Lecture on

New approaches to nonlinear control of robotic systems:

Lyapunov methods

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1. Outline

- **Adaptive fuzzy control based** on **differential flatness theory** for **multivariable control** (dive-plane control) of **autonomous submarines**.

- It is proven that the **dynamic model of the submarine**, having as state variables the vessel's depth and its pitch angle, is a **differentially flat** one. This means that all its state variables and its control inputs can be written as differential functions of the flat output and its derivatives.

- By exploiting differential flatness properties the system's dynamic model is written in the **multivariable linear canonical (Brunovsky) form**, for which the design of a state feedback controller becomes possible.

- After this **transformation**, the **new control in**puts of the system contain **unknown nonlinear parts**, which are **identified with the use of neurofuzzy approximators**.

- The **learning procedure** for these estimators is determined by the requirement the **first derivative of the closed-loop's Lyapunov function to be a negative one**.

- Moreover, **the Lyapunov stability analysis** shows that **H-infinity tracking performance** is succeeded for the feedback control loop and this assures improved **robustness** to the aforementioned model uncertainty as well as to external perturbations.

- The efficiency of the proposed adaptive fuzzy control scheme isconfirmed through simulation experiments.

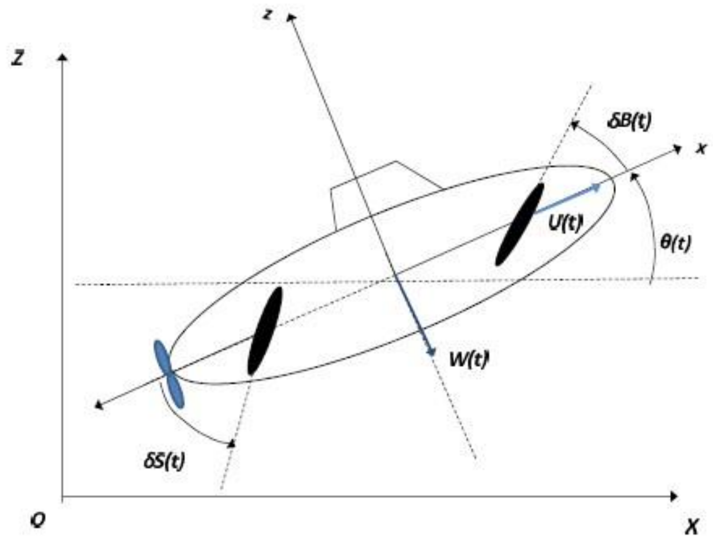
2. Problem statement

The **multivariable model** of the **submarine's dynamics** has as outputs

the **depth** of the submarine h
The **pitch angle** of the submarine θ

and as inputs

the **deflection angle of the hydroplanes** at the **front part** of vessel δB
the **deflection angle of the hydroplanes** located at the **rear** part of the vessel δS



The objective is to succeed **multivariable nonlinear feedback control** for the submarine's model, **without prior knowledge of the vessel's kinematic or dynamic model**

3. Dynamic model of the submarine

The **dynamic model of the submarine** is written as:

$$\dot{w}(t) = \frac{Z'_w U}{L m'_3} w(t) + \frac{1}{m'_3} \dot{Z}'_{\dot{\theta}} + m')U\dot{\theta}(t) + \frac{Z'_{\dot{Q}} L}{m'_z} \dot{Q}(t) + $$
$$+ \frac{Z'_{\delta B} U^2}{m'_3 L} \delta B(t) + \frac{Z'_{\delta S} U^2}{m'_3 L} \delta S(t) + \frac{Z_d(t)}{0.5\rho L^3 m'_3} + Z_\eta(w, q) \qquad (1)$$

$$\dot{Q}(t) = \frac{M'_{\dot{w}}}{L I'_2} \dot{w}(t) + \frac{M'_w U}{L^2 I'_2} w(t) + \frac{M'_{\dot{\theta}} U}{L I'_2} \dot{\theta}(t) + $$
$$+ \frac{M'_{\delta B} U^2}{L^2 I'_2} \delta B(t) + \frac{M'_{\delta S} U^2}{L^2 I'_2} \delta S(t) + \frac{2mg(z_G - z_B)}{\rho L^5 I'_2} \theta(t) + \frac{M_d(t)}{0.5\rho L^5 I'_2} + M_\eta(w, q) \qquad (2)$$

w is the velocity along the z-axis, of the body-fixed frame

h is the depth of the vessel measured in the inertial coordinates system,

θ is the pitch angle

$Q = \dot{\theta}$ is the rate of change of the pitch angle.

δB is the hydroplane deflection in the bow plane,

δS is the hydroplane deflection in the stern

Z_d, M_d are bounded disturbance inputs due to sea currents

$Z_\eta(w, q)$, $M_\eta(w, q)$ are disturbance inputs representing the vessel's cross-flow drag

$U = U_0$ denotes the x-axis (forward) velocity of the vessel.

3. Dynamic model of the submarine

Indicative values of the parameters of the submarine.s dynamic model are:

Table I [1]

Parameters of the Submarine's dynamic model

Parameter Value	Parameter Value	Parameter Value
$Z'_w = -0.0110$	$Z'_{\dot{w}} = -0.0075$	$Z'_\theta = -0.0045$
$Z'_\theta = -0.0002$	$Z'_{\delta B} = -0.0025$	$Z'_{\delta S} = -0.0050$
$M'_w = 0.0030$	$M'_{\dot{w}} = -0.0002$	$M'_\theta = -0.0025$
$M'_{\dot{\theta}} = -0.0004$	$M'_{\delta B} = 0.0005$	$M'_{\delta S} = -0.0025$
$I'_y = 5.6867 \cdot 10e^{-4}$	$L = 286\text{ft}$	$m = 1.52 \cdot 10^5\,\text{slug}$
$Z_g - Z_B = -1.5\text{ft}$	$U = 8.43\text{ft/s}$	$\rho = 2.0\text{slug}/\text{ft}^3$
$I'_2 = I'_y - M'_B$	$m = 2m/(\rho L^3))$	$m'_3 = m' - Z'_w$

[1] K.Lee and S,Singh,Journal of Systems and Control Engineering, vol. 328, no. 3, 2014

These can be obtained directly from the design characteristics of the vessel or indirectly through an **identification procedure** in the sense of nonlinear least squares or nonlinear Kalman Filtering

However, since **adaptive control is a model-free control method**, there is **no need about prior knowledge** of these parameters' values..

Adaptive control assures stability of the control loop **under** unknown dynamic model parameters and unknown **external perturbations and disturbances** ..

3. Dynamic model of the submarine

The **dynamic model of the submarine** can be written in **matrix form**:

$$\begin{pmatrix} \dot{w} \\ \dot{Q} \end{pmatrix} = \begin{pmatrix} f_W(w, \theta, Q, t) \\ f_\theta(w, \theta, Q, t) \end{pmatrix} + B_o u \qquad (3)$$

where the **control input vector** is: $u = [\delta B(t) \;\; \delta S(t)]^T$

and is generated by **electric actuators** that rotate the hydroplanes. Therefore the control input describes actually **voltage or current signals** that define the turn angle of the rotor of these electric actuators.

This indicates clearly the significance of **electric actuators** in the **submarine's propulsion**.

In this description:

$$\begin{pmatrix} f_w(w, \theta, Q, t) \\ f_\theta(f_W(w, \theta, Q, t) \end{pmatrix} = M^{-1} \begin{pmatrix} \frac{Z_w' U}{L m_3'} w(t) + \frac{1}{m_3'} \dot{Z}_\theta' + m') U \dot{\theta}(t) + \frac{Z_{\dot{Q}}' L}{m_z'} \dot{Q}(t) + \frac{Z_d(t)}{0.5 \rho L^3 m_3'} + Z_\eta(w, q) \\ \frac{M_{\dot{w}}'}{L I_0'} \dot{w}(t) + \frac{M_v' U}{L^2 I_0'} w(t) + \frac{M_{\dot{\theta}}' U}{L I_0'} \dot{\theta}(t) + \frac{2mg(z_G - z_B)}{\rho L^5 I_0'} \theta(t) + \frac{M_d(t)}{0.5 \rho L^5 I_0'} + M_\eta(w, q) \end{pmatrix}$$

while for matrices M and B_o it holds

$$M = \begin{pmatrix} 1 & -Z_{\dot{Q}} L / m_3' \\ -M_{\dot{w}} (L I_2'^{\,-1}) & 1 \end{pmatrix} \quad B_o = \begin{pmatrix} \frac{Z_{\delta B}' U^2}{m_3' L} & \frac{Z_{\delta S}' U^2}{m_3' L} \\ \frac{M' \delta B U^2}{L^2 I_2'} & \frac{M' \delta S U^2}{L^2 I_2'} \end{pmatrix}$$

3. Dynamic model of the submarine

It holds that the **depth of the vessel** measured **in the inertial reference frame** and the velocity **w** of the submarine along the z-axis of the **body-fixed frame** are related as follows:

$$\begin{gathered}\dot{h}=w\cos(\theta)-U_o\sin(\theta)\Rightarrow\\ \ddot{h}=\dot{w}\cos(\theta)-w\sin(\theta)\dot{\theta}-U_o\cos(\theta)\dot{\theta}\Rightarrow\\ \ddot{h}=\dot{w}\cos(\theta)-wQ\sin(\theta)-U_oQ\cos(\theta)\end{gathered} \quad (4)$$

From the above relation one can compute about the **diving speed** of the vessel:

$$w=(\cos(\theta)^{-1})(\dot{h}+U_o\sin(\theta)) \quad (5)$$

Moreover, from Eq (3) one has:

$$\begin{gathered}\dot{w}=f_w(w,\theta,Q,t)+B_{o_{11}}u_1+B_{o_{12}}u_2\\ \dot{Q}=f_\theta(w,\theta,Q,t)+B_{o_{21}}u_1+B_{o_{22}}u_2\end{gathered} \quad (6)$$

Substituting Eq. (5) and the first row of Eq. (6) into Eq. (4) one gets

$$\ddot{h}=[f_w(w,\theta,Q,t)+B_{o_{11}}u_1+B_{o_{12}}u_2]\cos(\theta)-\frac{(\dot{h}+U_0\sin(\theta))}{\cos(\theta)}Q\sin(\theta)-U_0Q\cos(\theta) \quad (7)$$

3. Dynamic model of the submarine

Next, by denoting:

$$f_w(w,\theta,Q,t)=g_h(h,\dot{h},\theta,\dot{\theta},t)$$
$$f_\theta(w,\theta,Q,t)=g_\theta(h,\dot{h},\theta,\dot{\theta},t)$$

And by substituting this relation in Eq. 7, together with $Q=\dot{\theta}$ one obtains:

$$\ddot{h}=g_h(h,\dot{h},\theta,\dot{\theta},t)cos(\theta)-\frac{(\dot{h}+U_0 sin(\theta))}{cos(\theta)}\dot{\theta}sin(\theta)-U_0\dot{\theta}cos(\theta)+$$
$$+B_{0_{11}}cos(\theta)u_1+B_{0_{12}}cos(\theta)u_2$$

$$\ddot{\theta}=g_\theta(h,\dot{h},\theta,\dot{\theta},t)+B_{0_{21}}u_1+B_{0_{22}}u_2$$

Then, by defining the **state vector** $x=[h,\dot{h},\theta,\dot{\theta}]^T$

$$\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_3\end{pmatrix}=\begin{pmatrix}g_b(x,t)cos(x_3)-\frac{x_4+U_0 sin(x_3)}{cos(x_3)}x_4 sin(x_3)-U_0x_4cos(x_3)\\ g_\theta(x,t)\end{pmatrix}+\begin{pmatrix}B_{0_{11}} & B_{0_{12}}\\ B_{0_{21}} & B_{0_{22}}\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} \quad (8)$$

From Eq. 8 one finally arrives at the **MIMO state-space description** of the submarine

$$\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_3\end{pmatrix}=\begin{pmatrix}f_1(x,t)\\ f_2(x,t)\end{pmatrix}+\begin{pmatrix}g_{11}(x,t) & g_{12}(x,t)\\ g_{21}(x,t) & g_{22}(x,t)\end{pmatrix}\begin{pmatrix}u_1\\ u_2\end{pmatrix} \quad (9)$$

4. Differential flatness of the submarine's dynamic model

Next, by denoting the **flat output of the submarine** as:

$$y = [x_1, x_3]^T = [h, \theta]^T$$

it can be proven that the submarine's dynamic model is a **differentially flat** one

This means that all its **state variables** and its **control inputs** can be expressed as **differential functions of the flat output**

From Eq. 9 one gets $x_2 = \dot{x}_1$ and $x_4 = \dot{x}_3$, which means

$$\begin{aligned} x_2 &= [1\ 0]\dot{y} \\ x_4 &= [0\ 1]\dot{y} \end{aligned} \qquad (10)$$

Again, from Eq. 9 one gets

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} g_{11}(x) & g_{12}(x) \\ g_{21}(x) & g_{22}(x) \end{pmatrix}^{-1} \left(\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} - \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} \right)$$

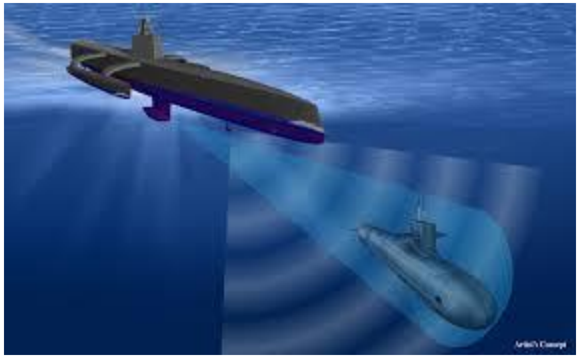
which means

$$\begin{aligned} u_1 &= f_a(y, \dot{y}, \ddot{y}) \\ u_2 &= f_b(y, \dot{y}, \ddot{y}). \end{aligned} \qquad (11)$$

Eq. 10 and Eq. 11 confirm that the submarine's model is a differentially flat one.

4. Differential flatness of the submarine's dynamic model

The differential flatness property of the submarine's model is important because it means that the vessel's model can be transformed into the **MIMO linear canonical (Brunovsky) form** through a change of its state variables (diffeomorphism)



By defining the **new state variables of the vessel**

$$y_1 = x_1, \quad y_2 = \dot{y}_1, \quad y_3 = x_2, \quad y_4 = \dot{y}_3$$

and by defining **the transformed control inputs** of the vessel

$$\begin{aligned} v_1 &= f_1(x,t) + g_{11}u_1 + g_{12}u_2 \\ v_2 &= f_2(x,t) + g_{21}u_1 + g_{22}u_2 \end{aligned} \quad (12)$$

one obtains the **linearized and decoupled state-space model** of the submarine

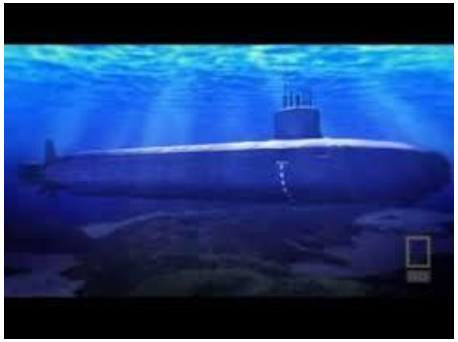
$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (13)$$

for which the design of a **state-feedback controller** is possible

5. Design of a stabilizing feedback controller for the submarine

For the **transformed state-space model** of the vessel

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \quad (13)$$



It is considered that the complete state vector is measurable

$$y = [h, \dot{h}, \theta, \dot{\theta}]$$

Then, to succeed tracking of the **reference setpoint**

$$y^d = [y_1^d, y_2^d, y_3^d, y_4^d]^T = [x_1^d, \dot{x}_1^d, x_2^d, \dot{x}_2^d]^T$$

the **feedback control inputs** should be chosen as

$$\begin{aligned} v_1 &= \ddot{y}_1^d - k_d^1(\dot{y}_1 - \dot{y}_1^d) - k_p^1(y_1 - y_1^d) \\ v_2 &= \ddot{y}_3^d - k_d^2(\dot{y}_3 - \dot{y}_3^d) - k_p^2(y_3 - y_3^d) \end{aligned} \quad (14)$$

5. Design of a stabilizing feedback controller for the submarine

By substituting Eq. 14 Into Eq. 13 one obtains the

tracking error dynamics for the submarine

$$\ddot{e}_1 + k_d^1 \dot{e}_1 + k_p^1 e_1 = 0 \qquad \ddot{e}_2 + k_d^2 \dot{e}_2 + k_p^2 e_2 = 0 \qquad (15)$$

where the tracking error is defined as $e_1 = y_1 - y_1^d, \quad e_2 = y_3 - y_3^d$

By selecting the feedback control gains $k_p^i, k_d^i \quad i = 1,2$ so as the **characteristic polynomials**

$$p_1(s) = s^2 + k_d^1 s + k_p^1 \qquad p_2(s) = s^2 + k_d^2 s + k_p^2 \qquad (16)$$

to have roots explicitly in the **left complex semiplane**, it is assured that

$$\lim_{t \to \infty} e_i(t) = 0 \qquad i = 1,2$$

Finally, the feedback **control input** that is **actually exerted on the submarine** is .

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} g_{11}(x,t) & g_{12}(x,t) \\ g_{21}(x,t) & g_{22}(x,t) \end{pmatrix}^{-1} [\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} - \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix}] \qquad (17)$$

6. Differential flatness of MIMO marine systems

- **Differential flatness theory** has been developed as a **global linearization control method** by M. Fliess (Ecole Polytechnique, France) and co-researchers (Lévine, Rouchon, Mounier, Rudolph, Petit, Martin, Zhu, Sira-Ramirez et. al)

- A dynamical system can be written in the ODE form $S_i(w, \dot{w}, \ddot{w}, ..., w^{(i)}),\ \ i=1,2,...,q$

  where $w^{(i)}$ stands for the i-th derivative of either a state vector element or of a control input

- The system is said to be **differentially flat** with respect to the **flat output**

$$y_i = \phi(w, \dot{w}, \ddot{w}, ..., w^{(a)}),\ i=1,...,m \quad \text{where} \quad y=(y_1, y_2, ..., y_m)$$

if the following two conditions are satisfied

(i) There does not exist any differential relation of the form

$$R(y, \dot{y}, \ddot{y}, ..., y^{(\beta)})=0$$

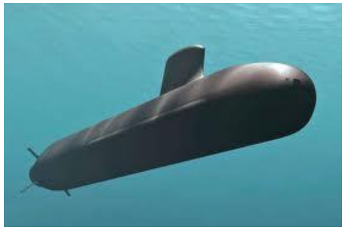
which means that **the flat output and its derivatives are linearly independent**

(ii) All system variables are **functions of the flat output and its derivatives**

$$w^{(i)} = \psi(y, \dot{y}, \ddot{y}, ..., y^{(\gamma_i)})$$

6. Differential flatness of MIMO marine systems

The proposed adaptive control method is based on the **transformation** of the vessel's model into the **linear canonical form**, and this transformation is succeeded by exploiting the system's differential flatness properties



- **All single input vessel models** are differentially flat and can be transformed into the linear canonical form

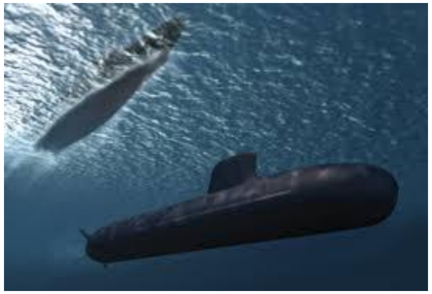
One has to define also which are the **MIMO vessel models** which are differentially flat.

- Differential flatness holds for **MIMO vessel models** that admit **static feedback linearization.**and which can be transformed into the linear canonical form through a change of variables (diffeomorphism) and feedback of the state vector. This **is the case of the submarine's model**

- Differential flatness holds for **MIMO vessel models** that admit **dynamic feedback linearization**, This **is the case of underactuated vessel models** (e.g. hovercraft) In the latter case the state vector of the system is extended by considering as additional flat outputs some of the control inputs and their derivatives

- Finally, a more rare case is the so-called **Liouvillian systems**. These are systems for which differential flatness properties hold for part of their state vector (constituting a flat subsystem) while the non-flat state variables can be obtained by integration of the elements of the flat subsystem.

7. Design of an adaptive controller for the submarine's model



For the **differentially flat MIMO model of the submarine** one has the dynamics

$$\ddot{x}_1 = f_1(x,t) + g_1(x,t)u + \tilde{d}_1$$
$$\ddot{x}_3 = f_2(x,t) + g_2(x,t)u + \tilde{d}_2$$

The following **control input is** considered
$$u = \begin{bmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{bmatrix} - \begin{bmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{bmatrix} - \begin{bmatrix} K_1^T \\ K_2^T \end{bmatrix} e + \begin{bmatrix} u_{c_1} \\ u_{c_2} \end{bmatrix} \right\}$$

where $\hat{f}$ and $\hat{g}$ stand for estimates of the unknown nonlinear terms f and g

These estimates are provided by neurofuzzy approximators or other nonlinear regressors

This results in **tracking error dynamics** of the form

$$\dot{e} = (A - BK^T)e + Bu_c + B\left\{ \begin{bmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t) \end{bmatrix} + \begin{bmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{bmatrix} \begin{bmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{bmatrix}^{-1} u + \tilde{d} \right\}$$

where matrices A,B,K are defined as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, K^T = \begin{bmatrix} K_1^1 & K_2^1 & K_3^1 & K_4^1 \\ K_1^2 & K_2^2 & K_3^2 & K_4^2 \end{bmatrix}$$

7. Design of an adaptive controller for the submarine's model

The **nonlinear regressors** (neurofuzzy approximators) **consist** of the **kernel functions** and **weights functions.** Unlike SISO systems, in the case of MIMO dynamics the kernel and weights functions are not represented as vectors but **take the form of matrices**. Thus one has:

$$\hat{f}(x|\theta_f)=\Phi_f(x)\theta_f \quad \text{and} \quad \hat{g}(x|\theta_g)=\Phi_g(x)\theta_g$$

Kernel and weights functions for the **approximation of the nonlinear dynamics *f*:**

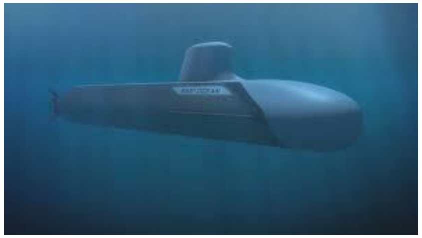
$$\Phi_f(x)=\begin{bmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{bmatrix} \qquad \theta_f^T=\begin{bmatrix}\theta_f^1 & \theta_f^2 & \cdots & \theta_f^N\end{bmatrix}$$

Kernel and weights functions for the **approximation of the nonlinear dynamics *g***:

$$\Phi_g(x)=\begin{bmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{bmatrix} \qquad \theta_g=\begin{bmatrix} \theta_{g1}^1 & \theta_{g1}^2 & \cdots & \theta_{g1}^p \\ \theta_{g2}^1 & \theta_{g2}^2 & \cdots & \theta_{g2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{gN}^1 & \theta_{gN}^2 & \cdots & \theta_{gN}^p \end{bmatrix}$$

7. Design of an adaptive controller for the submarine's model

The **weight functions** of the neurofuzzy approximators **are learned** through an **adaptation procedure** that is determined by **Lyapunov stability analysis** for the submarine's model.



The following quadratic **Lyapunov function** is defined:

$$V=\frac{1}{2}e^T Pe+\frac{1}{2\gamma_1}\tilde{\theta}_f^T\tilde{\theta}_f+\frac{1}{2\gamma_2}tr[\tilde{\theta}_g^T\tilde{\theta}_g]$$

e: state vector tracking error

$\tilde{\theta}_f=\theta_f-\theta_f^*$: Difference of the weights from the value that succeeds exact estimation of *f*

$\tilde{\theta}_g=\theta_g-\theta_g^*$: Difference of the weights from the value that succeeds exact estimation of *g*

Differentiating one obtains: $\dot{V}=\frac{1}{2}\dot{e}^T Pe+\frac{1}{2}e^T P\dot{e}+\frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f+\frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]$

The associated tracking error dynamics is:

$$\dot{e}=(A-BK^T)e+Bu_c+B\{\begin{bmatrix}f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{bmatrix}+\begin{bmatrix}g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{bmatrix}\begin{bmatrix}\hat{g}_1(x,t)\\ \hat{g}_2(x,t)\end{bmatrix}^{-1}u+\tilde{d}\}$$

The **effect of modelling errors** is denoted by:

$$w=\begin{bmatrix}f_1(x,t)-\hat{f}_1(x,t)\\ f_2(x,t)-\hat{f}_2(x,t)\end{bmatrix}+\begin{bmatrix}g_1(x,t)-\hat{g}_1(x,t)\\ g_2(x,t)-\hat{g}_2(x,t)\end{bmatrix}\begin{bmatrix}\hat{g}_1(x,t)\\ \hat{g}_2(x,t)\end{bmatrix}^{-1}u$$

7. Design of an adaptive controller for the submarine's model

Thus one obtains the following **tracking error dynamics**:

$$\dot{e} = (A - BK^T)e + Bu_c + B(w + \tilde{d})$$

The **first derivative of the Lyapunov function** becomes:

$$\dot{V} = \frac{1}{2}\{e^T(A - BK^T)^T + u_c^T B^T + (w + \tilde{d})^T B^T\}Pe + \frac{1}{2}e^T P\{(A - BK^T)e + Bu_c + B(w + \tilde{d})\}$$

$$+\frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

and after intermediate terms substitution one obtains:

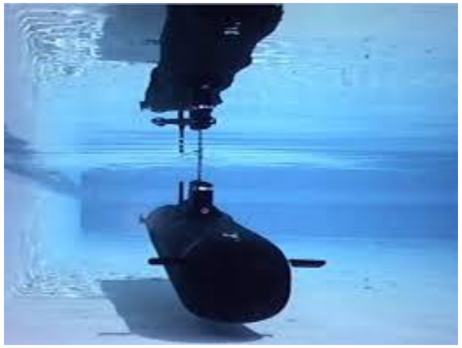
$$\dot{V} = \frac{1}{2}e^T\{(A - BK^T)^T P + P(A - BK^T)\}e + \frac{1}{2}2e^T PBu_c + \frac{1}{2}2B^T Pe(w + \tilde{d})$$

$$+\frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Assumption 1: the positive definite and symmetric matrix P is chosen as solution of the Riccati equation:

$$(A - BK^T)^T P + P(A - BK^T) - PB(\frac{2}{r} - \frac{1}{\rho^2})B^T P + Q = 0 \quad (18)$$

7. Design of an adaptive controller for the submarine's model



Using as supervisory control input $u_c = -\frac{1}{r} B^T Pe$ one obtains:

$$\dot{V} = \frac{1}{2} e^T \{-Q + PB(\frac{2}{r} - \frac{1}{\rho^2}) B^T P\} e + e^T PB\{-\frac{1}{r} B^T Pe\} + B^T P(w + \tilde{d}) +$$

$$+ \frac{1}{\gamma_1} \dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

which can be written in the form:

$$\dot{V} = -\frac{1}{2} e^T Qe - \frac{1}{2\rho^2} e^T PBB^T Pe + e^T PB(w + \tilde{d}) + \frac{1}{\gamma_1} \dot{\tilde{\theta}}_f^T \tilde{\theta}_f + \frac{1}{\gamma_2} tr[\dot{\tilde{\theta}}_g^T \tilde{\theta}_g]$$

Next, substituting: $\dot{\tilde{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f$ and $\dot{\tilde{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g$

i.e: $\dot{\theta}_f = -\gamma_1 \Phi(x)^T B^T Pe$ and $\dot{\theta}_g = -\gamma_2 \Phi(x)^T B^T Peu^T$

the following form of the **derivative of the Lyapunov function** is obtained:

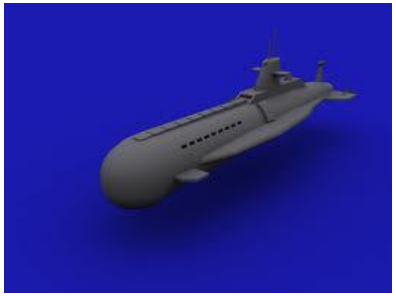
$$\dot{V} = -\frac{1}{2} e^T Qe - \frac{1}{2\rho^2} e^T PBB^T Pe + e^T PB(w + \tilde{d}) +$$

$$+ \frac{1}{\gamma_1} (-\gamma_1) e^T PB\Phi(x)(\theta_f - \theta_f^*) + \frac{1}{\gamma_2} (-\gamma_2) tr[ue^T PB\Phi(x)(\theta_g - \theta_g^*)]$$

7. Design of an adaptive controller for the submarine's model

Taking into account that $u \in R^{2\times 1}$ and $e^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)) \in R^{1\times 2}$

the following form is obtained for the **Lyapunov function derivative** :



$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + e^T PB(w + \tilde{d}) +$$

$$+\frac{1}{\gamma_1}(-\gamma_1)e^T PB\Phi(x)(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(-\gamma_2)tr[e^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u]$$

and since $e^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u \in R^{1\times 1}$ it holds that

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + e^T PB(w + \tilde{d}) +$$

$$+\frac{1}{\gamma_1}(-\gamma_1)e^T PB\Phi(x)(\theta_f - \theta_f^*) + \frac{1}{\gamma_2}(-\gamma_2)e^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u$$

Using the following description for the model approximation error:

$$w_a = [\hat{f}(x|\theta_f^*) - \hat{f}(x|\theta_f)] + [\hat{g}(x|\theta_f^*) - \hat{g}(x|\theta_f)]u$$

the equation of the Lyapunov function derivative becomes:

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + e^T PB(w + \tilde{d}) + e^T PBw_a$$

7. Design of an adaptive controller for the submarine's model

and denoting the disturbances and modelling error terms as: $w_1 = w + \tilde{d} + w_a$

one has: $$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T e + e^T PBw_1$$

or: $$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T e + \frac{1}{2}e^T PBw_1 + \frac{1}{2}w_1^T B^T Pe$$

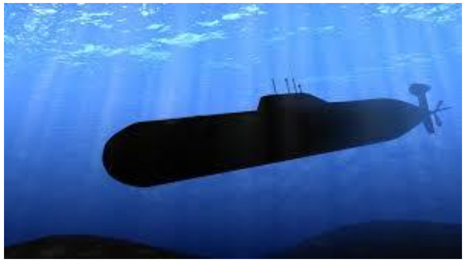
Next **the following inequality** is used:

Lemma: It holds that $$\frac{1}{2}e^T Pw_1 + \frac{1}{2}w_1^T B^T Pe - \frac{1}{2\rho^2}e^T PBB^T Pe \leq \frac{1}{2}\rho^2 w_1^T w_1 \qquad (19)$$

Proof:

The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow$$
$$ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2$$



By substituting $a = w_1$ and $b = \bar{e}^T P_2 B$ one gets

$$\frac{1}{2}w_1^T B^T P_2 \bar{e} + \frac{1}{2}\bar{e}^T P_2 B w_1 - \frac{1}{2\rho^2}\bar{e}^T P_2 BB^T P_2 \bar{e}$$
$$\leq \frac{1}{2}\rho^2 w_1^T w_1$$

7. Design of an adaptive controller for the submarine's model

By substituting Eq. 19 into the relation of the derivative of the Lyapunov function gives:

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 w_1^T w_1 \qquad (20)$$

This is the **H-infinity tracking performance criterion** which means that for bounded disturbance and modelling error the control law results in very small bounded tracking error:

It is noted that, by choosing the **attenuation coefficient ρ** to be sufficiently small, the right part of Eq. 20 can be always made to be upper bounded by zero.

In such a case the **asymptotic stability condition** is clear to hold..

The minimum value of ρ for which a solution of the Riccati Eq. 18 exists, is the one that provides the control loop with maximum robustness.

Moreover, if $\int_0^{\infty} \| w_1 \|^2 dt \leq M_w$ one has the following integral:

$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T \| e(t) \|^2 dt + \frac{1}{2}\rho^2 \int_0^T \| w_1 \|^2 \, dt \Rightarrow 2V(T) + \int_0^T \| e(t) \|_Q^2 \, dt \leq 2V(0) + \rho^2 \int_0^T \| w_1 \|^2 \, dt$$

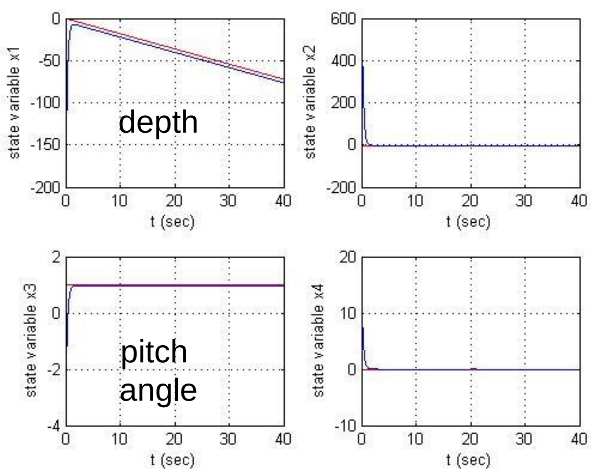
which means that: $\int_0^{\infty} \| e \|_Q^2 \, dt \leq 2V(0) + \rho^2 M_w$ and from **Barbalat's Lemma** one has that

$\lim_{t \to \infty} e(t) = 0$ which confirms again that the tracking error vanishes

8. Simulation tests

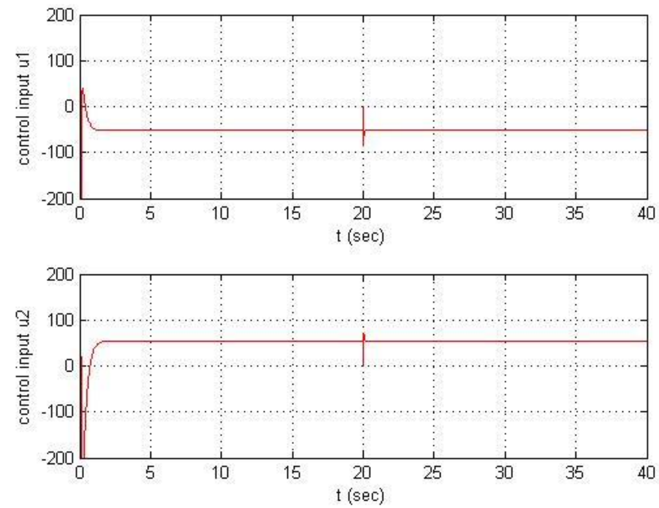
• In the simulation tests, the **dynamic model of the submarine** was considered to be **completely unknown** and was identified in real-time by the previously analyzed nonlinear regressors

• The **estimated unknown dynamics** of the system **was used in the computation of the control inputs** (generated by the electric actuators of the hydroplanes) which were finally exerted on the submarine's model.



state variables $x_i, \; i = 1, \cdots, 4$

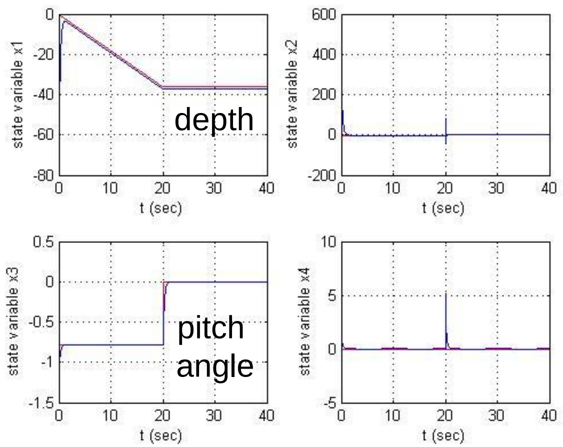
Setpoint 1:



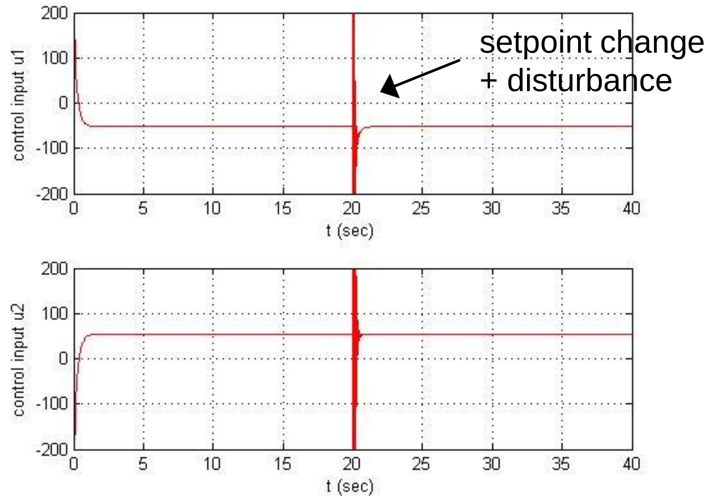
Variations of the control inputs

8. Simulation tests

Setpoint 2:

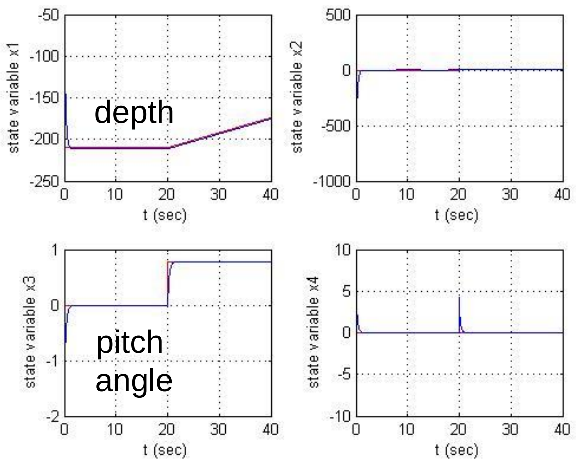


state variables $x_i,\ i = 1, \cdots, 4$

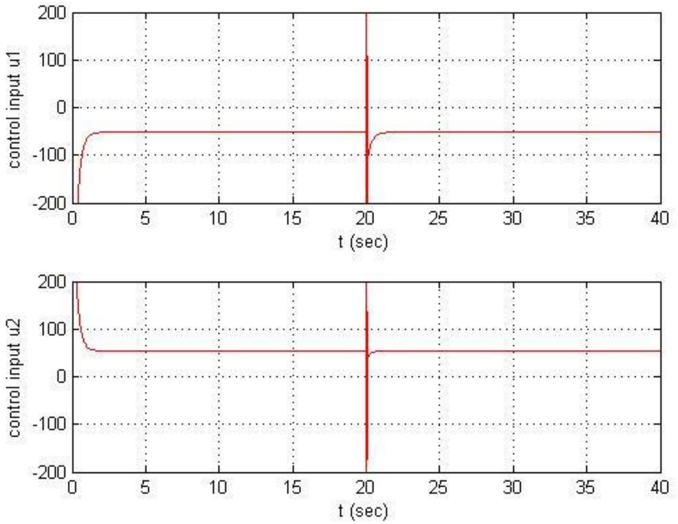


Variations of the control inputs

Setpoint 3:



state variables $x_i,\ i = 1, \cdots, 4$



Variations of the control inputs

9. Conclusions

• By exploiting the **differential flatness** properties of the **MIMO nonlinear model of the submarine** the system was transformed into the **linear canonical (Brunovsky) form.** For the latter description the design of a feedback controller was possible.

• Moreover, to cope with **unknown nonlinear terms** appearing in the new control inputs of the transformed state-space description of the submarine, the use of nonlinear regressors (neurofuzzy approximators) has been proposed..

• These estimators were online trained to identify the unknown dynamics of the system and the associated learning procedure was determined by the requirement the derivative of the system's Lyapunov function to be a negative one.

• Through **Lyapunov stability analysis** it was proven that the closed loop satisfies the **H-infinity tracking performance criterion**, and this assures improved robustness against model uncertainties and external perturbations.

• The computation of the control input required the **solution of an algebraic Riccati equation**. Suitable selection of the attenuation coefficient ρ in this equation assures asymptotic stability and provides maximum robustness.

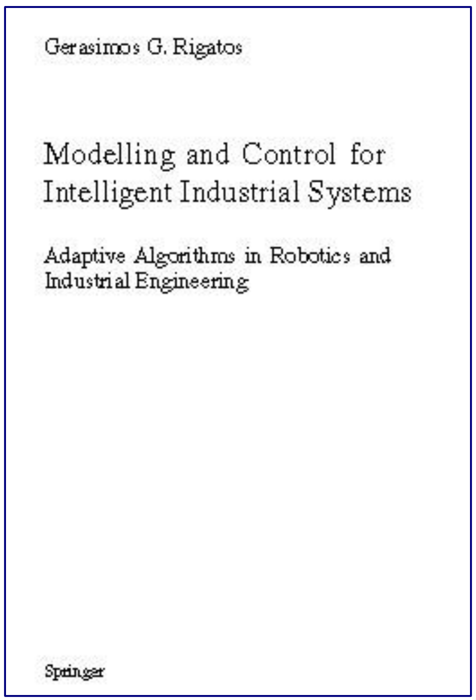
• The proposed flatness-based adaptive fuzzy control method is generic and **can be applied to a wide class of vessels**, such as surface vessels or AUVs and submersibles,inIcluding also the case of underactuated vessels.

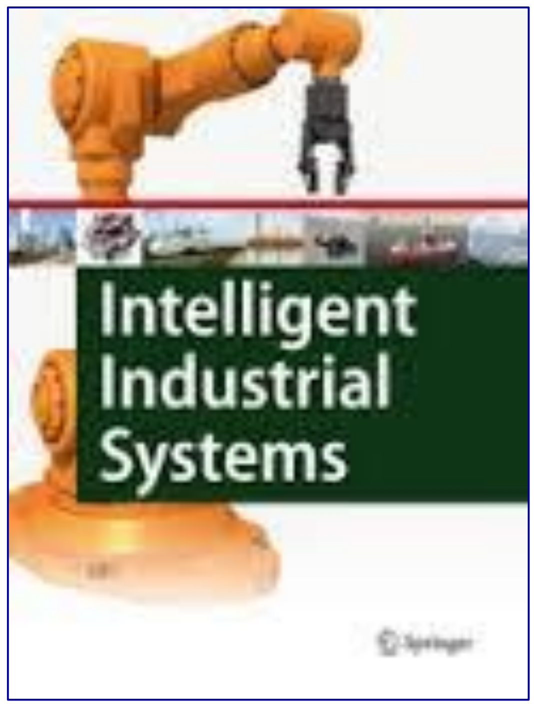
Conclusions

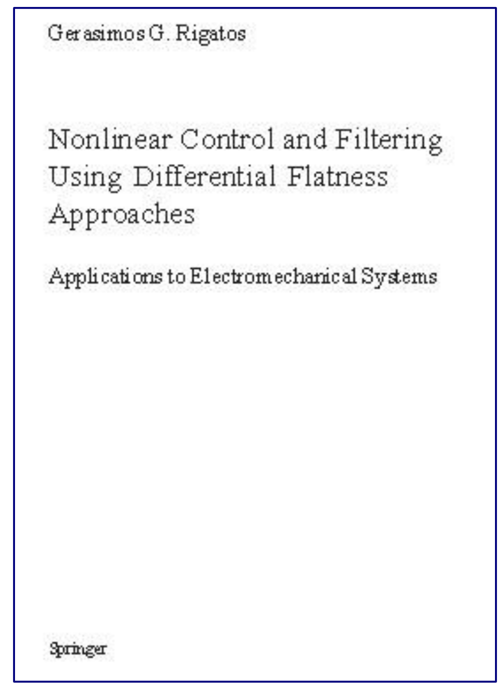
- **Adaptive control for marine systems dynamics** and **ship propulsion** is well addressed by **differential flatness theory-based methods**. Recommended books for future reference are:

[1] G. Rigatos, **Modelling and control for intelligent industrial systems: adaptive algorithms In Robotics and Industrial Engineering**, Springer, 2011

[2] G. Rigatos, **Nonlinear control and filtering using differential flatness proaches: applications to electromechanical systems**, Springer 2015.







Thank you for your attention