(x_1^2+x_2^2)}$$

Example 1: Nonlinear control and state estimation using global linearization

5.3. Differential flatness of the inverter

or equivalently.

$$\dot{y}_2 = -\omega y_1 + \frac{1}{C_f}x_4 - \frac{1}{C_f}\frac{p_f y_2 - q_f y_1}{(y_1^2+y_2^2)} + \omega y_1 + \\ -\frac{1}{C_f}\frac{\omega L_f y_1\{[-(\frac{y_2}{y_1})x_4 + f_a(y_1,\dot{y}_1,y_2,\dot{y}_2)]^2 + x_4^2\}}{(y_1^2+y_2^2)}$$

which finally gives.

$$x_4 = f_b(y_1,\dot{y}_1,y_2,\dot{y}_2) \qquad \text{H}$$

Moreover, by substituting Eq. H into Eq. G one gets

$$x_3 = -(\frac{y_2}{y_1})f_b(y_1,\dot{y}_1,y_2,\dot{y}_2) + f_4(y_1,\dot{y}_1,y_2,\dot{y}_2)$$

From the last two rows of the state-space equations one has that

$$\dot{x}_3 = \omega x_4 - \frac{1}{L_f}x_1 + \frac{1}{L_f}u_1 \qquad \text{I}$$

$$\dot{x}_4 = -\omega x_3 - \frac{1}{L_f}x_2 + \frac{1}{L_f}u_2 \qquad \text{J}$$

Thus, one obtains

$$u_1 = L_f\{\dot{x}_3 - \omega x_4 + \frac{1}{L_f}x_1\} \Rightarrow u_1 = f_c(y_1,\dot{y}_1,y_2,\dot{y}_2)\}$$

$$u_2 = L_f\{\dot{x}_4 - \omega x_3 + \frac{1}{L_f}x_2\} \Rightarrow u_2 = f_d(y_1,\dot{y}_1,y_2,\dot{y}_2)\}$$

This **confirms the differential flatness of the model**

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

By considering the **active and reactive power** of the inverter as **piecewise constant** and by deriving the first row of the state-space equations in time, one has

$$\ddot{x}_1 = \omega\dot{x}_2 + \frac{1}{C_f}\dot{x}_3 - \frac{1}{C_f}\frac{\{(p_f\dot{x}_1+q_f\dot{x}_2)(x_1^2+x_2^2)-(p_f x_1+q_f x_2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2\}} -$$

$$-\omega\dot{x}_2 + \frac{1}{C_f}\{\frac{\omega L_f\dot{x}_2(x_3^2+x_4^2)(x_1^2+x_2^2)+\omega L_f x_2(2x_3\dot{x}_3+2x_4\dot{x}_4)(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2}\}$$

$$-\frac{\omega L_f x_2(x_3^2+x_4^2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}$$

The time derivatives are substituted from the associated rows of the state-space equations.

$$\ddot{x}_1 = \omega\dot{x}_2 + \frac{1}{C_f}(\omega x_4 - \frac{1}{L_f}x_1 + \frac{1}{L_f}u_1) - \frac{1}{C_f}\frac{\{(p_f\dot{x}_1+q_f\dot{x}_2)(x_1^2+x_2^2)-(p_f x_1+q_f x_2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2\}} -$$

$$-\omega\dot{x}_2 + \frac{1}{C_f}\{\frac{\omega L_f\dot{x}_2(x_3^2+x_4^2)(x_1^2+x_2^2)+\omega L_f x_2(x_3^2+x_4^2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$+\frac{\omega L_f x_2}{C_f}\frac{2x_3(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2}(\omega x_4 - \frac{1}{L_f}x_1 + \frac{1}{L_f}u_1) + \frac{\omega L_f x_2}{C_f}\frac{2x_4(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2}(-\omega x_3 - \frac{1}{L_f}x_2 + \frac{1}{L_f}u_2)$$

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

The previous relation can be also written using the **notation of the Lie algebra-based linearization**

$$\ddot{x}_1 = L_f^2 h_1(x) + L_{g_a} L_f h_1(x) u_1 + L_{g_b} L_f h_1(x) u_2$$

where

$$L_f^2 h_1(x) = \omega \dot{x}_2 + \frac{1}{C_f}(\omega x_4 - \frac{1}{L_f} x_1) - \frac{1}{C_f}\{\frac{(p_f \dot{x}_1 + q_f \dot{x}_2)(x_1^2+x_2^2) - (p_f x_1 + q_f x_2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$-\omega \dot{x}_2 + \frac{1}{C_f}\{\frac{\omega L_f \dot{x}_2 (x_3^2+x_4^2)(x_1^2+x_2^2) - \omega L_f x_2 (x_3^2+x_4^2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$+\frac{\omega L_f x_2}{C_f} \cdot \frac{2x_3(\omega x_4 - \frac{1}{L_f} x_1)}{(x_1^2+x_2^2)} + \frac{\omega L_f x_2}{C_f} \cdot \frac{2x_1(-\omega x_3 - \frac{1}{L_f} x_2)}{(x_1^2+x_2^2)}$$

$$L_{g_a} L_f h_1(x) = \frac{1}{C_f}\{\frac{\omega L_f x_2 (2x_3)}{(x_1^2+x_2^2) L_f} + \frac{1}{L_f}\}$$

$$L_{g_b} L_f h_1(x) = \frac{\omega L_f x_2}{C_f} \frac{2x_4}{(x_1^2+x_2^2)} \frac{1}{L_f}$$

In a similar manner, by differentiating the second row of the state-space equations with respect to time one has

$$\ddot{x}_2 = -\omega \dot{x}_1 + \frac{1}{C_f}\dot{x}_4 - \frac{1}{C_f}\frac{(p_f \dot{x}_2 - g_f \dot{x}_1)(x_1^2+x_2^2) - (p_f x_2 - g_f x_1 (2x_1\dot{x}_1 + 2x_2\dot{x}_2))}{(x_1^2+x_2^2)^2} +$$

$$+\omega \dot{x}_1 - \frac{1}{C_f}\{\frac{\omega L_f \dot{x}_1 (x_3^2+x_4^2)(x_1^2+x_2^2) + \omega L_f x_1 (2x_3\dot{x}_3 + 2x_4\dot{x}_4)(x_1^2+x_2^2)}{(x_1^2+x_2^2)^2} -$$

$$\frac{-\omega L_f x_1 (x_3^2+x_4^2)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

The previous relation can be also written using the **notation of the Lie algebra-based linearization**

$$\ddot{x}_2 = L_f^2h_2(x) + L_{g_a}L_fh_2(x)u_1 + L_{g_b}L_fh_2(x)u_2$$

where

$$L_f^2h_2(x) = -\omega\dot{x}_1 + \frac{1}{C_f}(-\omega x_3) - \frac{1}{C_f}\{\frac{(p_f\dot{x}_2 - g_f\dot{x}_1)(x_1^2+x_2^2) - (p_fx_2 - g_fx_1)(2x_1\dot{x}_1 + 2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\} +$$

$$\omega\dot{x}_1 - \frac{1}{C_f}\{\frac{\omega L_f\dot{x}_1(x_3^2+x_4^2)(x_1^2+x_2^2) - \omega L_fx_1(x_3^2+x_4^2)(2x_1\dot{x}_1+2x_2\dot{x}_2)}{(x_1^2+x_2^2)^2}\}$$

$$-\frac{1}{C_f}\frac{\omega L_fx_12x_3(\omega x_4 - \frac{1}{L_f}x_1)}{(x_1^2+x_2^2)} - \frac{1}{C_f}\frac{\omega L_fx_12x_4(-\omega x_3 - \frac{1}{L_f}x_2)}{(x_1^2+x_2^2)}$$

$$L_{g_a}L_fh_2(x) = -\frac{1}{C_f}\frac{\omega L_fx_12x_3}{(x_1^2+x_2^2)}\frac{1}{L_f}$$

$$L_{g_b}L_fh_2(x) = -\frac{1}{C_f}\{\frac{\omega L_fx_12x_4}{(x_1^2+x_2^2)}\frac{1}{L_f} + \frac{1}{L_f}\}$$

Thus, one obtains **an input-output linearized description of the inverter**

$$\begin{aligned}\ddot{x}_1 &= L_f^2h_1(x) + L_{g_a}L_fh_1(x)u_1 + L_{g_b}L_fh_1(x)u_2\\ \ddot{x}_2 &= L_f^2h_2(x) + L_{g_a}L_fh_2(x)u_1 + L_{g_b}L_fh_2(x)u_2\end{aligned}$$

or equivalently

$$\begin{aligned}\ddot{x}_1 &= v_1\\ \ddot{x}_2 &= v_2\end{aligned}$$ with $$\begin{aligned}v_1 &= L_f^2h_1(x) + L_{g_a}L_fh_1(x)u_1 + L_{g_b}L_fh_1(x)u_2\\ v_2 &= L_f^2h_2(x) + L_{g_a}L_fh_2(x)u_1 + L_{g_b}L_fh_2(x)u_2\end{aligned}$$

Example 1: Nonlinear control and state estimation using global linearization

5.4. Flatness-based control of the inverter

For this form of the system's dynamics the design of a **state feedback controller** is easy. This takes the form

$$v_1 = \ddot{x}_1^d - k_d^1(\dot{x}_1 - \dot{x}_1^d) - k_p^1(x_1 - x_1^d)$$
$$v_2 = \ddot{x}_2^d - k_d^2(\dot{x}_2 - \dot{x}_2^d) - k_p^2(x_2 - x_2^d)$$

The **control input** that is actually applied to the inerter is given form $\tilde{v} = \tilde{f} + \tilde{M}u$

or equivalently
$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{pmatrix} + \begin{pmatrix} L_{g_a} L_f h_1(x) & L_{g_b} L_f h_1(x) \\ L_{g_a} L_f h_2(x) & L_{g_b} L_f h_2(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

which means that the control input that is finally applied to the system is

$$\tilde{u} = \tilde{M}^{-1}(\tilde{v} - \tilde{f})$$

Moreover, by defining the **new state variables** $z_1 = x_1,\ z_2 = \dot{x}_1,\ z_3 = x_2 \text{ and } z_4 = \dot{x}_2,$ the following state-space description is obtained

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\begin{pmatrix} z_1^m \\ z_2^m \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

Example 1: Nonlinear control and state estimation using global linearization

5.5. Equivalence between inverters and synchronous generators

Synchronization between parallel inverters is considered next. The functioning of the i-th inverter is shown to be equivalent to a synchronous generator with turn speed denoted as ω_i

The **deviation from the synchronous speed** is shown to be proportional to the deviation of the produced active power from a reference value

$$\Delta\delta_i = \omega_i - \omega_d = -k_{p_i}(P_i^m - P_i^d) \qquad \text{K}$$

P_i^m **measured active power** of the i-th power generation unit

P_i^d **desirable active power**

k_{p_i} "**droop" gain** which is practically computed by dividing the range of variation of the inverter's frequency $(\omega_{max}$-$\omega_{min})$ by the maximum active power $P_{i_{max}}$

Since the measured active power is obtained from the inverter's real active power with a **time delay in measurement**, it holds that

$P_i^m(s) = e^{-s\tau_{p_i}}P(s)$ or equivalently $\tau_{p_i}\dot{P}_i^m = -P_i^m + P_i$ L

Thus the i-th **inverter's dynamics** is expressed as

$$\Delta\dot{\delta}_i = \Delta\omega_i$$
$$\tau_{p_i}\dot{P}_i^m = -P_i^m + P_i$$

Example 1: Nonlinear control and state estimation using global linearization

5.5. Equivalence between inverters and synchronous generators

By differentiating Eq. K one obtains

$$\Delta\dot{\omega}_i = -k_{p_i}\dot{P}_i^m + k_{p_i}\dot{P}_i^d \qquad \text{M}$$

Moreover, from Eq. L one obtains

$$\dot{P}_i^m = -\frac{1}{\tau_{p_i}}P_i^m + \frac{1}{\tau_{p_i}}P_i \qquad \text{N}$$

By substituting Eq. N Into Eq. M one obtains

$$\Delta\dot{\omega}_i = -k_{p_i}\left(-\frac{1}{\tau_{p_i}}P_i^m + \frac{1}{\tau_{p_i}}P_i\right) + k_{p_i}\dot{P}_i^d$$

and using that $\dot{P}_i^d = 0$ one has

$\Delta\dot{\omega}_i = \frac{k_{p_i}}{\tau_{p_i}}P_i^m - \frac{k_{p_i}}{\tau_{p_i}}P_i$ or equivalently $J_i\Delta\dot{\omega}_i = P_i^m - P_i$ with $J_i = \tau_{p_i}/k_{p_i}$

Additionally, from Eq. K one has

$$\omega_i - \omega_d - k_{p_i}P_i^d = -k_{p_i}P_i^m \Rightarrow P_i^m = -\frac{1}{k_{p_i}}\omega_i + \frac{1}{k_{p_i}}\omega_d + P_i^d \Rightarrow$$
$$P_i^m = -\frac{1}{k_{p_i}}\Delta\omega_i + P_i^d$$

From the previous two equations one gets

$$\Delta\dot{\omega}_i = -k_{p_i}\dot{P}_i^m + k_{p_i}\dot{P}_i^d \quad \text{or} \quad J_i\Delta\dot{\omega}_i = -D_{p_i}\Delta\omega_i + P_i^d - P_i \qquad \text{O}$$

Example 1: Nonlinear control and state estimation using global linearization

5.5. Equivalence between inverters and synchronous generators

In ideal conditions there is no **interaction (power exchange) between distributed power units** connected to the same electricity grid.

However, frequently such interaction exists and in the latter case Eq. O should be enhanced by including an interaction term

$$J_i\Delta\dot{\omega}_i = -D_{p_i}\Delta\omega_i + (P_i^d - P_i) + \sum_{j=1,j\neq i}^{n} G_{ij}sin(\delta_i - \delta_j)$$

where δ_i is the virtual turn angle that is associated with the i-th power generation unit (inverter).

About the **coupling coefficients** G_{ij} these are functions of the conductance of the grid line which connects the *i-th* to the *j-th* power generation unit, as well as of the grid voltage that is measured at points *i* and *j* respectively

Thus, finally the **dynamics of the i-th power generation** unit (inverter) is **described as a synchronous generator,** which interacts with other generators In the grid

$$\begin{aligned} \Delta\dot{\delta}_i(t) &= \Delta\omega_i(t) \\ J_i\Delta\dot{\omega}_i(t) &= -D_{p_i}\Delta\omega_i(t) + (P_i^d(t) - P_i(t)) + \sum_{j=1}^{N} G_{ij}sin(\delta_i - \delta_j) \end{aligned} \qquad \text{P}$$

In this approach, it is considered that the **i-th local controller** not only processes measurements coming from the associated power generation unit, but also **uses measurements coming from the other power units** which are connected to the grid

Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

By **representing the inverter as a virtual synchronous generator** then one has that its dynamics is composed of two parts (i) the rotation part and (ii) the electrical part.

(i) **Rotation part**

$$\begin{array}{c} \Delta\dot{\delta}_i(t) = \Delta\omega_i(t) \\ J_i\Delta\dot{\omega}_i(t) = -D_{p_i}\Delta\omega_i(t) + (P_i^d(t) - P_i(t)) + \sum_{j=1, j\neq i}^{N} G_{ij} sin(\delta_i - \delta_j) \end{array} \qquad \text{Q}$$

(i) **Electrical part**

$$\frac{d}{dt}\begin{pmatrix} V_{L_d} \\ V_{L_q} \\ i_{I_d} \\ i_{I_q} \end{pmatrix} = \begin{pmatrix} \omega V_{L_q} + \frac{1}{C_f} i_{I_d} - \frac{1}{C_f}\frac{p_f V_{L_d} + q_f V_{L_q}}{V_{L_d}^2 + V_{L_q}^2} + \omega C_f V_{L_q} - \frac{\omega L_f V_{L_q}(i_{I_d}^2 + i_{I_q}^2)}{(V_{L_d}^2 + V_{L_q}^2)} \\ -\omega V_{L_d} + \frac{1}{C_f} i_{I_q} - \frac{1}{C_f}\frac{p_f V_{L_q} - q_f V_{L_d}}{V_{L_d}^2 + V_{L_q}^2} - \omega C_f V_{L_d} + \frac{\omega L_f V_{L_d}(i_{I_d}^2 + i_{I_q}^2)}{(V_{L_d}^2 + V_{L_q}^2)} \\ \omega i_{I_q} - \frac{1}{L_f} V_{L_d} \\ -\omega i_{I_d} - \frac{1}{L_f} V_{L_q} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L_f} & 0 \\ 0 & \frac{1}{L_f} \end{pmatrix} \begin{pmatrix} V_{I_d} \\ V_{I_q} \end{pmatrix} \qquad \text{R}$$

The **synchronizing control approach** for the i-th inverter makes use of Eq. Q and of the linearized inverter model given in Eq. R

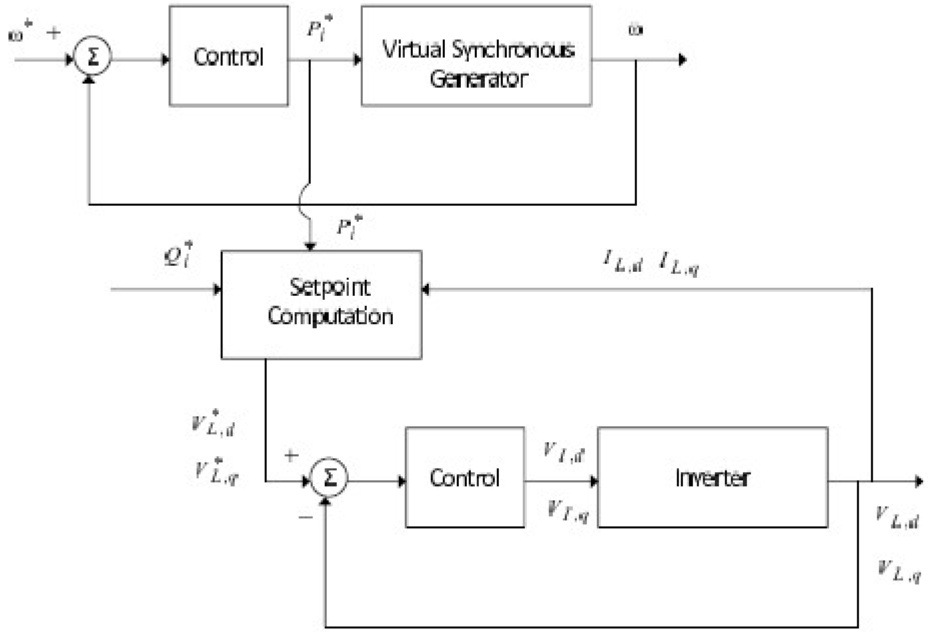
Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

First, the value of P_i, that is the **active power** that the i-th inverter should inject to the grid, is found from the solution of the control problem of Eq. Q

Subsequently P_i is used in the computation of the solution of the control problem of Eq. R

The **computation of setpoints** for the control of the **electric part of the inverter** is shown in the following diagram



Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

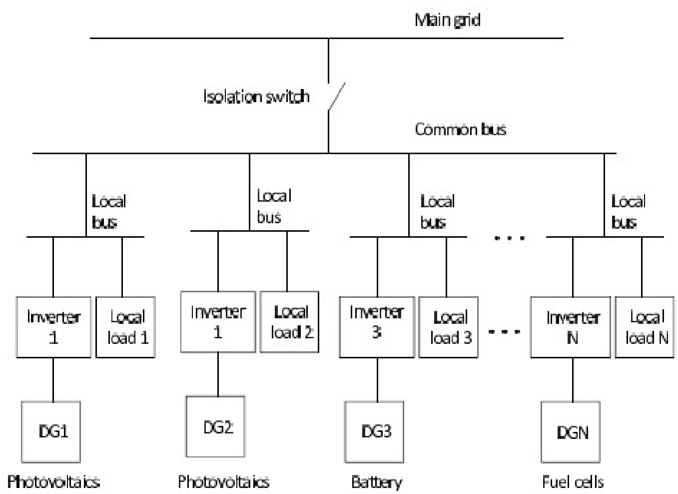
It can be proven that the model of **N-parallel inverters connected to the electricity grid** is a **differentially flat** one

By defining as **flat output** a **generalization of the state vector** of the stand-alone inverter, that is

$$Y = [y_1^1, y_2^1, y_3^1, y_1^2, y_2^2, y_3^2 \cdots, y_1^N, y_2^N, y_3^N]$$

or equivalently

$$Y = [\delta^1, V_{L_d}^1, V_{L_q}^1,\ \delta^2, V_{L_d}^2, V_{L_q}^2, \cdots, \delta^N, V_{L_d}^N, V_{L_q}^N]$$



It can be confirmed that **all state variables and control inputs** for **the model of the N coupled inverters** can be expressed as **functions of the aforementioned flat output Y** and of its derivatives.

Example 1: Nonlinear control and state estimation using global linearization

5.6. Control for parallel inverters connected to the grid

Using the previous flat output definition, and the state variables

$$z_1^i = y_1,\ z_2^i = \dot{y}_1,\ z_3^i = y_2,\ z_4^i = \dot{y}_2,\ \ z_5^i = y_3,\ z_6^i = \dot{y}_3$$

one has the state-space description

$$\begin{pmatrix} \dot{z}_1^i \\ \dot{z}_2^i \\ \dot{z}_3^i \\ \dot{z}_4^i \\ \dot{z}_5^i \\ \dot{z}_6^i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^i \\ z_2^i \\ z_3^i \\ z_4^i \\ z_5^i \\ z_6^i \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} v_1^i \\ v_2^i \\ v_3^i \end{pmatrix}$$

where the control inputs of this model are defined as

$$v_1^i = \frac{1}{J_i}[-D_{p_i}\Delta\omega_i(t) + (P_i^d(t) - P_i(t)) + \sum_{j=1}^N G_{ij} sin(\delta_i - \delta_j)] \qquad \text{S}$$

$$\begin{aligned} v_2^i &= L_f^2 h_1{}^i(x) + L_{g_a} L_f h_1^i(x) u_1^i + L_{g_b} L_f h_1^i(x) u_2^i \\ v_3^i &= L_f^2 h_2{}^i(x) + L_{g_a} L_f h_2^i(x) u_1^i + L_{g_b} L_f h_2^i(x) u_2^i \end{aligned} \qquad \text{T}$$

The above mean that for the **synchronization of the i-th virtual generator (inverter)** the **control input** (in the form of active power) is finally given by

$$\begin{aligned} P_i = -J_i \ddot{x}_{1,i}^d - D_{p_1} x_2 + P_i^d + \sum_{j=1}^N G_{ij} sin(x_{1,i} - x_{1,j}) + \\ + J_i K_{d_i}(\dot{x}_{1,i} - \dot{x}_{1,i}^d) + J_i K_{p_i}(x_{1,i} - x_{1,i}^d) \end{aligned} \qquad \text{U}$$

Example 1: Nonlinear control and state estimation using global linearization

5.7. Disturbances estimation with Kalman Filtering

A **state estimator for each local power generation unit** can be also designed in the form of a disturbance observer.

It is considered that the **linearized model of the i-th inverter** is affected by **additive input disturbances**

$$\ddot{z}_1^i = v_1^i + \tilde{d}_1^i$$
$$\ddot{z}_3^i = v_2^i + \tilde{d}_2^i$$
$$\ddot{z}_5^i = v_3^i + \tilde{d}_3^i$$

The **disturbances' dynamics can be represented by the n-th order derivative** of the disturbances variables together with the associated initial conditions.

Thus the **additive disturbances** are equivalently described in the form

$$\tilde{d}_1^{(n)} = f_{d_1},\ \tilde{d}_2^{(n)} = f_{d_2} \text{ and } \tilde{d}_3^{(n)} = f_{d_3}$$

The state vector is extended by **including as additional state variables** the disturbances and their derivatives. Thus, one has

$$\dot{z}_1^i = z_2^i,\ \dot{z}_2^i = z_7 + v_1^i,\ \dot{z}_3^i = z_4^i,\ \dot{z}_4^i = z_9 + v_2^i,\ \dot{z}_5^i = z_6^i,\ \dot{z}_6^i = z_{11}^i + v_3^i,$$

$$\dot{z}_7^i = z_8^i,\ \dot{z}_8^i = f_{d_1}^i,\ \dot{z}_9^i = z_{10}^i, \quad z_{11}^i = \tilde{d}_3 \text{ and } z_{12}^i = \dot{\tilde{d}}_3.$$

Example 1: Nonlinear control and state estimation using global linearization

5.7. Disturbances estimation with Kalman Filtering

Therefore, one has the system's dynamics in the **extended spate-space form**

$$\dot{z}_e = A_e \hat{z}_e + B_e v_e$$

where the **extended inputs vector** is $v_e = [v_1^i, v_2^i, v_3^i, f_{d_1}, f_{d_2}, f_{d_3}]^T$

while

$$A_e = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \quad B_e = \begin{pmatrix} 0&0&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&1&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&1&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&1 \end{pmatrix} \quad C_e^T = \begin{pmatrix} 1&0&0 \\ 0&0&0 \\ 0&1&0 \\ 0&0&0 \\ 0&0&1 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \\ 0&0&0 \end{pmatrix}$$

For the extended state-space description of the system the **state observer** becomes

$$\dot{\hat{z}}_e = A_o \hat{z}_e + B_o v_e + K_f (C_o z_e - C_o)\hat{z}_e$$

where $A_o = A_e$ and $C_o = C_e$

while B_o differs from B_e In the elements of the 10th and 12th rows which are all set to 0

Example 1: Nonlinear control and state estimation using global linearization

5.7. Disturbances estimation with Kalman Filtering

For the **linearized model** of the parallel inverters, **state estimation** is performed with the use of the **Kalman Filter (Derivative-free nonlinear Kalman Filter)**

In the filter's algorithm, the previously defined matrices A_e, B_e and C_e are substituted by their discrete-time equivalents A_{e_d}, B_{e_d} and C_{e_d} This is done through common discretization methods

The **filter's recursion** is:

$measurement\ update:$

$$
\begin{aligned}
K_f(k) &= P^-(k) C_d^T [C_{e_d} P^-(k) C_{e_d}^T + R(k)]^{-1} \\
\hat{x}(k) &= \hat{x}^-(k) + K_f(k)[C_{e_d} z(k) - C_{e_d} \hat{z}(k)] \\
P(k) &= P^-(k) - K(k) C_{e_d} P^-(k)
\end{aligned}
$$

$time\ update:$

$$
\begin{aligned}
P^-(k+1) &= A_{e_d} P(k) A_{e_d}^T + Q(k) \\
\hat{x}^-(k+1) &= A_{e_d} \hat{x}(k) + B_{e_d} v(k)
\end{aligned}
$$

After identifying the disturbance terms, the control input of the inverter is modified as follows:

$$
\begin{aligned}
v_1^i &= \ddot{z}_1 - k_d^1(\dot{z}_1 - \dot{z}_1^d) - k_p^1(z_1 - z_1^d) - \hat{z}_7 \\
v_2^i &= \ddot{z}_3 - k_d^2(\dot{z}_3 - \dot{z}_3^d) - k_p^2(z_3 - z_3^d) - \hat{z}_9 \\
v_3^i &= \ddot{z}_5 - k_d^3(\dot{z}_5 - \dot{z}_5^d) - k_p^3(z_5 - z_5^d) - \hat{z}_{11}
\end{aligned}
$$

The inclusion of the **disturbance estimation term** $\hat{z}_7, \hat{z}_9$ and $\hat{z}_{11}$ in the feedback control inputs enables to compensate for effects of the perturb $\tilde{d}_1, \tilde{d}_2$ and $\tilde{d}_3$

Example 1: Nonlinear control and state estimation using global linearization

5.8. Simulation tests

The performance of the proposed **distributed control scheme for the synchronization of parallel inverters** was tested through simulation experiments. A **model of *N* = *3* distributed power generation units** was considered, while each one of these units was connected to the grid through an inverter

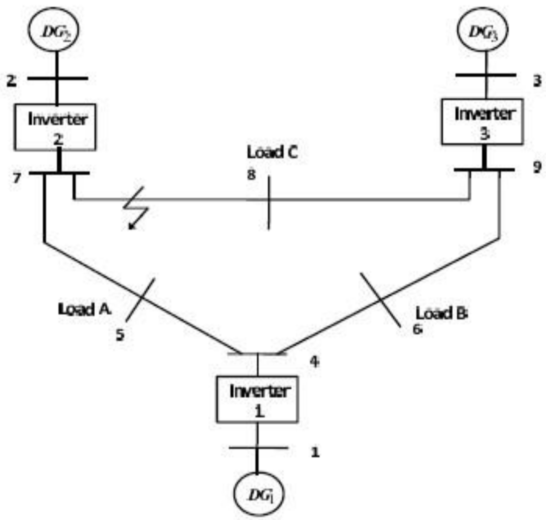


Table I
Parameters of the Inverters

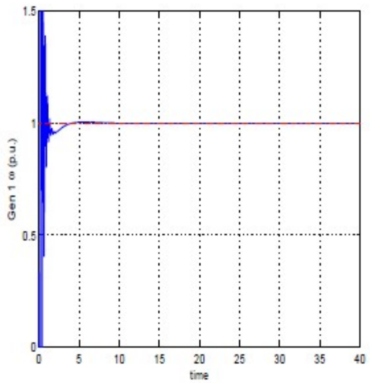
	Inv_1	Inv_2	Inv_3
$L_f\ (mH)$	10.5	10.3	10.1
$C_f\ (mF)$	0.04	0.03	0.02
$p_f\ (Kw)$	21.1	22.3	23.6

The three interconnected inverters, shown in Fig. 4, are assumed to have different model parameters which are described in **Table I.**

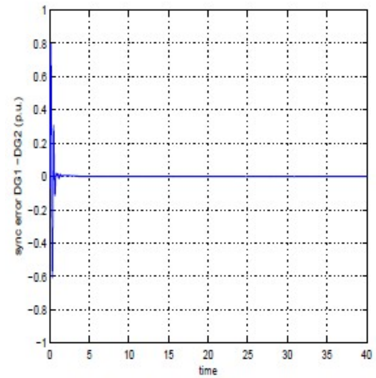
The objective is that all inverters (virtual synchronous generators) finally attain the same frequency ω_i

Example 1: Nonlinear control and state estimation using global linearization

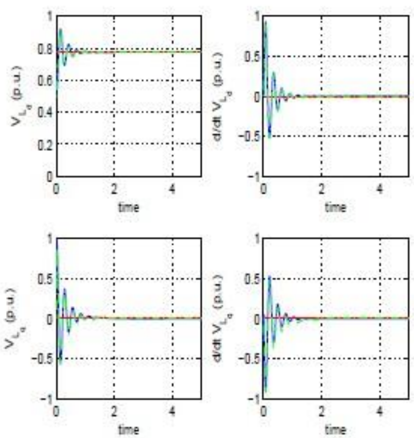
5.8. Simulation tests



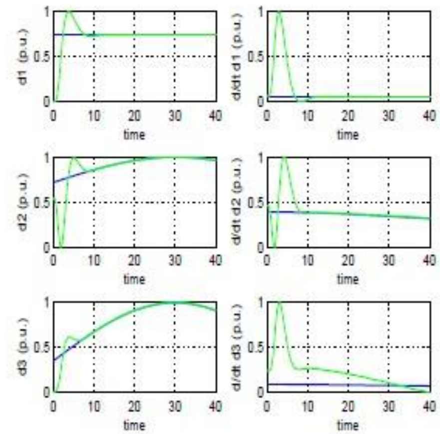
Test 1: (a) Angular speed of power generation unit 1



Test 1: synchronization error between power generation units 1 and 2



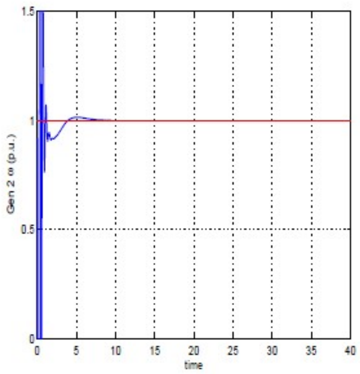
Test 1: Voltage components (in dq frame) and their derivatives



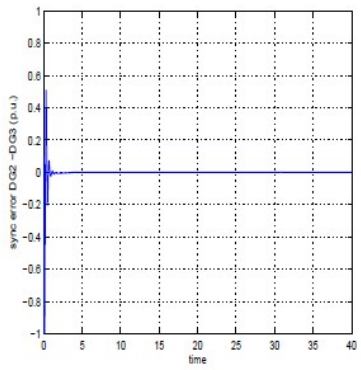
Test 1: Estimation of disturbance inputs

Example 1: Nonlinear control and state estimation using global linearization

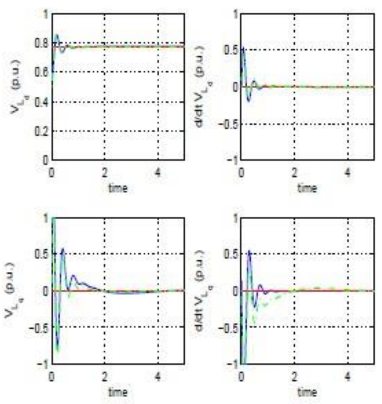
5.8. Simulation tests



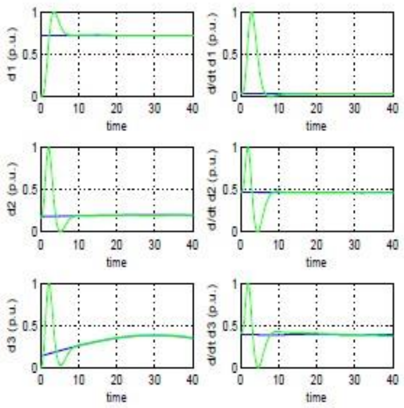
Test 2: (a) Angular speed of power generation unit 2



Test 2: synchronization error between power generation units 2 and 3



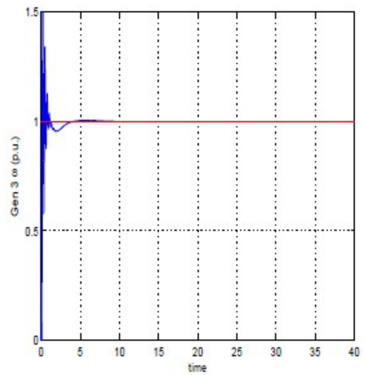
Test 2: Voltage components (in dq frame) and their derivatives



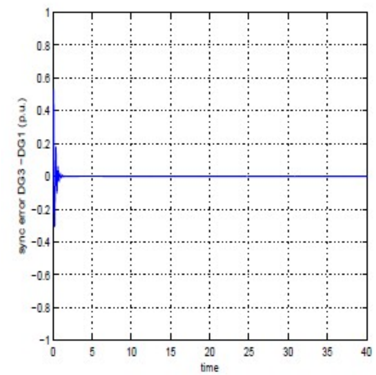
Test 2: Estimation of disturbance inputs

Example 1: Nonlinear control and state estimation using global linearization

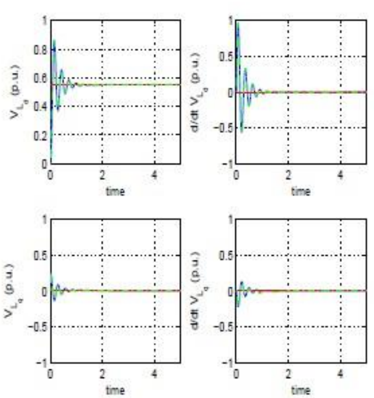
5.8. Simulation tests



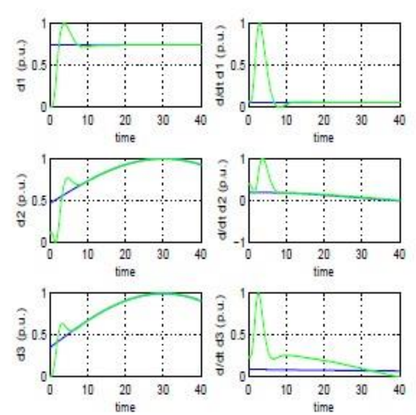
Test 3: (a) Angular speed of power generation unit 3



Test 3: synchronization error between power generation units 3 and 1



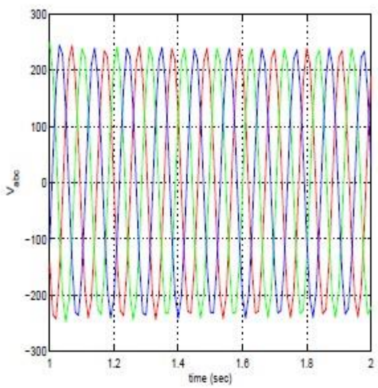
Test 3: Voltage components (in dq frame) and their derivatives



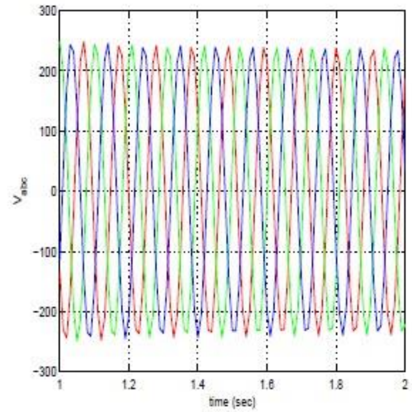
Test 3: Estimation of disturbance inputs

Example 1: Nonlinear control and state estimation using global linearization

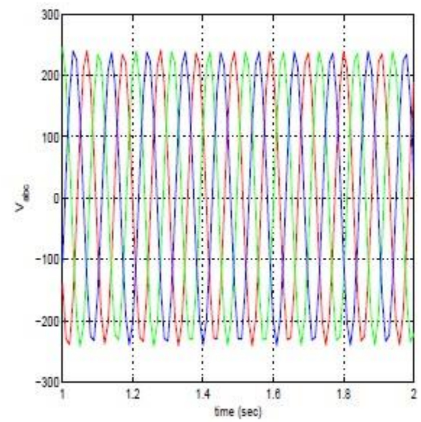
5.8. Simulation tests



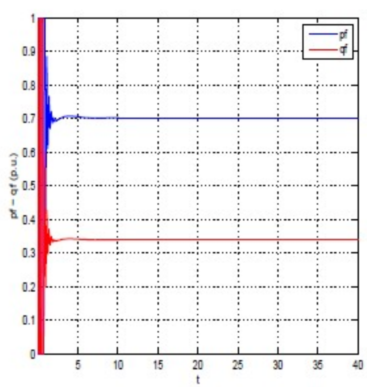
Test 1: Three-phase voltage variables



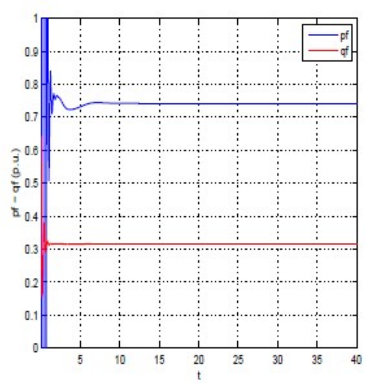
Test 2: Three-phase voltage variables



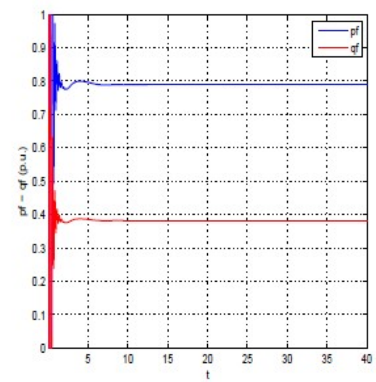
Test 3: Three-phase voltage variables



Test 1: Active and reactive power of the inverter



Test 2: Active and reactive power of the inverter



Test 3: Active and reactive power of the inverter

Example 1: Nonlinear control and state estimation using global linearization

5.8. Simulation tests

The presented simulation experiments demonstrated the **efficiency of the control method** in tracking rapidly changing reference setpoints while also achieving good transients. The associated results are outlined in Table II

Table II
RMSE for the distributed inverters

	$RMSE_1$	$RMSE_2$	$RMSE_3$
ω	0.0225	0.0427	0.0199
V_{L_d}	0.0180	0.0008	0.0003
V_{L_q}	0.0246	0.0020	0.0010

The disturbances appearing in the simulation experiments could be met in **adverse operating conditions** of the distributed power generation system.

Even for the latter case the good performance of the control loop is confirmed.

Such **disturbances can be due to modelling errors** (e.g. parametric changes in the inverters' model) or **due to external perturbations** (e.g. grid faults or disturbances due to the connection or disconnection from the grid of power generation units).

Example 1: Nonlinear control and state estimation using global linearization

5.9. Conclusions

• The **inverter's model satisfies differential flatness properties**, which allows to transform the inverter's model to the **linear canonical form**.

• Next, the problem of **control and synchronization of parallel inverters** connected to the grid was analysed. It has been shown that, the **dynamics of each inverter** can be written in a form that is equivalent to the **model of the synchronous power generator.**

• Using the latter description one can compute **the active power** that each inverter should be contributing so as **to remain synchronized** with the reference frequency of the grid.

• The **active power** and the **frequency associated with the inverter** were used next to **compute the control input** that is applied to the inverter's electrical model.

• Thus, finally the **synchronization problem of each local inverter** was turned into a problem of **nonlinear feedback control** for the associated inverter's electrical model.

• To compensate for **additive disturbance terms** that affect the local inverters' models, the **Derivative-free nonlinear Kalman Filter** was redesigned as a **disturbance observer**.

• The performance of the proposed distributed feedback control scheme for parallel inverters was tested through simulation experiments

Example 2: Nonlinear control and state estimation using approximate linearization

6.1. Outline

- A **nonlinear optimal (H-infinity) control** method is proposed for the model of a hybrid excited synchronous generator.

- The generator has **primary excitation at stator through AC/DC and DC/AC** converter, and **auxiliary excitation at a secondary winding** fed by an AC to DC converter.

- Through the **hybrid excitation scheme** more control inputs are applied to the generator, thus **achieving better performance** for the system's control loop.

- The **dynamic model of the generator** undergoes **approximate linearization** around a **temporary operating point** which is recomputed at each time-step of the algorithm.

- The **linearization procedure** relies on **Taylor series expansion** and on the computation of the associated **Jacobian matrices.**

- For the approximately linearized model of the hybrid excited synchronous generator a **stabilizing H-infinity feedback controller** is designed.

- To compute the **controller's feedback gains** an **algebraic Riccati equation** is **repetitively solved** at each iteration of the control method.

- The **global stability properties** of the control scheme are proven through **Lyapunov stability analysis**

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

The **hybrid excited synchronous machine** receives **double excitation through** (i) **an AC/DC and DC/AC converter** which is **connected to the stator's windings**, (ii) **an AC/DC converter** which is **connected to an auxiliary winding of the stator.**

The **diagram** of the **hybrid excited synchronous generator** is given next

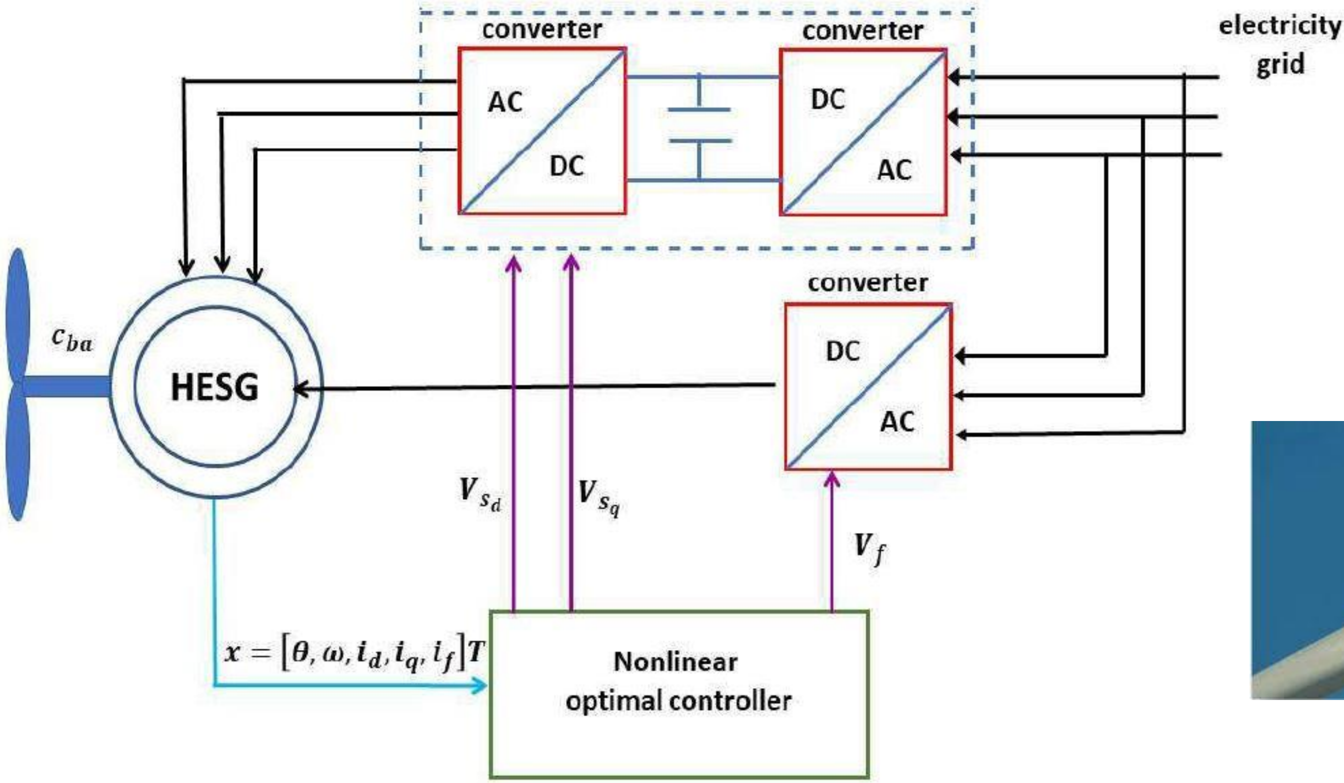


Fig. 1: Diagram of the hybrid excited synchronous generator

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

- **Hybrid excited synchronous machines** differ from typical synchronous machines because they **receive double excitation at their stator**.

- Moreover, their **rotor does not have any windings** and can be also **free of permanent magnets**, as in the case of **reluctance machines**.

- By providing **double excitation at the stator** one can **control more efficiently such machines**, and particularly their magnetic flux

- The **dynamic model of the hybrid excited synchronous machines** remains a **nonlinear and multivariable** one, therefore their control is a non-trivial problem.

- Hybrid excited synchronous machines **can be used as generators** thus making them suitable for **renewable energy applications**.

- Besides, they can be used as **electric motors in the traction systems of trains and electric vehicles**.

- Other uses of hybrid excited synchronous generators are in **aircraft power supply and in wind power systems**.

- Due to the **complexity of the dynamic model of such machines**, stability proofs are often left as open research problems

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

The **state-space model of the Hybrid Excited Synchonous Generator** HESG is given in the following set of equations

$$\dot{\theta} = \omega \quad (1)$$

$$\frac{d\omega}{dt} = -\frac{b}{J}\omega + \frac{P_n}{J}(L_d - L_q)i_d i_q + \frac{P_n \Psi_a}{J} i_q + \frac{P_n M_f}{J} i_q i_f - \frac{1}{J} T_L \quad (2)$$

$$\frac{di_d}{dt} = -L_f R K i_d + L_f L_q K P_n \omega i_q + M_f R_f K i_f + L_f K v_{s_d} - M_f K v_f \quad (3)$$

$$\frac{di_q}{dt} = -\frac{L_d}{L_q} P_n \omega i_d - \frac{R}{L_q} i_q - \frac{M_f}{L_q} P_n \omega i_f - \frac{\Psi_a}{L_q} P_n \omega + \frac{1}{L_q} v_{s_q} \quad (4)$$

$$\frac{di_f}{dt} = M_f R K i_d - M_f L_q K P_n \omega i_q - L_d R_f K i_f - M_f K v_{s_d} + L_d K v_f \quad (5)$$

To describe the **electric dynamics of the machine**, the **synchronously rotating dq reference** frame is used

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

The **parameters and state variables** of the **HESG model** are defined as follows:

$\mathbf{v_{sd}}$ is **the d-axis component of the stator voltage**

$\mathbf{v_{sq}}$ is the **q-axis component of the stator voltage**

$\mathbf{v_f}$ is the **excitation (field) voltage** at an auxiliary winding of the stator

$\mathbf{i_d}$ is the **d-axis component of the stator current**

$\mathbf{i_q}$ is the **q-axis component of the stator current**

$\mathbf{i_f}$ is the **current at the auxiliary excitation circuit** of the stator

R is the per-phase **resistance of the stator**

$\mathbf{R_f}$ is the **resistance of the auxiliary excitation winding**

$\mathbf{L_d}$ is the **d-axis component of the stator inductance**

$\mathbf{L_q}$ is the **q-axis component of the stator inductance**

$\mathbf{L_f}$ is the **inductance of the auxiliary excitation winding**.

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

Additional parameters of the HESG model are:

M_f is the **mutual inductance between** the **auxiliary excitation winding** and the **d-axis winding of the stator**,

Ψ_n is the **flux linkage** of the **rotor's permanent magnet**,

P_n is the **number of poles** of the machine,

ω is the **rotor's turn speed**, **J is the rotor's moment of inertia**,

b is the **friction coefficient** in the turn motion of the rotor

T_L is the **mechanical torque** exerted on the machine due to wind.

Moreover, coefficient K is defined as $K = 1 / (L_d L_f - M_f^2)$ $K = 1/(L_d L_f - M_f^2)$

The **state vector of the hybrid excited synchronous** machine is defined as **$x = [\theta, \omega, i_d, i_q, i_f]^T$** ,

while the **control inputs vector** of the machine is given by **$v = [c_{b\alpha}, v_d, v_q, v_f]^T$** ,

c_{ba} is an additional **control input** that depends on the **pitch angle of the turbine's blades** of the generator.

Example 2: Nonlinear control and state estimation using approximate linearization

2. Dynamic model of the hybrid excited synchronous generator

About the **mechanical torque** that is **applied to the rotor of the generator** and which is **due to wind** one has

$$T_L = \frac{1}{2}\rho\pi R^3 c_{ba}(\lambda,\beta)v^2 \Rightarrow T_L = c_{ba}(\lambda,\beta)T_m \quad (6)$$

where v is the speed of the wind, β is the blades pitch angle

λ is the tip-speed-v ratio is $\lambda = \omega_t R/v$, ω_t is the turbine's turn speed.

Using the previous notation about the **state variables** and the **control inputs** of the hybrid excited synchronous generator one has the following **state-space description**

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{b}{J}x_2 + \frac{P_n}{J}(L_d - L_q)x_3x_4 + \frac{P_n\Psi_a}{J}x_4 + \frac{P_nM_f}{J}x_4x_5 - \frac{T_m}{J}u_1 \quad (7)$$

$$\dot{x}_3 = -L_fRKx_3 + L_fL_qKP_nx_2x_4 + M_fR_fKx_5 + L_fKu_2 - M_fKu_4$$

$$\dot{x}_4 = -\frac{L_d}{L_q}P_nx_2x_3 - \frac{R}{L_q}x_4 - \frac{M_f}{L_q}P_nx_2x_5 - \frac{\Psi_a}{L_q}P_nx_2 + \frac{1}{L_q}u_4$$

$$\dot{x}_5 = M_fRKx_3 - M_fL_qKP_nx_2x_4 - L_dR_fKx_5 - M_fKu_3 + L_dKu_4$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

As a result of the above one arrives at the following, **affine-in-the-input, state-space description** for the **dynamic model** of the **hybrid excited synchronous generator**

$$\dot{x} = f(x) + g(x)u \qquad (8)$$

with $x \in R^{5\times 1}$, $f(x) \in R^{5\times 1}$, $g(x) \in R^{5\times 4}$ and $u \in R^{4\times 1}$

In particular **vector f(x) and matrix g(x)** are given by

$$f(x) = \begin{pmatrix} x_2 \\ -\frac{b}{J}x_2 + \frac{P_n}{J}(L_d - L_q)x_3x_4 + \frac{P_n\Psi_a}{J}x_4 + \frac{P_nM_f}{J}x_4x_5 \\ -L_fRKx_3 + L_fL_qKP_nx_2x_4 + M_fR_fKx_5 \\ -\frac{L_d}{L_q}P_nx_2x_3 - \frac{R}{L_q}x_4 - \frac{M_f}{L_q}P_nx_2x_5 - \frac{\Psi_a}{L_q}P_nx_2 \\ M_fRKx_3 - M_fL_qKP_nx_2x_4 - L_dR_fKx_5 \end{pmatrix} \qquad (9)$$

$$g(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{T_m}{J} & 0 & 0 & 0 \\ 0 & L_fK & 0 & -M_fK \\ 0 & 0 & \frac{1}{L_q} & 0 \\ 0 & 0 & -M_fK & L_dK \end{pmatrix} \qquad (10)$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.2. Dynamic model of the hybrid excited synchronous generator

The **definition of abbreviated parameters' names** in the dynamic model and the control loop of the **hybrid excited synchronous generator** is outlined in the following **Table**:

Table I: Definition of parameters

HESG	Hybrid Excited Synchronous Generator
v_{s_d}	d-axis component of the stator voltage
v_{s_q}	q-axis component of the stator voltage
v_f	excitation (field) voltage at an auxiliary winding of the stator
i_d	d-axis component of the stator current
i_q	q-axis component of the stator current
i_f	current at the auxiliary excitatory circuit of the current
R	per-phase resistance of the stator
R_f	resistance of the auxiliary excitation winding
L_d	d-axis component of the stator inductance
L_q	q-axis component of the stator inductance
L_f	inductance of the auxiliary excitation winding
M_f	mutual inductance between excitation and d-axis winding of stator
Ψ_a	flux linkage of the rotor's permanent magnet
P_n	number of poles of the machine
ω	rotor's turn speed
J	rotor's moment of inertia
b	friction coefficient of the rotor
T_L	mechanical torque exerted on the machine due to wind

Example 2: Nonlinear control and state estimation using approximate linearization

6.3. Approximate linearization of the Hybrid Excited Synchronous Generator

The **state-space model of the generator** undergoes **approximate linearization** around the **temporary operating point** (equilibrium) (x^*, u^*), where

x^* is the **present value of the system's state vector** and
u^* is **the last sampled value of the control inputs vector**

For the **linearized state-space model** of the svstem it holds that

$$\dot{x} = Ax + bu + \tilde{d} \qquad (11)$$

where $\tilde{d}$ is **the cumulative disturbance vector** due to approximate linearization and **truncation of higher-order terms in the Taylor series expansion**, and

$$A = \nabla_x[f(x) + g(x)u] \mid_{(x^*,u^*)} \Rightarrow A = \nabla_x[f(x)] \mid_{(x^*,u^*)} \qquad (12)$$

$$B = \nabla_u[f(x) + g(x)u] \mid_{(x^*,u^*)} \Rightarrow B = g(x) \mid_{(x^*,u^*)} \qquad (13)$$

About the **Jacobian matrix** $\nabla_x[f(x)] \mid_{(x^*,u^*)}$ one has

First row of the **Jacobian matrix** $\nabla_x[f(x)] \mid_{(x^*,u^*)}$

$$\frac{\partial f_1}{\partial x_1} = 0, \ \frac{\partial f_1}{\partial x_2} = 1, \ \frac{\partial f_1}{\partial x_3} = 0, \ \frac{\partial f_1}{\partial x_4} = 0, \text{ and } \frac{\partial f_1}{\partial x_5} = 0.$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.3. Approximate linearization of the Hybrid Excited Synchronous Generator

The **state-space model of the generator** undergoes **approximate linearization** around the **temporary operating point** (equilibrium) (x*, u*), where

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$$A = \nabla_x[f(x) + g(x)u] \mid_{(x^*,u^*)} \Rightarrow A = \nabla_x[f(x)] \mid_{(x^*,u^*)} \qquad (12)$$

$$B = \nabla_u[f(x) + g(x)u] \mid_{(x^*,u^*)} \Rightarrow B = g(x) \mid_{(x^*,u^*)} \qquad (13)$$

About the **Jacobian matrix** $\nabla_x[f(x)] \mid_{(x^*,u^*)}$ one has

First row of the **Jacobian matrix** $\nabla_x[f(x)] \mid_{(x^*,u^*)}$

$$\frac{\partial f_1}{\partial x_1} = 0, \; \frac{\partial f_1}{\partial x_2} = 1, \; \frac{\partial f_1}{\partial x_3} = 0, \; \frac{\partial f_1}{\partial x_4} = 0, \text{ and } \frac{\partial f_1}{\partial x_5} = 0.$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.3. Approximate linearization of the Hybrid Excited Synchronous Generator

Second row of the Jacobian matrix $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_2}{\partial x_1}=0,\ \frac{\partial f_2}{\partial x_2}=-\frac{b}{J},\ \frac{\partial f_2}{\partial x_3}=\frac{P_n}{J}(L_d-L_q)x_4,$$

$$\frac{\partial f_2}{\partial x_4}=\frac{P_n}{J}(L_d-L_q)x_3+\frac{P_n\Psi_a}{J}+\frac{P_nM_f}{J}x_5,\ \text{and}\ \frac{\partial f_2}{\partial x_5}=\frac{P_nM_f}{J}x_4$$

Third row of the Jacobian matrix $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_3}{\partial x_1}=0,\ \frac{\partial f_3}{\partial x_2}=L_fL_qKP_nx_4,\ \frac{\partial f_3}{\partial x_3}=-L_fRK,$$

$$\frac{\partial f_3}{\partial x_4}=L_fL_qKP_mx_2,\ \frac{\partial f_3}{\partial x_5}=-M_fR_fK$$

Fourth row of the Jacobian matrix $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_4}{\partial x_1}=0,\ \frac{\partial f_4}{\partial x_2}=-\frac{L_d}{L_q}P_nx_3-\frac{M_f}{L_q}P_nx_5-\frac{\Psi_n}{L_q}L_q,$$

$$\frac{\partial f_4}{\partial x_4}=-\frac{R}{L_q}\quad \text{and}\ \frac{\partial f_4}{\partial x_5}=-\frac{M_f}{L_q}P_nx_2.$$

Fifth row of the Jacobian matrix $\nabla_x[f(x)]\,|_{(x^*,u^*)}$

$$\frac{\partial f_5}{\partial x_1}=0,\ \frac{\partial f_5}{\partial x_2}=-M_fL_qKP_nx_4,\ \frac{\partial f_5}{\partial x_3}=M_fRK,$$

$$\frac{\partial f_5}{\partial x_4}=-M_fL_qKP_nx_2,\ \text{and}\ \frac{\partial f_5}{\partial x_5}=-L_dR_fK$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.4. Design of the H-infinity feedback controller

The **state vector notation** *x* is used for the model of Eq. 14

At every time instant the control input u^* is assumed to differ from the control input u appearing above by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \quad (15)$$

The **dynamics of the system** of Eq. 11 can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad (16)$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad (17)$$

By subtracting Eq. 15 from Eq. 16 one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad (18)$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$ the **tracking error dynamics** becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad (19)$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.4. Design of the H-infinity feedback controller

The initial model of the Hybrid Excited Synchronous Generator is assumed to be in the form

$$\dot{x} = f(x,u) \quad x \in R^n, \; u \in R^m$$

Linearization of the system is performed at each iteration of the control algorithm round its present operating point

$$(x^*, u^*) = (x(t), u(t - T_s)).$$

The **linearized equivalent of the system** is described by

$$\dot{x} = Ax + Bu + Ld \quad x \in R^n, \; u \in R^m, \; d \in R^q$$

where matrices A and B are obtained from the **computation of the Jacobians**

20

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Bigg|_{(x^*, u^*)} \qquad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Bigg|_{(x^*, u^*)}$$

and vector d denotes disturbance terms due to linearization errors.

The problem of **disturbance rejection** for the linearized model that is described by

$$\dot{x} = Ax + Bu + Ld$$
$$y = Cx$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.4. Design of the H-infinity feedback controller

where $x \in R^n$, $u \in R^m$, $d \in R^q$ and $y \in R^p$,cannot be handled efficiently if the classical LQR control scheme is applied. This because of the existence of the perturbation term d.

In the $H\infty$ control approach, a **feedback control scheme** is designed for **trajectory tracking** by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner

The disturbances' effect are incorporated in the following **quadratic cost function**

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2\tilde{d}^T(t)\tilde{d}(t)]dt, \quad r,\rho > 0 \tag{21}$$

The coefficient r determines **the penalization of the control input** and the weight coefficient ρ determines the **reward of the disturbances**' effects. It is assumed that

Then, the **optimal feedback control** law is $u(t) = -Kx(t)$ vith $K = \frac{1}{r}B^TP$

where P **is a positive semi-definite symmetric matrix** which is obtained from the solution of the **Riccati equation**

$$A^TP + PA + Q - P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P = 0 \tag{22}$$

where Q is also a positive definite symmetric matrix.

The parameter ρ in Eq. (15), is an **indication of the closed-loop system robustness**. If the values of ρ> 0 are excessively decreased with respect to r, then the solution of the **Riccati equation** is no longer a **positive definite matrix**. Consequently, there is a lower bound ρ_{min} of for which the H-infinity control problem has a solution.

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

The **tracking error dynamics** for the **Hybrid Excited Synchronous Generator** is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \qquad (23)$$

where in the **Hybrid Excited Synchronous Generator** $\boldsymbol{L} = \boldsymbol{I} \in \boldsymbol{I}^{5x5}$ with I being the identity matrix. The following **Lyapunov function** is considered

$$V = \frac{1}{2}e^T P e \qquad (24)$$

$$e = x - x_d$$

$$\dot{V} = \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow$$

$$\dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T] P e + \\ + \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}e^T A^T P e + \frac{1}{2}u^T B^T P e + \frac{1}{2}\tilde{d}^T L^T P e + \\ \frac{1}{2}e^T P A e + \frac{1}{2}e^T P B u + \frac{1}{2}e^T P L \tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(A^TP + PA)e + (\frac{1}{2}u^TB^TPe + \frac{1}{2}e^TPBu) + \\ +(\frac{1}{2}\tilde{d}^TL^TPe + \frac{1}{2}e^TPL\tilde{d})$$

Assumption: For given positive definite matrix Q *and coefficients* r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \quad (25)$$

Moreover, the following **feedback control law** is applied to the **Synchronous Reluctance Machine**

$$u = -\frac{1}{r}B^TPe \quad (26)$$

By substituting Eq. (25) and Eq. (26) one obtains

$$\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P]e + \\ +e^TPB(-\frac{1}{r}B^TPe + e^TPL\tilde{d} \Rightarrow$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

Continuing with computations one obtains

$$\dot{V} = -\frac{1}{2}e^TQe + (\frac{1}{r}PBB^TPe - \frac{1}{2\rho^2}e^TPLL^T)Pe$$
$$-\frac{1}{r}e^TPBB^TPe + e^TPL\tilde{d}$$

which next gives

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe + e^TPL\tilde{d}$$

or equivalently

$$\dot{V} = -\frac{1}{2}e^TQe - \frac{1}{2\rho^2}e^TPLL^TPe +$$
$$+\frac{1}{2}e^TPL\tilde{d} + \frac{1}{2}\tilde{d}^TL^TPe \quad (27)$$

Lemma: The following inequality holds

$$\frac{1}{2}e^TL\tilde{d} + \frac{1}{2}\tilde{d}L^TPe - \frac{1}{2\rho^2}e^TPLL^TPe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d}$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

Proof : *The binomial* $(\rho a - \frac{1}{\rho} b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T P L$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2} e^T P L \tilde{d} - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (28)$$

Eq. 28 is substituted in Eq. 27 and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2} e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \quad (29)$$

Eq. 30 shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T ||\tilde{d}||^2 dt \Rightarrow$$
$$2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||\tilde{d}||^2 dt$$

Example 2: Nonlinear control and state estimation using approximate linearization

6.5. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d$$

then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded.

Moreover, V(T) is bounded and from the definition of the Lyapunov function V it becomes clear that **e(t) will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}.$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$lim_{t \to \infty} e(t) = 0.$$

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Example 2: Nonlinear control and state estimation using approximate linearization

6.6. Robust state estimation with the use of the H-infinity Kalman Filter

- The control loop has to be implemented with the use of information provided by a **small number of sensors** and by processing only a small number of state variables.

- Actually, one can implement feedback control by **measuring only the stator currents**. To reconstruct the missing information about the state vector of the Hybrid Excited Synchronous Generator one cam use **use a filter** and based on it to apply state **estimation-based control** .

- The **recursion of the H-infinity Kalman Filter**, for the **Hybrid Excited Synchronous Generator**, can be formulated in terms of a measurement update and a time update part

Measurement update

$$D(k) = [I - \theta W(k)P^-(k) + C^T(k)R(k)^{-1}C(k)P^-(k)]^{-1}$$
$$K(k) = P^-(k)D(k)C^T(k)R(k)^{-1}$$
$$\hat{x}(k) = \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)] \quad (32)$$

Time update

$$\hat{x}^-(k+1) = A(k)x(k) + B(k)u(k)$$
$$P^-(k+1) = A(k)P^-(k)D(k)A^T(k) + Q(k) \quad (33)$$

where it is assumed that parameter θ is sufficiently small to assure that the **covariance matrix**

$$P^-(k) - \theta W(k) + C^T(k)R(k)^{-1}C(k)$$

Is **positive definite**

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

- The performance of the proposed nonlinear **H-infinity control scheme** for the **system of the Hybrid Excited Synchronous Generator** is tested through simulation:

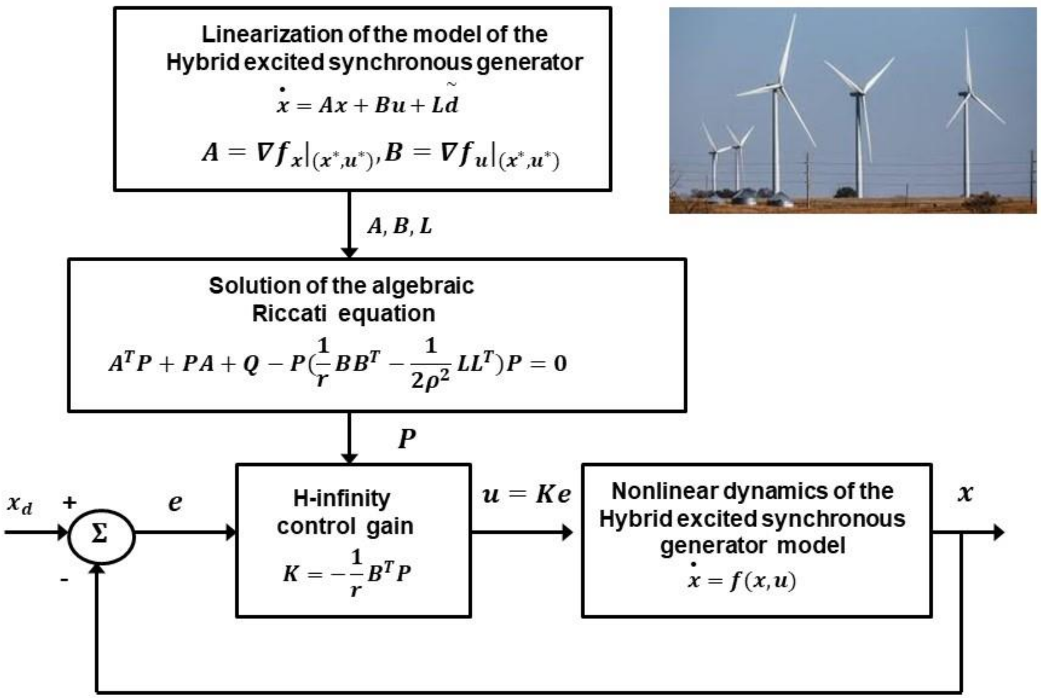


Fig.2 Diagram of the nonlinear optimal control for the Hybrid Excited Synchronous Generator

With the use of the H-infinity control method, **fast and accurate tracking of the reference setpoints** of the state variables of the Hybrid Excited Synchronous Generator was achieved

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

Test 1 for the hybrid excited synchronous generator:

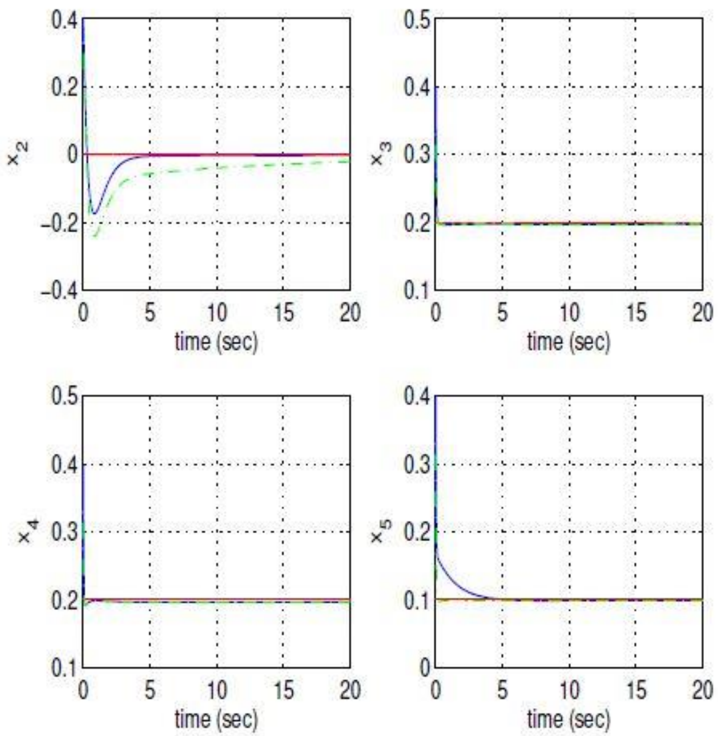


Fig3(a) Convergence of the state variables x2 to x5 (blue lines) to the reference setpoints (red lines) and state estimates provided by the Kalman Filter (green lines)

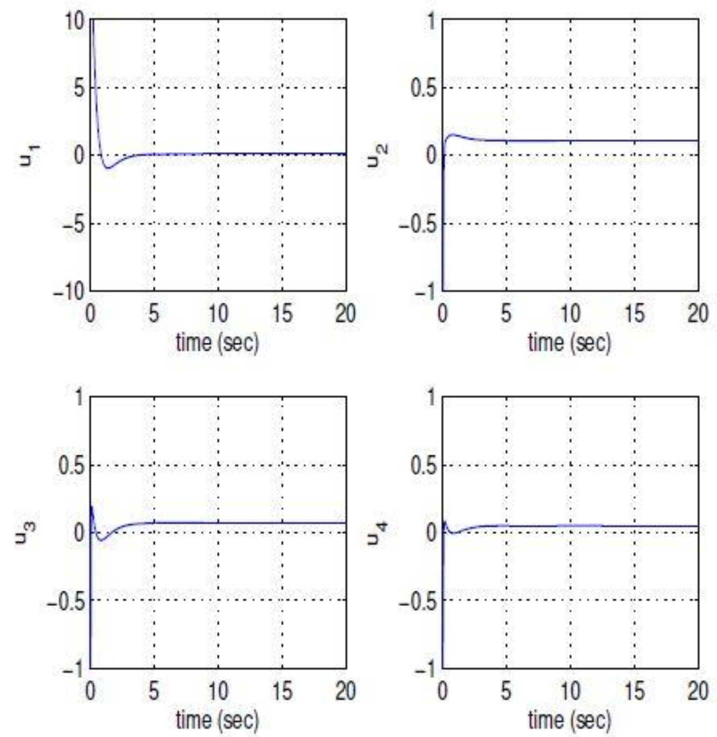


Fig 3(b) Control inputs ui, i = 1, …, 4 applied to the hybrid excited synchronous generator

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

Test 2 for the hybrid excited synchronous generator:

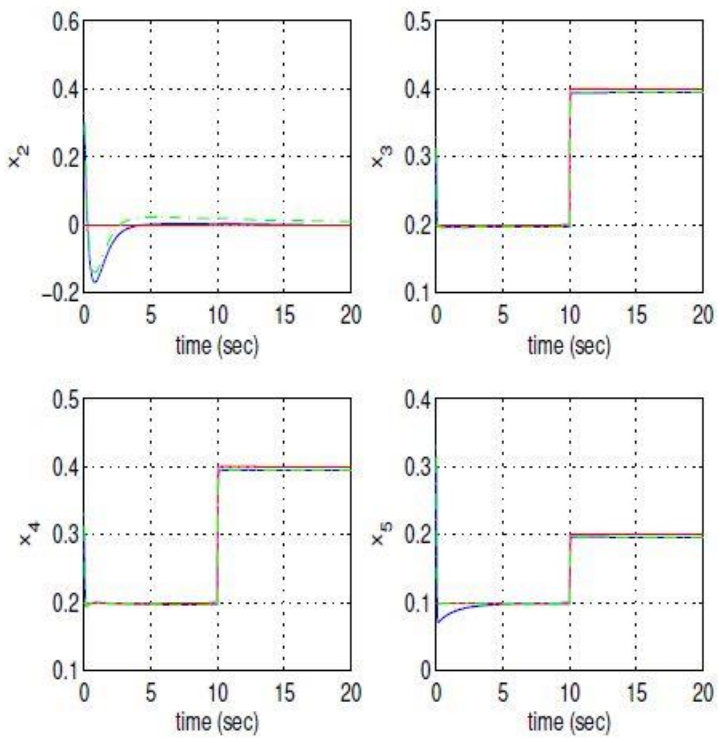


Fig4(a) Convergence of the state variables x2 to x5 (blue lines) to the reference setpoints (red lines) and state estimates provided by the Kalman Filter (green lines)

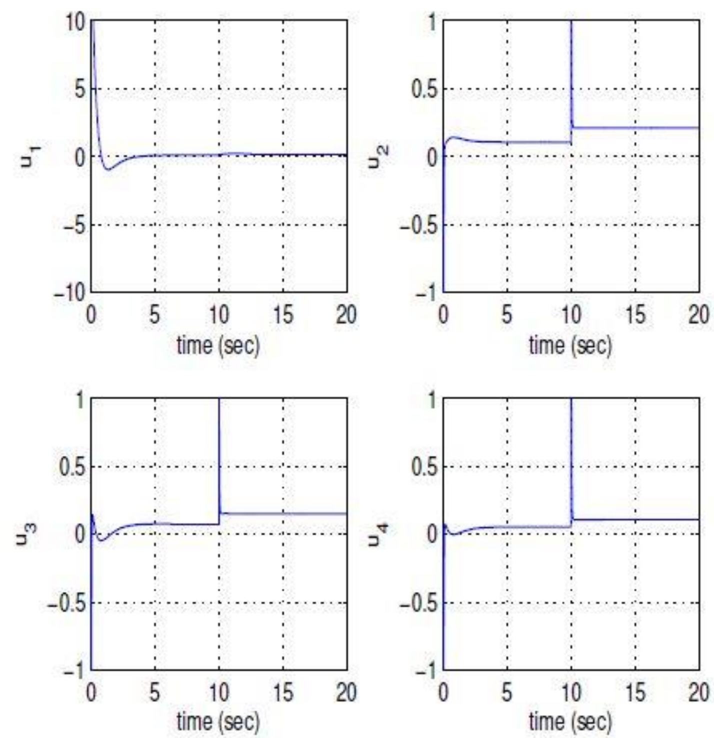
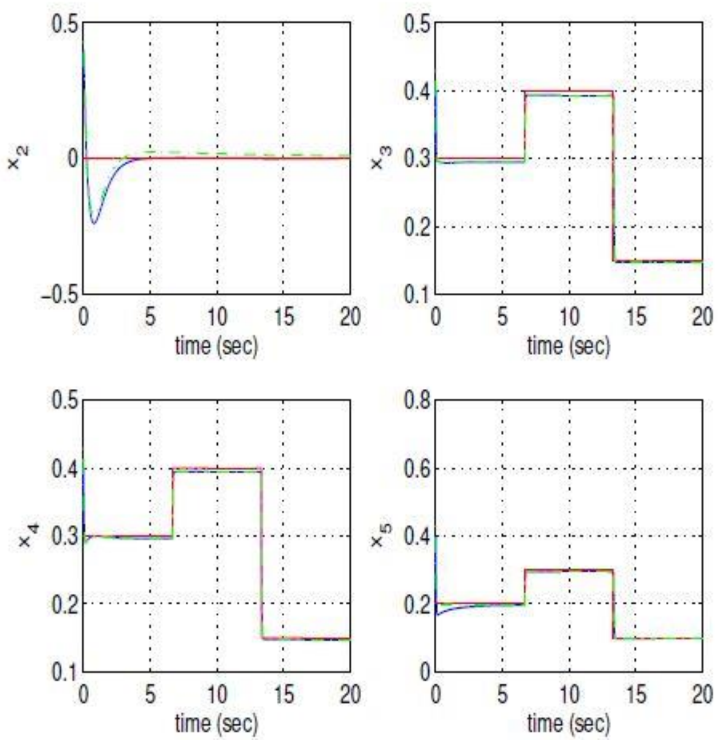


Fig 4(b) Control inputs ui, i = 1, …, 4 applied to the hybrid excited synchronous generator

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

Test 3 for the hybrid excited synchronous generator:



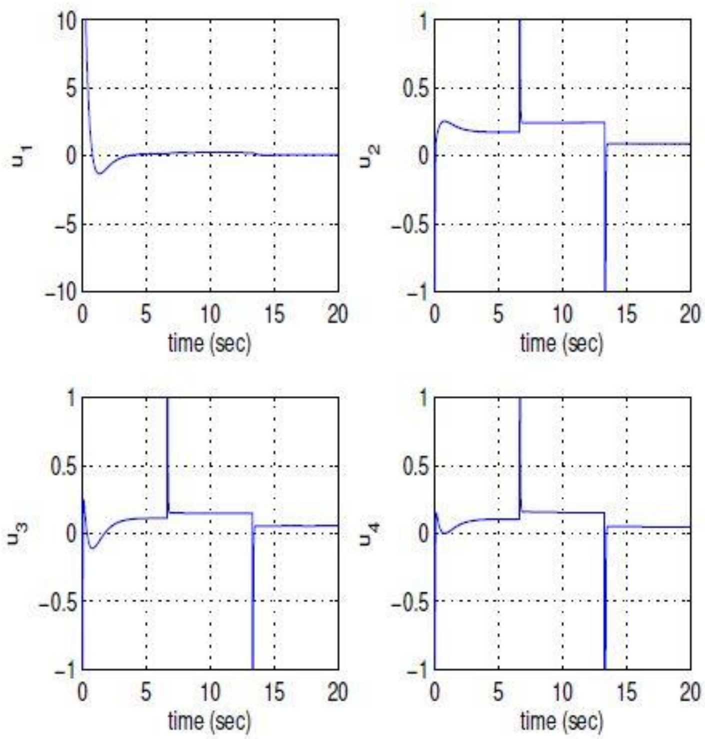


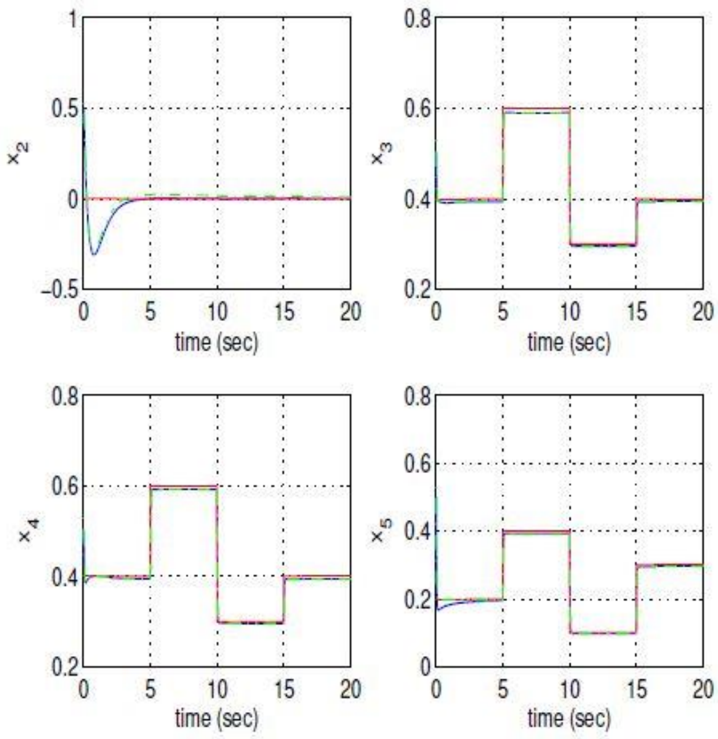
Fig5(a) Convergence of the state variables x2 to x5 (blue lines) to the reference setpoints (red lines) and state estimates provided by the Kalman Filter (green lines)

Fig 5(b) Control inputs u_i, i = 1, ..., 4 applied to the hybrid excited synchronous generator

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

Test 4 for the hybrid excited synchronous generator:



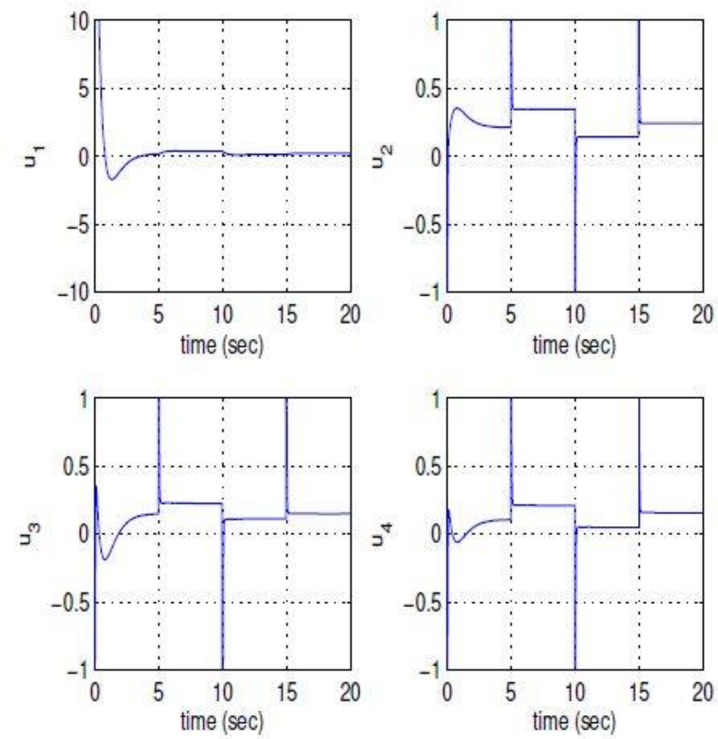


Fig6(a) Convergence of the state variables x2 to x5 (blue lines) to the reference setpoints (red lines) and state estimates provided by the Kalman Filter (green lines)

Fig 6(b) Control inputs ui, i = 1, …, 4 applied to the hybrid excited synchronous generator

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

Test 5 for the hybrid excited synchronous generator:

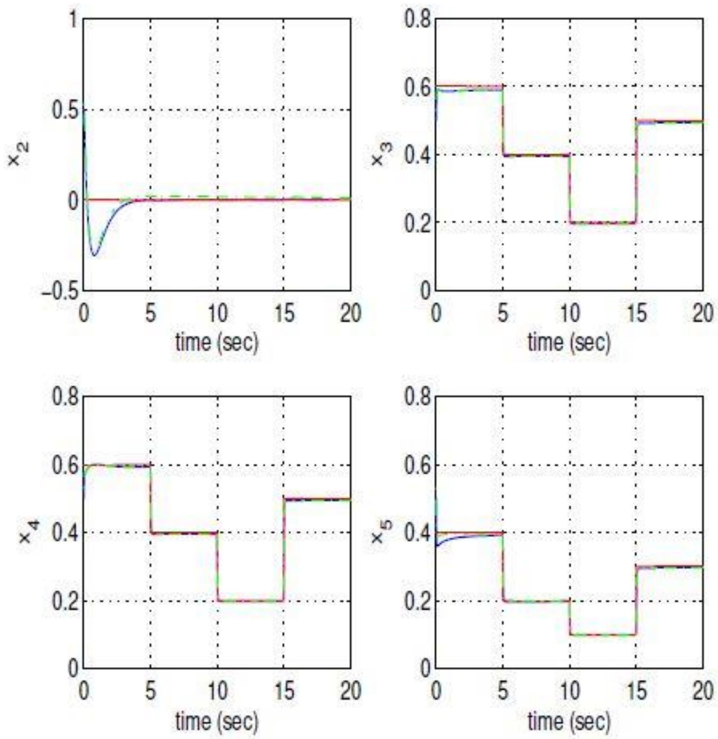


Fig7(a) Convergence of the state variables x2 to x5 (blue lines) to the reference setpoints (red lines) and state estimates provided by the Kalman Filter (green lines)

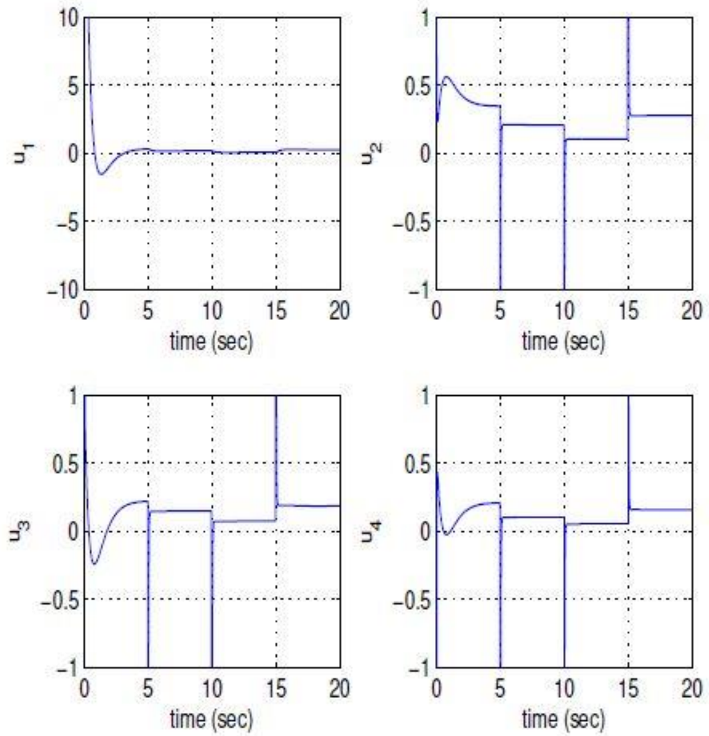


Fig 7(b) Control inputs ui, i = 1, …, 4 applied to the hybrid excited synchronous generator

Example 2: Nonlinear control and state estimation using approximate linearization

6.7. Simulation tests

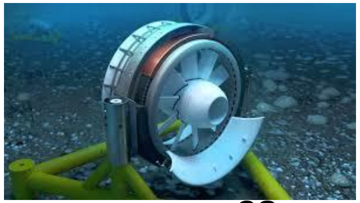
• Because of the **nonlinearity of the state-space model** of the HESG other approaches to solve the associated optimal control problem, such as the typical **model predictive control (MPC)** and the **nonlinear model predictive control, (NMPC)** are of questionable performance.

• Thus, it is widely acknowledged that **MPC is a linear control method** which in the case of the nonlinear dynamics of the hybrid excited synchronous generator **cannot assure the stability of the control loop.**

• Besides, it is known that the **NMPC's iterative search for an optimum** is **dependent on initial parametrization** and is **not always of assured convergence**.

• On the other side the use of **global linearization-based methods** for the control of the considered HESG requires the definition of the **linearizing outputs** in a case-based manner and the application of **complicated change of state-space variables**].

• Moreover, such methods may come against **singularity problems** due to including also additional **transformations being-based on matrices inversions**.



• Finally, **sliding-mode control** cannot be directly applied to the considered model of the hybrid power generator because this is **not found in a canonical linear form** and consequently there is no systematic manner to define a sliding surface

Example 2: Nonlinear control and state estimation using approximate linearization

6.8. Conclusions

- A **nonlinear optimal control approach** has been introduced for **hybrid excited synchronous generators**.

- This type of generator receives **double excitation**, (i) from the **stator's windings** through voltage that is provided by a **AC/DC and DC/AC converter**, and (ii) from an **auxiliary excitation circuit** at the stator that is fed by an **AC to DC converter.**

- The nonlinear dynamic model of the hybrid excited synchronous generator has undergone **approximate linearization** around a **temporary operating point** that was recomputed at **each time-step** of the control algorithm.

- The **linearization procedure** relied on **Taylor series expansion** and through the computation of **Jacobian matrices**. For the approximately linearized model of the generator a **stabilizing H-infinity feedback controller** has been designed.

- For the computation of the **controller's feedback gains** an **algebraic Riccati equation** had to be **repetitively solved** at each iteration of the control algorithm.

- The **global stability and robustness properties** of the control method have been proven through **Lyapunov analysis**.

- To implement **state estimation-based control** without the need to measure its entire state vector, the **H-infinity Kalman Filter** has been used as a **robust state estimator.**

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.1. Outline

● An adaptive control approach is proposed that is capable of compensating for model uncertainty and parametric changes of the distributed synchronous generators, as well as for the lack of measurements about the distributed SG's state vector elements.

● First it is proven that the distributed SG's model is a differentially flat one. By exploiting differential flatness properties it is shown that the distributed SG's model can be transformed into the linear canonical form.

● For the latter description, the new control inputs comprise unknown nonlinear functions which can be identified with the use of neurofuzzy approximators. The estimated dynamics of the machine is used by a feedback controller thus establishing an indirect adaptive control scheme.

● Moreover, to enforce the robustness of the control loop, a supplementary control term is computed using H-infinity control theory.

● Another problem that has to be dealt with comes from partial measurements of the state vector of the generator. Thus, a state observer is implemented in the control loop.

● The stability of the considered observer-based adaptive control approach is proven using Lyapunov analysis. Moreover, the performance of the control scheme is evaluated through simulation experiments.

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the distributed synchronous generatorss

The **dynamic model of the distributed power generation units** is assumed to consist of multiple synchronous generators. The modelling approach is also applicable to PMSGs (permanent magnet synchronous generators) which are a special case of synchronous electric machines.

$$\dot{\delta} = \omega$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \frac{\omega_0}{2J}(P_m - P_e) \qquad (1)$$

δ : turn angle of the rotor

ω : turn speed of the rotor

ω_0 : synchronous speed

J : moment of inertia of the rotor

P_e : active electrical power of the machine

P_m : mechanical power of the machine

D : damping coefficient

T_e : electromagnetic torque

The **generator's electrical dynamics** is:

$$\dot{E}_q' = \frac{1}{T_{d_o}}(E_f - E_q) \qquad (2)$$

E_q' is the quadrature-axis transient voltage (a variable related to the magnetic flux)

E_q is quadrature axis voltage of the generator

T_{d_o} is the direct axis open-circuit transient time constant

E_f is the equivalent voltage in the excitation coil

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the distributed synchronous generators

The **synchronous generator's model** is complemented by a set of algebraic equations:

$$E_q = \frac{x_{d\Sigma}}{x'_{d\Sigma}}E'_q - (x_d - x'_d)\frac{V_s}{x'_{d\Sigma}}cos(\Delta\delta)$$
$$I_q = \frac{V_s}{x'_{d\Sigma}}sin(\Delta\delta)$$
$$I_d = \frac{E'_q}{x'_{d\Sigma}} - \frac{V_s}{x'_{d\Sigma}}cos(\Delta\delta)$$
$$P_e = \frac{V_s E'_q}{x'_{d\Sigma}}sin(\Delta\delta)$$
$$Q_e = \frac{V_s E'_q}{x'_{d\Sigma}}cos(\Delta\delta) - \frac{V_s^2}{x_{d\Sigma}}$$
$$V_t = \sqrt{(E'_q - X'_d I_d)^2 + (X'_d I_q)^2}$$

3

where: $x_{d\Sigma} = x_d + x_T + x_L$ $\quad x'_{d\Sigma} = x'_d + x_T + x_L$

x_d : direct-axis synchronous reactance

x_T : reactance of the transformer

x'_d : direct-axis transient reactance

x_L : transmission line reactance

I_d and I_q : direct and quadrature axis currents

V_s : infinite bus voltage

Q_e : reactive power of the generator

V_t : terminal voltage of the generator

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the synchronous generator

From Eq. 1 and Eq. 2 one obtains the **dynamic model of the synchronous generator**:

$$\dot{\delta} = \omega - \omega_0$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \omega_0\frac{P_m}{2J} - \omega_0\frac{1}{2J}\frac{V_s E_q^{'}}{x_{d_\Sigma}^{'}}sin(\Delta\delta)$$
$$\dot{E}_q^{'} = -\frac{1}{T_d^{'}}E_q^{'} + \frac{1}{T_{d_o}}\frac{x_d - x_d^{'}}{x_{d_\Sigma}^{'}}V_s cos(\Delta\delta) + \frac{1}{T_{d_o}}E_f$$

Moreover, the generator can be written in a **state-space form**:

$$\dot{x} = f(x) + g(x)u$$

where the state vector is $x = (\Delta\delta \quad \Delta\omega \quad E_q^{'})^T$ and

$$f(x) = \begin{pmatrix} \omega - \omega_0 \\ -\frac{D}{2J}(\omega - \omega_0) + \omega_0\frac{P_m}{2J} - \omega_0\frac{1}{2J}\frac{V_s E_q^{'}}{x_{d\Sigma}^{'}}sin(\Delta\delta) \\ -\frac{1}{T_d^{'}}E_q^{'} + \frac{1}{T_{do}}\frac{x_d - x_d^{'}}{x_{d\Sigma}^{'}}V_s cos(\Delta\delta) \end{pmatrix}$$

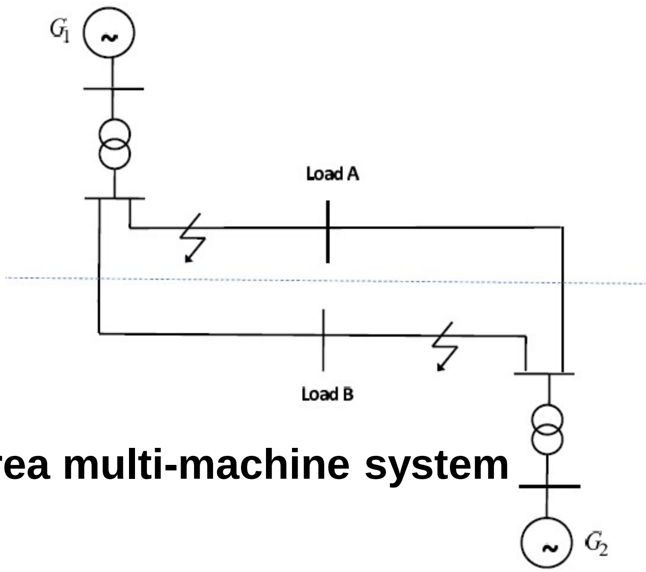
$$g(x) = (0 \quad 0 \quad \frac{1}{T_{do}})^T$$

while the system's output is $y = h(x) = \delta - \delta_0$

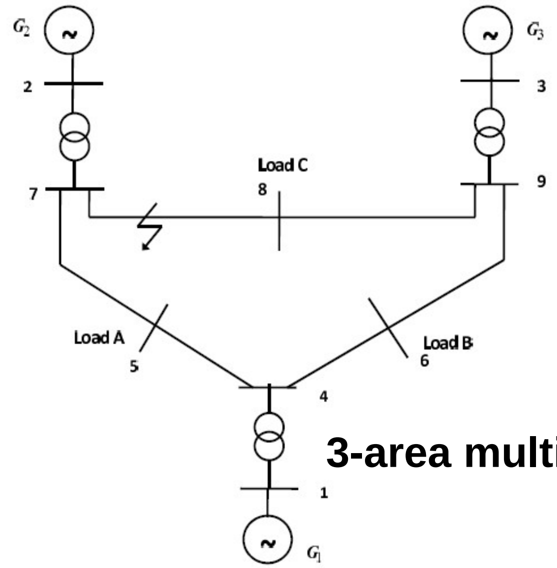
Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the synchronous generator

The **interconnection between distributed power generators** results into a **multi-area multi-machine power system** model



2-area multi-machine system



3-area multi-machine system

The dynamic model of a power system that comprises **n-interconnected power generators** is

$$\dot{\delta}_i = \omega_i - \omega_0$$

$$\dot{\omega}_i = -\frac{D_i}{2J_i}(\omega_i - \omega_0) + \omega_0 \frac{P_{m_i}}{2J_i} -$$

$$-\omega_0 \frac{1}{2J_i}[G_{ii} E_{qi}^{'\,2} + E_{qi}^{'} \sum_{j=1, j \neq i}^{n} E_{qj}^{'} G_{ij} sin(\delta_i - \delta_j - \alpha_{ij})]$$

$$\dot{E}_{q_i}^{'} = -\frac{1}{T_{d_i}^{'}} E_{q_i}^{'} + \frac{1}{T_{do_i}} \frac{x_{d_i} - x_{d_i}^{'}}{x_{d\Sigma i}^{'}} V_{s_i} cos(\Delta\delta_i) + \frac{1}{T_{do_i}} E_{f_i}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the synchronous generator

The **active power** associated with the i-th power generator is given by:

$$P_{e_i} = G_{ii}{E'_{qi}}^2 + E'_{qi}\sum_{j=1,j\neq i}^{n} E'_{qj}G_{ij}sin(\delta_i - \delta_j - \alpha_{ij})$$

The state vector of the distributed power system is given by $x = [x^1, x^2, \cdots, x^n]^T$

where $x^i = [x^i_1, x^i_2, x^i_3]^T$ with $x^i_1 = \Delta\delta_i$ $x^i_2 = \Delta\omega_i$ and $x^i_3 = E'_{qi}$ $i = 1, 2, \cdots, n$

Next, **differential flatness** is proven for the model of the **stand-alone synchronous generator**.

In state-space form one has:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{D}{2J}x_2 + \omega_0\frac{P_m}{2J} - \frac{\omega_0}{2J}\frac{V_s}{x'_{d\Sigma}}x_3 sin(x_1)$$
$$\dot{x}_3 = -\frac{1}{T'_d}x_3 + \frac{1}{T_{do}}\frac{x_d - x'_d}{x'_{d\Sigma}}V_s cos(x_1) + \frac{1}{T_{do}}u$$

The **flat output** is taken to be $y = x_1$

It holds that $x_1 = y$ $x_2 = \dot{y}$ and for $x_1 \neq \pm n\pi$,

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.2. Dynamic model of the synchronous generator

while for the **generator's control input** one has

$$u = T_{do}[\dot{x}_3 + \frac{1}{T_d'}x_3\frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(x_1)], \ \text{or}$$
$$u = f_b(y, \dot{y}, \ddot{y})$$

Consequently, **all state variables** and the **control input** of the synchronous generator are written as **differential functions** of the flat output and thus the differential flatness of the model is confirmed.

By defining the **new state variables** $y_1 = y,\ y_2 = \dot{y},\ y_3 = \ddot{y}$

the generator's model is transformed into the **canonical (Brunovsky) form**:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} v$$

with $v = f_c(y, \dot{y}, \ddot{y}) + g_c(y, \dot{y}, \ddot{y})u$ where

$$f_c(y, \dot{y}, \ddot{y}) = (\frac{D}{2J})^2\dot{y} - \omega_0\frac{D}{2J}\frac{P_m}{2J} + \omega_0\frac{D}{(2J)^2}\frac{V_s}{x_{d\Sigma}'}x_3 sin(\dot{y}) +$$
$$+\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}\frac{1}{T_d'}x_3 sin(y) - \frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}\frac{1}{T_{do}}\frac{x_d - x_d'}{x_{d\Sigma}'}V_s cos(y) sin(y) -$$
$$-\frac{\omega_0}{2J}\frac{V_s}{x_{d\Sigma}'}x_3 cos(y)\dot{y}$$

and

$$g_c(y, \dot{y}, \ddot{y}) = -\frac{\omega_0}{2J}\frac{1}{T_{do}}\frac{V_s}{x_{d\Sigma}'}sin(y)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Dynamic model of the distributed synchronous generators

Differential flatness can be also proven for the **model of the n-interconnected power generators**

The **flat output** is taken to be the vector of the turn angles of the n-power generators

$$y = [y_1^1, y_1^2, \cdots, y_1^n] \text{ or } y = \Delta\delta^1, \Delta\delta^2, \cdots, \Delta\delta^n$$

For the n-machines power generation system it holds

$$x_1^1 = y^1,\ x_1^2 = y^2,\ x_1^3 = y^3,\ \cdots,\ x_1^n = y^n$$

$$x_2^1 = \Delta\omega^1 = \dot{y}^1,\ x_2^2 = \Delta\omega^2 = \dot{y}^2,\ x_2^3 = \Delta\omega^3 = \dot{y}^3,\ \cdots,\ x_2^n = \Delta\omega^n = \dot{y}^n$$

Moreover, it holds

$$\dot{x}_2^i = -\frac{D_i}{2J_i}x_2^i + \frac{\omega_0}{2J_i}P_{mi} - \\ -\frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2 + x_3^i\sum_{j=1,j\neq i}^n [x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] \qquad 4$$

or using the flat outputs notation

$$\ddot{y}^i = -\frac{D_i}{2J_i}\dot{y}^i + \frac{\omega_0}{2J_i}P_{mi} - \\ -\frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2 + x_3^i\sum_{j=1,j\neq i}^n [x_3^j G_{ij} sin(y^i - y^j - \alpha_{ij})]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Dynamic model of the distributed synchronous generators

The **external mechanical torque** P_{mi} is considered to be a piecewise constant variable

From Eq. 4 and for one $i=1,2,\cdots,n$ has a system of n equations which can be solved with respect to the variables $x_3^i, i=1,2,\cdots,n$

Actually, all variables x_3^i, can be expressed as **differential functions of the flat outputs**

$$y^i,\ i=1,2,\cdots,n$$

and thus one has

$$x_3^i = f_{x_3}(y^1,y^2,\cdots,y^n)$$

Moreover, from

$$\dot{E}_{q_i} = -\frac{1}{T_{d_i}}E'_{q_i} + \frac{1}{T_{d_{oi}}}\frac{x_{d_i}-x'_{d_i}}{x_{d_{\Sigma_i}}}V_{s_i}cos(\Delta\delta_i)+\frac{1}{T_{d_{oi}}}E_{f_i}$$

one can demonstrate that the control inputs $u_i = E_{f_i}$ can be expressed as **differential functions of the flat outputs** $y^i,\ i=1,2,\cdots,n$

Consequently, all state variables and the control inputs of the distributed power system can be expressed as differential functions of the flat outputs, and **the system is a differentially flat one.**

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Dynamic model of the distributed synchronous generators

Next, the **external mechanical torque** P_{mi} is considered to be time-varying

The effect of this torque is viewed as a **disturbance** to each power generator

In such a case for a model of *n=2* interconnected generators one obtains the **input-output linearized dynamics**

$$\dot{z}_3^i = a^i(x) + {b_1}^i g_1 u_1 + {b_2}^i g_2 u_2 + \tilde{d}^i \quad \text{where} \quad z_3^i = \ddot{\delta}^i = \dot{\omega}^i$$

and

$$a^i = (\frac{D_i}{2J_i})^2 x_2^i + \frac{D_i \omega_0}{(2J_i)^2}[G_{ii}{x_3^i}^2 + x_3^i \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] -$$
$$-\frac{\omega_0}{2J_i}[G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i} - x'_{d_i}}{x_{d\Sigma_i}'}V_{s_i} cos(x_1^i))] -$$
$$-\frac{\omega_0}{2J_i}x_3^i \sum_{j=1,j\neq i}^n G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i} - x'_{d_i}}{x_{d\Sigma_i}'}V_{s_i} cos(x_1^i)) -$$
$$-\frac{\omega_0}{2J_i}x_3^i \sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij}) x_2^i \frac{\omega_0}{2J_i} x_3^i \sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij}) x_2^j$$

and

$$b_1^i = -\frac{\omega_0}{2J_i}[2G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})]\frac{1}{T_{d_o i}}$$
$$b_2^i = -\frac{\omega_0}{2J_i}G_{i2} sin(x_1^i - x_1^2 - \alpha_{i2})\frac{1}{T_{d_o 2}}$$

while

$$\tilde{d}^i = -\frac{D_i \omega_0}{2{J_i}^2}P_m^i + \frac{\omega_0}{2J_i}\dot{P}_m^i$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Dynamic model of the distributed synchronous generators

For the **two interconnected generators** (*i=1,2*) one has the linearized dynamics

$$\begin{aligned}\dot{z}_1^i &= z_2^i \\ \dot{z}_2^i &= z_3^i \\ \dot{z}_3^i &= a^i(x) + {b_1}^i g_1 u_1 + {b_2}^i g_2 u_2 + \tilde{d}^i\end{aligned}$$

It is used that

$$\begin{aligned}\dot{z}_3^1 &= a^1(x) + {b_1}^1 g_1 u_1 + {b_2}^1 g_2 u_2 + \tilde{d}^1 \\ \dot{z}_3^2 &= a^2(x) + {b_1}^2 g_1 u_1 + {b_2}^2 g_2 u_2 + \tilde{d}^2\end{aligned}$$

or in matrix form

$$\dot{z}_3 = f_a(x) + Mu + \tilde{d}$$

where

$$z_3 = [z_3^1, z_3^2]^T,\ u = [u_1, u_2]^T \text{ and } \tilde{d} = [\tilde{d}_1, \tilde{d}_2]^T$$

and

$$f_a(x) = \begin{pmatrix} a^1(x) \\ a^2(x) \end{pmatrix},\quad M = \begin{pmatrix} {b_1}^1 g_1 & {b_2}^1 g_2 \\ {b_1}^2 g_1 & {b_2}^2 g_2 \end{pmatrix}$$

Setting, $v = f_a(x) + Mu + \tilde{d}$ one obtains

$$\begin{pmatrix} \dot{z}_1^i \\ \dot{z}_2^i \\ \dot{z}_3^i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^i \\ z_2^i \\ z_3^i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (v^i + \tilde{d}^i)$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.3. Dynamic model of the distributed synchronous generators

For **the model of the 2-area distributed power generation system** it holds that

$$x_{1,1}^{(3)} = f_1(x,t) + g_1(x,t)u + d_1$$
$$x_{1,2}^{(3)} = f_2(x,t) + g_2(x,t)u + d_2$$

By denoting

$$x_1 = x_{1,1},\ x_2 = \dot{x}_{1,1},\ x_3 = \ddot{x}_{1,1}$$

$$x_2 = x_{2,1},\ x_5 = \dot{x}_{2,1},\ x_6 = \ddot{x}_{2,1}$$

the **Brunovsky (canonical form) of the distributed power system** is obtained

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

where

$$v_1 = f_1(x) + g_{11}(x)u_1 + g_{12}(x)u_2$$
$$v_2 = f_2(x) + g_{21}(x)u_1 + g_{22}(x)u_2$$

• For the 2-area **MIMO nonlinear system** of the distributed SGs differential flatness properties hold and one can apply an adaptive fuzzy control scheme using only output feedback.

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

It is assumed now that **after defining the flat outputs of the initial MIMO nonlinear system**, and after expressing the system state variables and control inputs as functions of the flat output and of the associated derivatives, the system can be **transformed in the Brunovsky canonical form**

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\cdots \\
\dot{x}_{r_1-1} &= x_{r_1} \\
\dot{x}_{r_1} &= f_1(x) + \sum_{j=1}^{p} g_{1_j}(x) u_j + d_1 \\
\\
\dot{x}_{r_1+1} &= x_{r_1+2} \\
\dot{x}_{r_1+2} &= x_{r_1+3} \\
&\cdots \\
\dot{x}_{p-1} &= x_p \\
\dot{x}_p &= f_p(x) + \sum_{j=1}^{p} g_{p_j}(x) u_j + d_p
\end{aligned}
$$

$$
\begin{aligned}
y_1 &= x_1 \\
y_2 &= x_{r_1-1} \\
&\cdots \\
y_p &= x_{n-r_p+1}
\end{aligned}
$$

$x = [x_1, \cdots, x_n]^T$: is the state vector

$u = [u_1, \cdots, u_p]^T$: is the inputs vector

$y = [y_1, \cdots, y_p]^T$: is the outputs vector

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Next **the following vectors and matrices** can be defined

$$f(x)=[f_1(x), \quad ..., \quad f_n(x)]^T$$

$$g(x)=[g_1(x), \quad ..., \quad g_n(x)]^T$$

with $g_i(x)=[g_{1i}(x), \quad ..., \quad g_{pi}(x)]^T$

$$A=diag[A_1,...,A_p], \; B=diag[B_1,...,B_p]$$

$$C^T=diag[C_1,...,C_p], \; d=[d_1,...,d_p]^T$$

where matrix A has the **MIMO canonical form**, i.e. with elements

$$A_i=\begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... \\ 0 & 0 & 0 & ... & 1 \\ 0 & 0 & 0 & ... & 0 \end{bmatrix}_{r_i \times r_i}$$

$$B_i^T=[0 \quad 0 \quad ... \quad 0 \quad 1]_{1\times r_i} \qquad C_i=[1 \quad 0 \quad ... \quad 0 \quad 0]_{1\times r_i}$$

Thus, the initial nonlinear system can be writtenin the **state-space form**

$$\dot{x}=Ax+B[f(x)+g(x)u+\tilde{d}]$$

$$y=Cx$$

or equivalently in the state space form

$$\dot{x}=Ax+Bv+B\tilde{d}$$

$$y=Cx$$

where $\quad v=f(x)+g(x)u$

For the case of the **MIMO distributed SGs model** it is assumed that the functions $f(x)$ and $g(x)$ are unknown and have to be approximated by **neuro-fuzzy networks**

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.1. Transformation of MIMO nonlinear systems into the Brunovsky form

Thus, the nonlinear system can be written in **state-space form**

$$\dot{x} = Ax + B[f(x) + g(x)u + \bar{d}]$$
$$y = C^T x$$

which equivalently can be written as

$$\dot{x} = Ax + Bv + B\bar{d}$$
$$y = C^T x$$

where $v = f(x) + g(x)u.$

The **reference setpoints** for the system's outputs $y_1, \cdots, y_p$ are denoted as $y_{1m}, \cdots, y_{pm}$ and the associated tracking errors are defined as

$$e_1 = y_1 - y_{1m}$$
$$e_2 = y_2 - y_{2m}$$
$$\cdots$$
$$e_p = y_p - y_{pm}$$

The **error vector of the outputs** of the transformed MIMO system is denoted as

$$E_1 = [e_1, \cdots, e_p]^T$$
$$y_m = [y_{1m}, \cdots, y_{pm}]^T$$
$$\cdots$$
$$y_m^{(\nu)} = [y_{1m}^{(\nu)}, \cdots, y_{pm}^{(\nu)}]^T$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.2. Control law

The **control signal of the MIMO nonlinear system** contains the **unknown nonlinear functions** f(x) and g(x) which can be approximated by

$$\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f, \quad \hat{g}(x|\theta_g) = \Phi_g(x)\theta_g$$

where $\Phi_f(x) = \left(\xi_f^1(x), \xi_f^2(x), \cdots \xi_f^n(x)\right)^T,$

$$\xi_f^i(x) = \left(\phi_f^{i,1}(x), \phi_f^{i,2}(x), \cdots, \phi_f^{i,N}(x)\right)$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_f{}^T = \left(\theta_f^1, \theta_f^2, \cdots \theta_f^N\right).$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.2. Control law

Similarly, it holds $\Phi_g(x) = \left(\xi_g^1(x), \xi_g^2(x), \cdots \xi_g^N(x)\right)^T$.

$$\xi_g^i(x) = \left(\phi_g^{i,1}(x), \phi_g^{i,2}(x), \cdots, \phi_g^{i,N}(x)\right).$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix}$$

while the weights vector is defined as $\theta_g = \left(\theta_g^1, \theta_g^2, \cdots, \theta_g^p\right)^T$.

However, here each row of θ_g is vector thus giving

$$\theta_g = \begin{pmatrix} \theta_{g1}^1 & \theta_{g1}^2 & \cdots & \theta_{g1}^p \\ \theta_{g2}^1 & \theta_{g2}^2 & \cdots & \theta_{g2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{gN}^1 & \theta_{gN}^2 & \cdots & \theta_{gN}^p \end{pmatrix}$$

If the state variables of the system are available for measurement then a **state-feedback control law can be formulated** as

$$u = \hat{g}^{-1}(x|\theta_g)\left[-\hat{f}(x|\theta_f) + y_m^{(r)} + K_c^T e + u_c\right]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.2. Estimation of the state vector

The control of the system described by becomes more complicated **when the state vector x is not directly measurable** and has to be reconstructed through a state observer. The following definitions are used

$e = x - x_m$: is the error of the state vector

$\hat{e} = \hat{x} - x_m$ is the error of the estimated state vector

$\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$ is the observation error

When an **observer is used to reconstruct the state vector**, the control law

$$u = \hat{g}^{-1}(\hat{x}|\theta_g)[-\hat{f}(\hat{x}|\theta_f) + y_m^{(r)} - K^T\hat{e} + u_c]$$

By applying the previous feedback control law one obtains the closed-loop dynamics

$$y^{(n)} = f(x) + g(x)\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(n)} = f(x) + [g(x) - \hat{g}(\hat{x}) + \hat{g}(\hat{x})]\hat{g}^{-1}(\hat{x})[-\hat{f}(\hat{x}) + y_m^{(n)} - K^T\hat{e} + u_c] + d \Rightarrow$$
$$y^{(n)} = [f(x) - \hat{f}(\hat{x})] + [g(x) - \hat{g}(\hat{x})]u + y_m^{(n)} - K^T\hat{e} + u_c + d$$

It holds $e = x - x_m \Rightarrow y^{(n)} = e^{(n)} + y_m^{(n)}$

and by substituting $y^{(n)}$ n the **previous tracking error dynamics** gives

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.4. Design of an adaptive neurofuzzy controller for the distributed SG system

7.4.2. Estimation of the state vector

the **new tracking error dynamics**

$$e^{(n)} + y_m^{(n)} = y_m^{(n)} - K^T\hat{e} + u_c + [f(x) - \hat{f}(\hat{x})] + \\ + [g(x) - \hat{g}(\hat{x})]u + d$$

or equivalently

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\{[f(x) - \hat{f}(\hat{x})] + \\ + [g(x) - \hat{g}(\hat{x})]u + d\} \tag{A}$$

$$e_1 = C^T e$$

where $e = [e^1, e^2, \cdots, e^p]^T$ with $e^i = [e_i, \dot{e}_i, \ddot{e}_i, \cdots, e_i^{(r_i-1)}]^T,\ i = 1, 2, \cdots, p$

and equivalently $\hat{e} = [\hat{e}^1, \hat{e}^2, \cdots, \hat{e}^p]^T$ with $\hat{e}^i = [\hat{e}_i, \dot{\hat{e}}_i, \ddot{\hat{e}}_i, \cdots, \hat{e}_i^{(r_i-1)}]^T,\ i = 1, 2, \cdots, p.$

A state observer is designed as:

$$\dot{\hat{e}} = A\hat{e} - BK^T\hat{e} + K_o[e_1 - C^T\hat{e}] \tag{B}$$

$$\hat{e}_1 = C^T\hat{e}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the distributed SG system

7.5.1. Tracking error dynamics under feedback control

By **applying differential flatness theory, and in the presence of disturbances**, the dvnamic model of the distributed SGs comes to the form

$$\begin{pmatrix}\ddot{x}_1\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}f_1(x,t)\\ f_2(x,t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)\\ g_2(x,t)\end{pmatrix}u+\begin{pmatrix}d_1\\ d_2\end{pmatrix} \qquad \text{C}$$

The following **control input** is defined:

$$u=\begin{pmatrix}\hat{g}_1(x,t)\\ \hat{g}_2(x,t)\end{pmatrix}^{-1}\left\{\begin{pmatrix}\ddot{x}_1^d\\ \dot{x}_3^d\end{pmatrix}-\begin{pmatrix}\hat{f}_1(x,t)\\ \hat{f}_2(x,t)\end{pmatrix}-\begin{pmatrix}K_1^T\\ K_2^T\end{pmatrix}e+\begin{pmatrix}u_{c_1}\\ u_{c_2}\end{pmatrix}\right\} \qquad \text{D}$$

where: $[u_{c_1}\ u_{c_2}]^T$ is a **robust control term** that is used for the compensation of the model's uncertainties as well as of the external disturbances

and: $K_i^T=[k_1^i,k_2^i,\cdots,k_{n-1}^i,k_n^i]$ is the feedback gain

Substituting the control input D into the system C one obtains

$$\begin{pmatrix}\ddot{x}_1\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}f_1(x,t)\\ f_2(x,t)\end{pmatrix}+\begin{pmatrix}g_1(x,t)\\ g_2(x,t)\end{pmatrix}\begin{pmatrix}\hat{g}_1(x,t)\\ \hat{g}_2(x,t)\end{pmatrix}^{-1}\cdot$$

$$\cdot\left\{\begin{pmatrix}\ddot{x}_1^d\\ \dot{x}_3^d\end{pmatrix}-\begin{pmatrix}\hat{f}_1(x,t)\\ \hat{f}_2(x,t)\end{pmatrix}-\begin{pmatrix}K_1^T\\ K_2^T\end{pmatrix}e+\begin{pmatrix}u_{c_1}\\ u_{c_2}\end{pmatrix}\right\}+\begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the distributed SG system

7.5.1. Tracking error dynamics under feedback control

Moreover, using again Eq. D one obtains the **tracking error dynamics**

$$\begin{pmatrix}\ddot{e}_1\\ \dot{e}_3\end{pmatrix} = \begin{pmatrix} f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u - \begin{pmatrix}K_1^T\\ K_2^T\end{pmatrix}e + \begin{pmatrix}u_{c_1}\\ u_{c_2}\end{pmatrix} + \begin{pmatrix}d_1\\ d_2\end{pmatrix}$$

The **approximation error** is defined $w = \begin{pmatrix} f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{pmatrix}u$

Using matrices A,B,K, $A = \begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\quad B = \begin{pmatrix}0&0\\1&0\\0&1\end{pmatrix}\quad K^T = \begin{pmatrix}K_1^1 & K_2^1 & K_3^1\\ K_1^2 & K_2^2 & K_3^2\end{pmatrix}$

and considering that **the estimated state vector is used in the control loop** the following description of the tracking error dynamics is obtained:

$$\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\left\{\begin{pmatrix} f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u + \tilde{d}\right\}$$

When the **estimated state vector** is used in the loop the **approximation error is written as**

$$w = \begin{pmatrix} f_1(x,t)-\hat{f}_1(\hat{x},t)\\ f_2(x,t)-\hat{f}_2(\hat{x},t)\end{pmatrix} + \begin{pmatrix} g_1(x,t)-\hat{g}_1(\hat{x},t)\\ g_2(x,t)-\hat{g}_2(\hat{x},t)\end{pmatrix}u$$

while the **tracking error dynamics becomes** $\dot{e} = Ae - BK^T\hat{e} + Bu_c + Bw + B\tilde{d}$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5.2. Dynamics of the observation error

The **observation error** is defined as: $\bar{e} = e - \hat{e} = x - \hat{x}$.

By subtracting Eq. B from Eq. A one obtains:

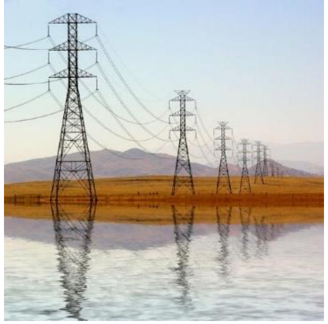
$$\dot{e} - \dot{\hat{e}} = A(e - \hat{e}) + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + \\ +[g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T(e - \hat{e})$$

$$e_1 - \hat{e}_1 = C^T(e - \hat{e})$$

or equivalently:

$$\dot{\bar{e}} = A\bar{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} - K_o C^T \bar{e}$$

$$\bar{e}_1 = C^T \bar{e}$$



which can be also written as:

$$\dot{\bar{e}} = (A - K_o C^T)\bar{e} + Bu_c + Bw + \tilde{d}\}$$

$$\bar{e}_1 = C^T \bar{e}$$

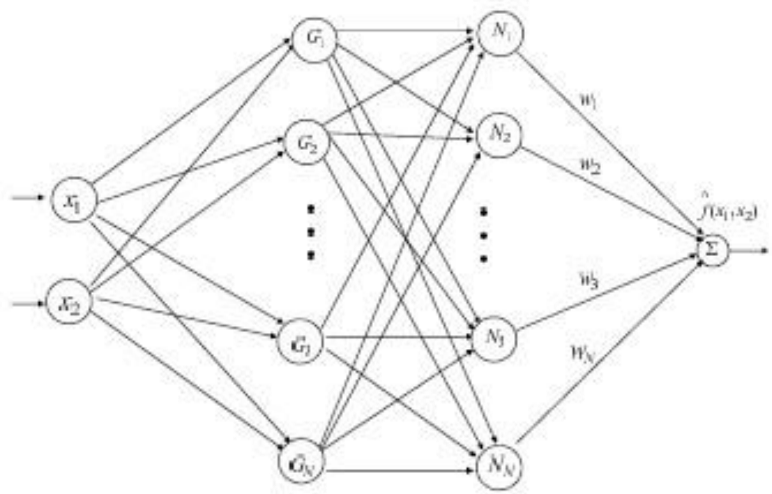
Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the distributed SG system

7.5.3. Approximation of functions f(x,t) and g(x,t)

Next, the **first of the approximators** of the unknown system dynamics is defined

$$\hat{f}(\hat{x}) = \begin{pmatrix} \hat{f}_1(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_1(\hat{x}|\theta_f) & \in & R^{1\times 1} \\ \hat{f}_2(\hat{x}|\theta_f) & \hat{x}\in R^{4\times 1} & \hat{f}_2(\hat{x}|\theta_f) & \in & R^{1\times 1} \end{pmatrix}$$



containing kernel functions $\phi_f^{i,j}(\hat{x}) = \frac{\prod_{j=1}^{n}\mu_{A_j^i}(\hat{x}_j)}{\sum_{l=1}^{N}\prod_{j=1}^{n}\mu_{A_j^i}(\hat{x}_j)}$

where $\mu_{A_j^i}(\hat{x})$ are fuzzy membership functions

appearing in the antecedent part of the *l-th* fuzzy rule

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the distributed SG system

7.5.3. Approximation of functions f(x,t) and g(x,t)

Similarly, **the second of the approximators** of the unknown system dynamics is defined

$$\hat{g}(\hat{x}) = \begin{pmatrix} \hat{g}_1(\hat{x}|\theta_g) & \hat{x}\in R^{4\times 1} & \hat{g}_1(\hat{x}|\theta_g) & \in & R^{1\times 2} \\ \hat{g}_2(\hat{x}|\theta_g) & \hat{x}\in R^{4\times 1} & \hat{g}_2(\hat{x}|\theta_g) & \in & R^{1\times 2} \end{pmatrix}$$

The **values of the weights that result in optimal approximation** are

$$\theta_f^* = arg\ min_{\theta_f\in M_{\theta_f}}[sup_{\hat{x}\in U_{\hat{x}}}(f(x)-\hat{f}(\hat{x}|\theta_f))]$$
$$\theta_g^* = arg\ min_{\theta_g\in M_{\theta_g}}[sup_{\hat{x}\in U_{\hat{x}}}(g(x)-\hat{g}(\hat{x}|\theta_g))]$$

The variation ranges for the weights are given by

$$M_{\theta_f} = \{\theta_f\in R^h:\ ||\theta_f||\leq m_{\theta_f}\}$$
$$M_{\theta_g} = \{\theta_g\in R^h:\ ||\theta_g||\leq m_{\theta_g}\}$$

The **value of the approximation error** that corresponds to the optimal values of the weights vectors is

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f^*)\right) + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g^*)\right)u$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.5. Application of adaptive neurofuzzy control to the distributed SG system

7.5.3. Approximation of functions f(x,t) and g(x,t)

which is next written as

$$w = \left(f(x,t) - \hat{f}(\hat{x}|\theta_f) + \hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)\right) + \\ + \left(g(x,t) - \hat{g}(\hat{x}|\theta_g) + \hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)\right) u$$

which can be also written in the following form

$$w = \ (w_a + w_b)$$

with

$$w_a = \{[f(x,t) - \hat{f}(\hat{x}|\theta_f)] + [g(x,t) - \hat{g}(\hat{x}|\theta_g)]\} u$$

and

$$w_b = \{[\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x},\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]\} u$$

Moreover, the following **weights error vectors are defined**

$$\bar{\theta}_f = \theta_f - \theta_f^*$$
$$\bar{\theta}_g = \theta_g - \theta_g^*$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

The **following Lyapunov function** is considered:

$$V = \frac{1}{2}\hat{e}^T P_1 \hat{e} + \frac{1}{2}\bar{e}^T P_2 \bar{e} + \frac{1}{2\gamma_1}\bar{\theta}_f^T \bar{\theta}_f + \frac{1}{2\gamma_2} tr[\bar{\theta}_g^T \bar{\theta}_g]$$

The selection of the **Lyapunov function** is based on the following principle of indirect adaptive control

$$\hat{e} : \lim_{t\to\infty} \hat{x}(t) = x_d(t)$$
$$\bar{e} : \lim_{t\to\infty} \hat{x}(t) = x(t).$$

this results into

$$\lim_{t\to\infty} x(t) = x_d(t)$$

By deriving the **Lyapunov function** with respect to time one obtains:

$$\dot{V} = \frac{1}{2}\dot{\hat{e}}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \dot{\hat{e}} + \frac{1}{2}\dot{\bar{e}}^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 \dot{\bar{e}} + \\ + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g] \Rightarrow$$

$$\dot{V} = \frac{1}{2}\{(A - BK^T)\hat{e} + K_o C^T \bar{e}\}^T P_1 \hat{e} + \frac{1}{2}\hat{e}^T P_1 \{(A - BK^T)\hat{e} + K_o C^T \bar{e}\} + \\ + \frac{1}{2}\{(A - K_o C^T)\bar{e} + B u_c + B\tilde{d} + Bw\}^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P_2 \{(A - K_o C^T)\bar{e} + B u_c + B\tilde{d} + Bw\} + \\ + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g] \Rightarrow$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

The equation is rewritten as:

$$\begin{aligned}\dot{V} = &\tfrac{1}{2}\{\hat{e}^T(A-BK^T)^T+\tilde{e}^TCK_o^T\}P_1\hat{e}+\tfrac{1}{2}\hat{e}^TP_1\{(A-BK^T)\hat{e}+K_oC^T\tilde{e}\}+\\ &+\tfrac{1}{2}\{\tilde{e}^T(A-K_oC^T)^T+u_c^TB^T+w^TB^T+\tilde{d}^TB^T\}P_2\tilde{e}+\\ &\tfrac{1}{2}\tilde{e}^TP_2\{(A-K_oC^T)\tilde{e}+Bu_c+Bw+B\tilde{d}\}+\tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f+\tfrac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\Rightarrow\end{aligned}$$

which finally takes the form:

$$\begin{aligned}\dot{V} = &\tfrac{1}{2}\hat{e}^T(A-BK^T)^TP_1\hat{e}+\tfrac{1}{2}\tilde{e}^TCK_o^TP_1\hat{e}+\\ &+\tfrac{1}{2}\hat{e}^TP_1(A-BK^T)\hat{e}+\tfrac{1}{2}\hat{e}^TP_1K_oC^T\tilde{e}+\\ &+\tfrac{1}{2}\tilde{e}^T(A-K_oC^T)^TP_2\tilde{e}+\tfrac{1}{2}(u_c^T+w^T+\tilde{d}^T)B^TP_2\tilde{e}+\\ &+\tfrac{1}{2}\tilde{e}^TP_2(A-K_oC^T)\tilde{e}+\tfrac{1}{2}\tilde{e}^TP_2B(u_c+w+\tilde{d})+\\ &+\tfrac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f+\tfrac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{aligned}$$

Assumption 1: For given positive definite matrices Q1 and Q2 there exist positive definite matrices P1 and P2, which are the solution of the following **Riccati equations**

$$(A-BK^T)^TP_1+P_1(A-BK^T)+Q_1=0$$

$$\begin{aligned}&(A-K_oC^T)^TP_2+P_2(A-K_oC^T)-\\ &-P_2B(\tfrac{2}{r}-\tfrac{1}{\rho^2})B^TP_2+Q_2=0\end{aligned}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

By substituting the conditions from the previous Riccati equations into the derivative of the Lyapunov function one gets:

$$\dot{V} = \frac{1}{2}\hat{e}^T\{(A-BK^T)^T P_1 + P_1(A-BK^T)\}\hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} +$$
$$+\frac{1}{2}\bar{e}^T\{(A-K_oC^T)^T P_2 + P_2(A-K_oC^T)\}\bar{e} +$$
$$+\bar{e}^T P_2 B(u_c + w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]$$

or:

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2}\bar{e}^T\{Q_2 - P_2 B(\frac{2}{r} - \frac{1}{\rho^2})B^T P_2\}\bar{e} +$$
$$+\bar{e}^T P_2 B(u_c + w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]$$

- The **supervisory control term** u_c consists of two terms: u_a and u_b.

$$u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$$

where assuming that the measurable elements of vector $\tilde{e}$ are $\{\tilde{e}_1, \tilde{e}_3, \cdots, \tilde{e}_k\}$,

the term Δu_a Is given by

$$-\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a = -\frac{1}{r}\begin{pmatrix} p_{11}\tilde{e}_1 + p_{13}\tilde{e}_3 + \cdots + p_{1k}\tilde{e}_k \\ p_{13}\tilde{e}_1 + p_{33}\tilde{e}_3 + \cdots + p_{3k}\tilde{e}_k \\ \cdots \cdots\cdots \\ p_{1k}\tilde{e}_1 + p_{3k}\tilde{e}_3 + \cdots + p_{kk}\tilde{e}_k \end{pmatrix}$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

- The control teri u_b. Is given by

$$u_b = -[(P_2B)^T(P_2B)]^{-1}(P_2B)^T C K_o^T P_1 \hat{e}$$

u_a is an H-infinity control used for the compensation of the approximation error w and the additive disturbance $\tilde{d}$.

Its first component $-\frac{1}{r}\tilde{e}^T P_2 B$ has been chosen so as to compensate for the term $\frac{1}{r}\tilde{e}^T P_2 B B^T P_2 \tilde{e}$, which appears in the previously computed function about $\dot{V}$.

By including also the second component Δu_a one has that u_a is computed based on the feedback only the measurable variables $\{\tilde{e}_1, \tilde{e_3}, \cdots, \tilde{e_k}\}$ out of the complete vector $\{\tilde{e}_1, \tilde{e_3}, \cdots, \tilde{e_k}\}$

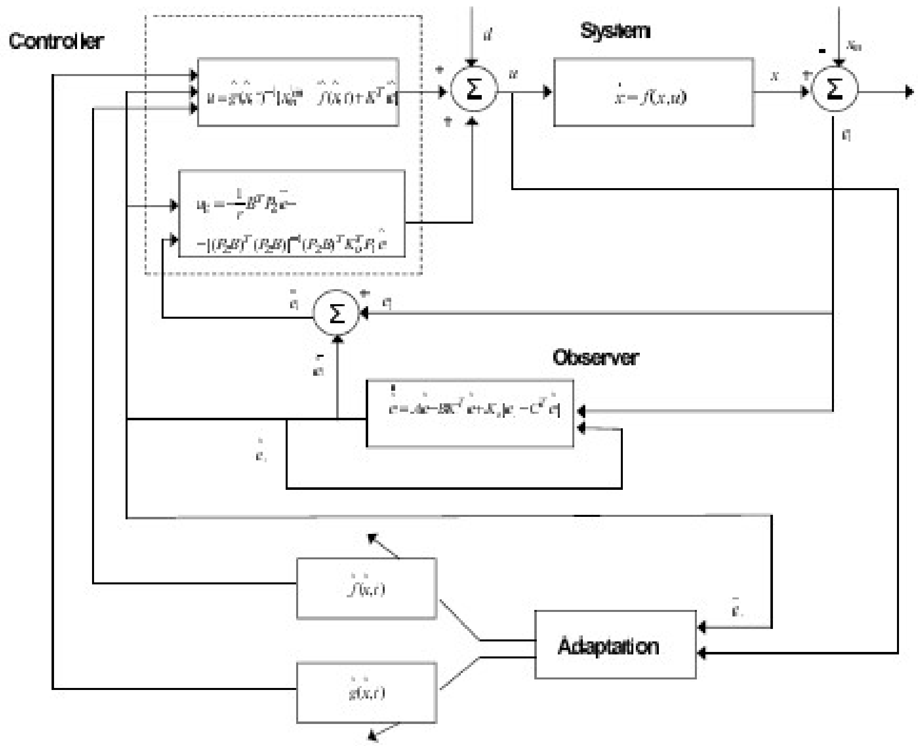
Eq. $u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$ finally rewritten as $u_a = -\frac{1}{r}\tilde{e}^T P_2 B + \Delta u_a$.

- u_b is a control used for the compensation of the observation error (the control term has been chosen so as to satisfy the condition

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

The control scheme is depicted in the following diagram



By substituting the supervisory control term in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} + \bar{e}^T C K_o^T P_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{r}\bar{e}^T P_2 B B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} +$$

$$+\bar{e}^T P_2 B u_a + \bar{e}^T P_2 B u_b + \bar{e}^T P_2 B (w + \bar{d}) + \frac{1}{\gamma_1}\dot{\bar{\theta}}_f^T \bar{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\bar{\theta}}_g^T \bar{\theta}_g]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

or equivalently

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \bar{e}^T P_2 B(w + \tilde{d}) + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Besides, about the adaptation of the weights of the neurofuzzy network it holds

$$\dot{\tilde{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f \qquad \dot{\tilde{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g.$$

and also

$$\dot{\theta}_f = -\gamma_1 \Phi(\hat{x})^T B^T P_2 \bar{e}$$
$$\dot{\theta}_g = -\gamma_2 \Phi(\hat{x})^T B^T P_2 \bar{e} u^T$$

By substituting the above relations in the derivative of the Lyapunov function one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + d) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[u\bar{e}^T P_2 B \Phi(\hat{x})(\theta_g - \theta_g^*)]$$

or

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + B^T P_2 \bar{e}(w + \tilde{d}) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\bar{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[u\bar{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))]$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Taking into account that $u \in R^{2\times 1}$ and $\tilde{e}^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1\times 2}$

one gets

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + \tilde{d}) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2) tr[\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u]$$

Since $\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u \in R^{1\times 1}$

it holds

$$tr(\tilde{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)u) = \\ = \tilde{e}^T P_2 B(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u$$

Therefore, one finally obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\tilde{e}^T Q_2 \tilde{e} - \frac{1}{2\rho^2}\tilde{e}^T P_2 B B^T P_2 \tilde{e} + B^T P_2 \tilde{e}(w + \tilde{d}) + \\ + \frac{1}{\gamma_1}(-\gamma_1)\tilde{e}^T P_2 B \Phi(\hat{x})(\theta_f - \theta_f^*) + \\ + \frac{1}{\gamma_2}(-\gamma_2)\tilde{e}^T P_2 B(\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*))u$$

Next, the following **approximation error** is defined

$$w_\alpha = [\hat{f}(\hat{x}|\theta_f^*) - \hat{f}(\hat{x}|\theta_f)] + [\hat{g}(\hat{x}|\theta_g^*) - \hat{g}(\hat{x}|\theta_g)]u$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Thus, one obtains

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + B^T P_2 \bar{e}(w + \tilde{d}) + \bar{e}^T P_2 B w_a$$

Denoting the **aggregate approximation e**rror and disturbances vector as

$$w_1 = w + \tilde{d} + w_a$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \bar{e}^T P_2 B w_1$$

which in turn is written as

$$\dot{V} = -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} + \\ + \frac{1}{2}\bar{e}^T P B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e}$$

Lemma: The following inequality holds

$$\frac{1}{2}\bar{e}^T P_2 B w_1 + \frac{1}{2} w_1^T B^T P_2 \bar{e} - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e} \\ \leq \frac{1}{2}\rho^2 w_1^T w_1$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Proof:

The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow$$
$$\frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow$$
$$\frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$

By substituting $a = w_1$ and $b = \bar{e}^T P_2 B$ one gets

$$\frac{1}{2}w_1^T B^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 B w_1 - \frac{1}{2\rho^2}\bar{e}^T P_2 B B^T P_2 \bar{e}$$
$$\leq \frac{1}{2}\rho^2 w_1^T w_1$$

Moreover, by substituting the above inequality into the derivative of the Lyapunov function one gets

$$\dot{V} \leq -\frac{1}{2}\hat{e}^T Q_1 \hat{e} - \frac{1}{2}\bar{e}^T Q_2 \bar{e} + \frac{1}{2}\rho^2 w_1^T w_1$$

which is also written as $\dot{V} \leq -\frac{1}{2}E^T Q E + \frac{1}{2}\rho^2 w_1^T w_1$

with $E = \begin{pmatrix} \hat{e} \\ \bar{e} \end{pmatrix}$, $Q = \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix} = diag[Q_1, Q_2]$

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.6. Lyapunov stability analysis

Hence, the H_∞ performance criterion is derived. For sufficiently small ρ the inequality will be true and the H_∞ tracking criterion will be satisfied. In that case, the integration of ˙V from 0 to T gives

$$\int_0^T \dot{V}(t)\,dt \leq -\frac{1}{2}\int_0^T ||E||^2\,dt + \frac{1}{2}\rho^2\int_0^T ||w_1||^2\,dt \Rightarrow$$
$$2V(T) - 2V(0) \leq -\int_0^T ||E||_Q^2\,dt + \rho^2\int_0^T ||w_1||^2\,dt \Rightarrow$$
$$2V(T) + \int_0^T ||E||_Q^2\,dt \leq 2V(0) + \rho^2\int_0^T ||w_1||^2\,dt$$

It is assumed that there exists a positive constant $M_w > 0$ such that

$$\int_0^\infty ||w_1||^2\,dt \leq M_w$$

Therefore for the integral $\int_0^T ||E||_Q^2\,dt$ one gets

$$\int_0^\infty ||E||_Q^2\,dt \leq 2V(0) + \rho^2 M_w$$

Thus, the integral $\int_0^\infty ||E||_Q^2\,dt$ is bounded and **according to Barbalat's Lemma**

$$\lim_{t\to\infty} e(t) = 0$$

Example 3: Nonlinear control and state estimation using Lyapunov methods

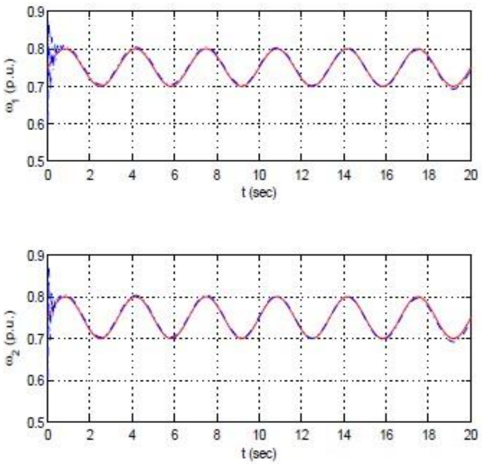
7.7. Simulation tests

The dynamic model of the distributed SGs was taken to be completely unknown, while the state vector could be partially measured

setpoint 1

ω_1

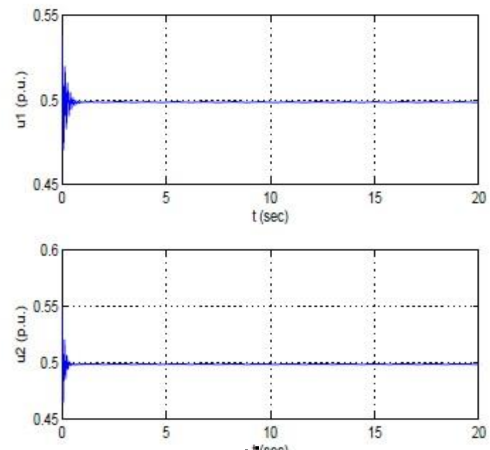
ω_2



time

u_1

u_2

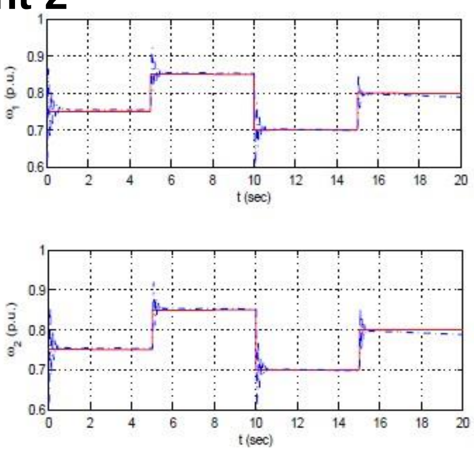


time

setpoint 2

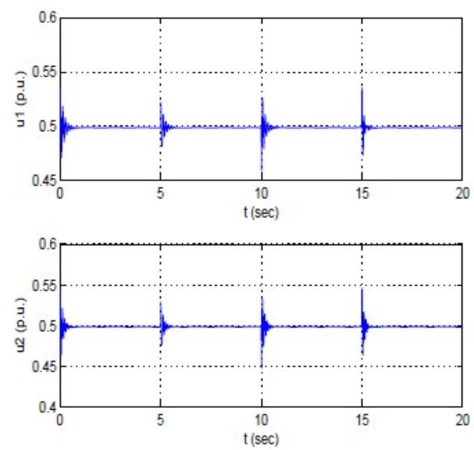
ω_1

ω_2



u_1

u_2

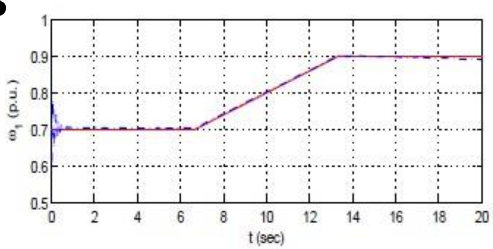


Example 3: Nonlinear control and state estimation using Lyapunov methods

7.7. Simulation tests

setpoint 3

ω_1



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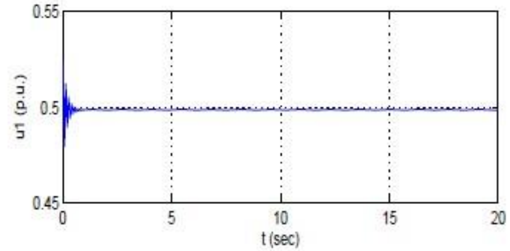
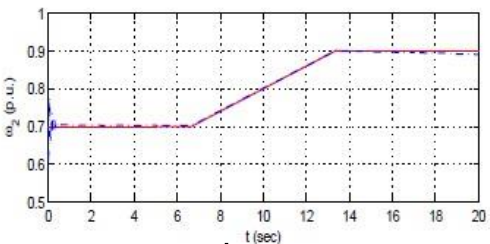
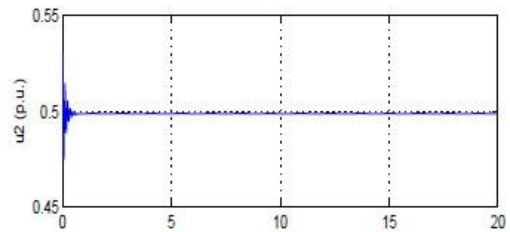


ω_2



u_2

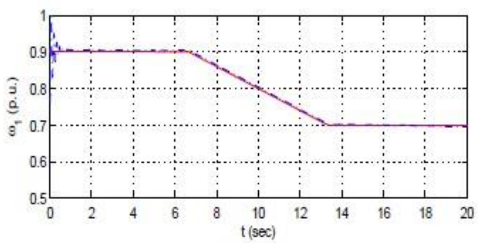


time

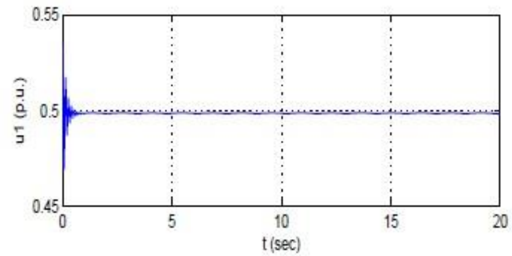
time

setpoint 4

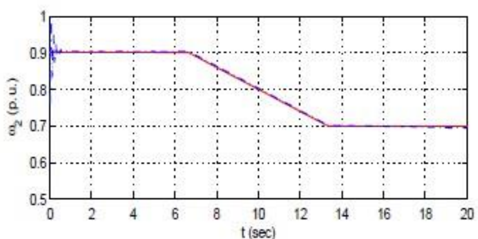
ω_1



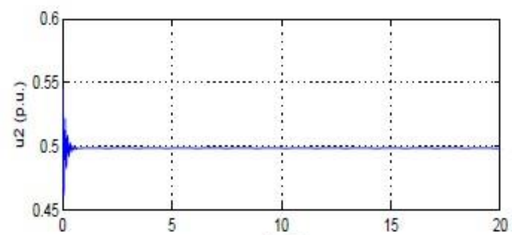
u_1



ω_2



u_2



time

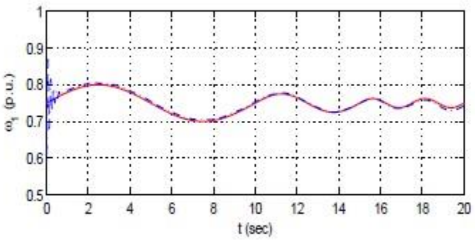
time

Example 3: Nonlinear control and state estimation using Lyapunov methods

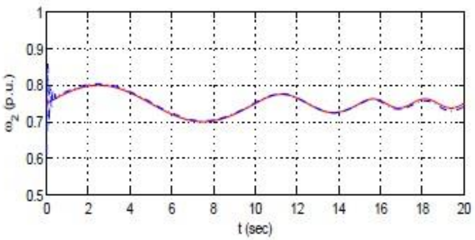
7.7. Simulation tests

setpoint 5

ω_1

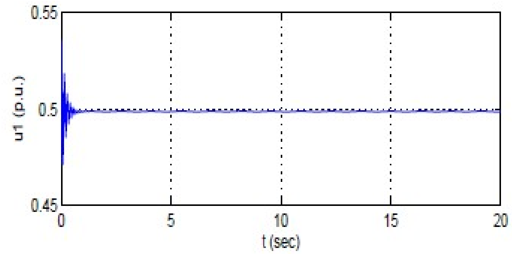


ω_2

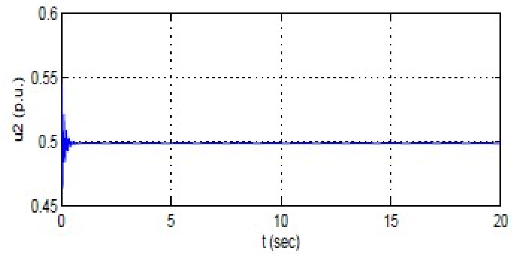


time

u_1



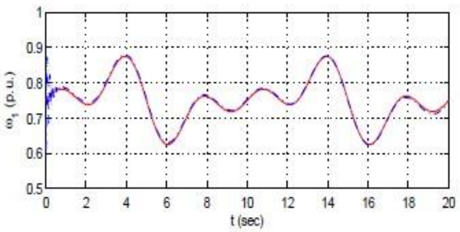
u_2



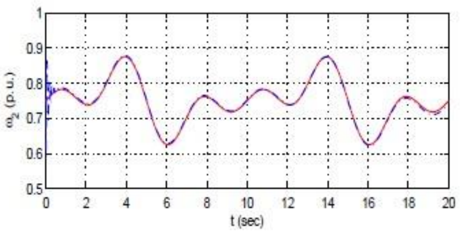
time

setpoint 6

ω_1

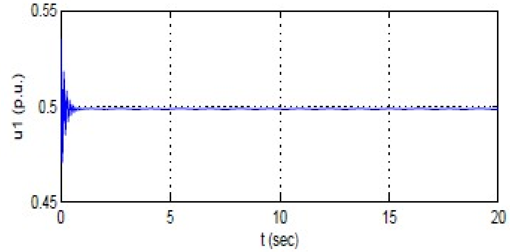


ω_2

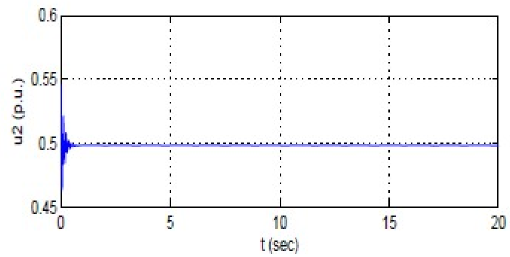


time

u_1



u_2



time

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.7. Simulation tests

Table I: RMSE of the power generator's state variables

parameter	ω_1	$\dot{\omega}_1$	ω_2	$\dot{\omega}_2$
$RMSE_1$	0.0035	0.0002	0.0034	0.0002
$RMSE_2$	0.0123	0.0545	0.0118	0.0602
$RMSE_3$	0.0035	0.0020	0.0035	0.0020
$RMSE_4$	0.0031	0.0020	0.0026	0.0020
$RMSE_5$	0.0034	0.0003	0.0033	0.0002
$RMSE_6$	0.0035	0.0003	0.0033	0.0002

The **tracking accuracy** of the control method was remarkable despite the fact that

(i) the **dynamic model** of the systems was **completely unknown**,

(ii) **only output feedback** was used in the implementation of the control scheme.

It has been also confirmed that the transient characteristics of the control scheme are quite satisfactory

The proposed **optimization-based modelling and control method** is of generic use and can be applied to a **wide class of nonlinear dynamical** systems of unknown model

Example 3: Nonlinear control and state estimation using Lyapunov methods

7.8. Conclusions

● A solution to the problem of model-free adaptive control for distributed synchronous generators has been proposed

● It was proven that the dynamic model of the distributed SGs is a differentially flat one. The flat outputs of the model were taken to be the rotor's turn speed and the currents of the secondary (control) winding of the stator.

● By proving differential flatness properties for the distributed SGs the transformation of its model to the linear canonical form was achieved.

● In this new linearized description the control inputs comprised nonlinear terms which were related to the system's unknown dynamics.

● These terms were dynamically identified with the use of neurofuzzy approximators. These estimates of the unknown dynamics were used in turn in the computation of a feedback control input, thus establishing an indirect adaptive control scheme.

● It was also assumed that only the output of the distributed SGs could be directly measured and that the rest of the state vector elements of the machine had to be computed with the use of a state-observer.

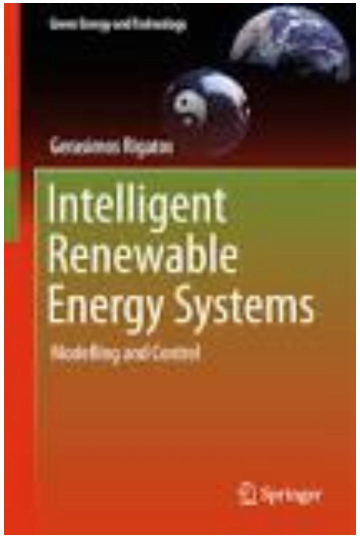
● The stability of the control loop was proven with the use of Lyapunov analysis.

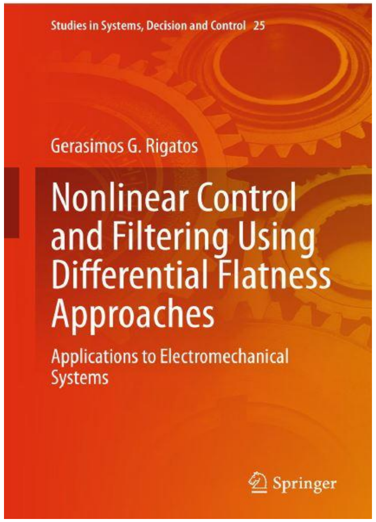
8. Final Conclusions

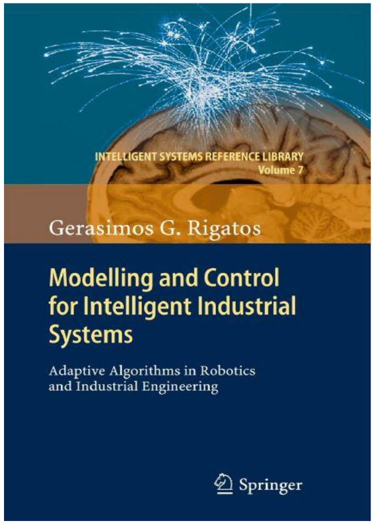
● Methods for nonlinear control and state estimation in electric power systems have been developed

● The main approaches for nonlinear control have been: (i) **control with global linearization** method (ii) **control with approximate (asymptotic) linearization** methods (iii) **control with Lyapunov theory methods (adaptive control)** in case that the dynamic model of the electric power system is unknown

● The main approaches for nonlinear state estimation are: (i) nonlinear state estimation with methods of global linearization (ii) nonlinear state estimation with methods of approximate (asymptotic) linearization







grigat@ieee.org