Lecture on

New approaches to nonlinear control of electric power systems:

Approximate linearization methods

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1. Outline

• A **new nonlinear H-infinity control method** for stabilization and synchronization of **distributed interconnected synchronous generators**.

• At first stage **local linearization** of the distributed generators' model is performed round its present operating point.

• The **approximation error** that is introduced to the linearized model is due to **truncation of higher-order terms in the performed Taylor series expansion** and is represented as a disturbance.

• The control problem is now formulated as **a mini-max differential game** in which the control input tries to minimize the state vector's tracking error while the disturbances affecting the model try to maximize it.

• Using the linearized description of the distributed generators' dynamics an **H-infinity feedback controller** is designed through the solution of a **Riccati equation** at each step of the control algorithm.

• The **inherent robustness properties of H-infinity control** assure that the disturbance effects will be eliminated and the state variables of the individual power generators will converge to the desirable setpoints.

• The proposed method, stands for a reliable solution to the problem of **nonlinear control** and stabilization for **interconnected synchronous generators**.

2. The model of the distributed synchronous generators

The **dynamic model of the distributed power generation units** is assumed to be that of synchronous generators. The modelling approach is also applicable to PMSGs (permanent magnet synchronous generators) which are a special case of synchronous electric machines.

$$\dot{\delta} = \omega$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \frac{\omega_0}{2J}(P_m - P_e) \qquad (1)$$

δ : turn angle of the rotor

ω : turn speed of the rotor

ω_0 : synchronous speed

J : moment of inertia of the rotor

P_e : active electrical power of the machine

P_m : mechanical power of the machine

D : damping coefficient

T_e : electromagnetic torque

The **generator's electrical dynamics** is:

$$\dot{E}_q' = \frac{1}{T_{d_o}}(E_f - E_q) \qquad (2)$$

E_q' is the quadrature-axis transient voltage (a variable related to the magnetic flux)

E_q is quadrature axis voltage of the generator

T_{d_o} is the direct axis open-circuit transient time constant

E_f

2. The model of the distributed synchronous generators

The **synchronous generator's model** is complemented by a set of algebraic equations:

$$E_q = \frac{x_{d_\Sigma}}{x'_{d_\Sigma}} E'_q - (x_d - x'_d)\frac{V_s}{x'_{d_\Sigma}} cos(\Delta\delta)$$
$$I_q = \frac{V_s}{x'_{d_\Sigma}} sin(\Delta\delta)$$
$$I_d = \frac{E'_q}{x'_{d_\Sigma}} - \frac{V_s}{x'_{d_\Sigma}} cos(\Delta\delta) \qquad (3)$$
$$P_e = \frac{V_s E'_q}{x'_{d_\Sigma}} sin(\Delta\delta)$$
$$Q_e = \frac{V_s E'_q}{x'_{d_\Sigma}} cos(\Delta\delta) - \frac{V_s^2}{x_{d_\Sigma}}$$
$$V_t = \sqrt{(E'_q - X'_d I_d)^2 + (X'_d I_q)^2}$$

where: $x_{d_\Sigma} = x_d + x_T + x_L \qquad x'_{d_\Sigma} = x'_d + x_T + x_L$

x_d : direct-axis synchronous reactance

x_T : reactance of the transformer

x'_d : direct-axis transient reactance

x_L : transmission line reactance

I_d and I_q : direct and quadrature axis currents

V_s : infinite bus voltage

Q_e : reactive power of the generator

V_t : terminal voltage of the generator

2. The model of the distributed synchronous generators

From Eq. 1 and Eq. 2 one obtains the **dynamic model of the synchronous generator**:

$$\dot{\delta} = \omega - \omega_0$$
$$\dot{\omega} = -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E_q'}{x_{d_\Sigma}'} sin(\Delta\delta)$$
$$\dot{E}_q' = -\frac{1}{T_d'} E_q' + \frac{1}{T_{d_o}} \frac{x_d - x_d'}{x_{d_\Sigma}'} V_s cos(\Delta\delta) + \frac{1}{T_{d_o}} E_f$$

Moreover, the generator can be written in a **state-space form**:

$$\dot{x} = f(x) + g(x)u$$

where the state vector is $x = (\Delta\delta \quad \Delta\omega \quad E_q')^T$ and

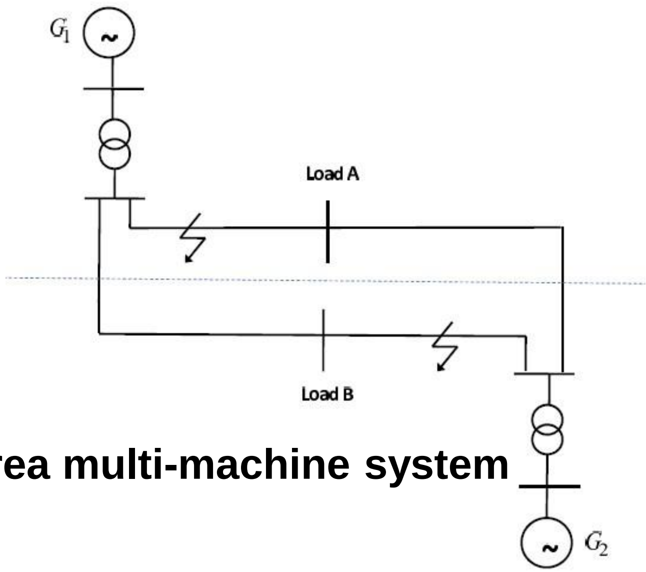
$$f(x) = \begin{pmatrix} \omega - \omega_0 \\ -\frac{D}{2J}(\omega - \omega_0) + \omega_0 \frac{P_m}{2J} - \omega_0 \frac{1}{2J} \frac{V_s E_q'}{x_{d\Sigma}'} sin(\Delta\delta) \\ -\frac{1}{T_d'} E_q' + \frac{1}{T_{do}} \frac{x_d - x_d'}{x_{d\Sigma}'} V_s cos(\Delta\delta) \end{pmatrix}$$

$$g(x) = (0 \quad 0 \quad \frac{1}{T_{do}})^T$$

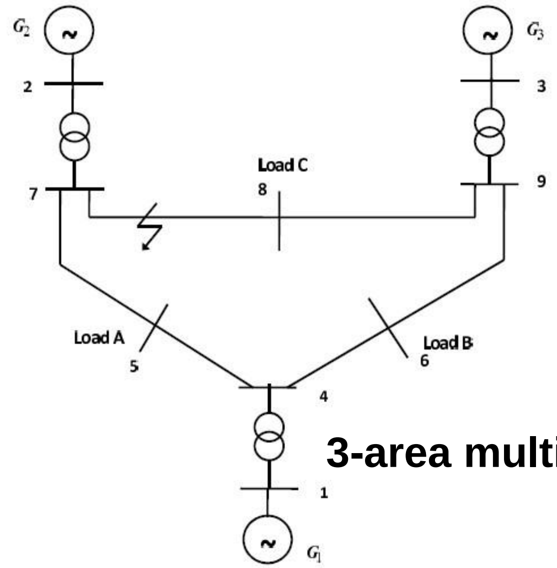
while the system's output is $y = h(x) = \delta - \delta_0$

2. The model of the distributed synchronous generators

The **interconnection between distributed power generators** results into a **multi-area multi-machine power system** model



2-area multi-machine system



3-area multi-machine system

The dynamic model of a power system that comprises **n-interconnected power generators** is

$$\dot{\delta}_i = \omega_i - \omega_0$$

$$\dot{\omega}_i = -\frac{D_i}{2J_i}(\omega_i - \omega_0) + \omega_0 \frac{P_{m_i}}{2J_i} -$$

$$-\omega_0 \frac{1}{2J_i}[G_{ii}E_{qi}^{'\,2} + E_{qi}^{'} \sum_{j=1, j\neq i}^{n} E_{qj}^{'} G_{ij} sin(\delta_i - \delta_j - \alpha_{ij})]$$

$$\dot{E}_{q_i}^{'} = -\frac{1}{T_{d_i}^{'}} E_{q_i}^{'} + \frac{1}{T_{do_i}} \frac{x_{d_i} - x_{d_i}^{'}}{x_{d\Sigma_i}^{'}} V_{s_i} cos(\Delta\delta_i) + \frac{1}{T_{do_i}} E_{f_i}$$

2. The model of the distributed synchronous generators

For the case of the three interconnected generators one has the state-space equations:

$$\dot{x}_1 = x_2 - \omega_0$$

$$\dot{x}_2 = -\frac{D_1}{2J_1}(x_2 - \omega_0) + \omega_0\frac{P_{m_1}}{2J_1} -$$
$$-\frac{\omega_0}{2J_1}\{G_{11}x_3^2 + x_3[x_6 G_{12} sin(x_1 - x_4 - a_{12}) + x_9 G_{13} sin(x_1 - x_7 - a_{13})]$$

$$\dot{x}_3 = -\frac{1}{T'_{d_1}}x_3 + \frac{1}{T_{d_{o1}}}\frac{x_{d_1} - x'_{d_1}}{x'_{d_{\Sigma_1}}}V_s cos(x_1) + \frac{1}{T_{d_{o1}}}u_1$$

$$\dot{x}_4 = x_5 - \omega_0$$

$$\dot{x}_5 = -\frac{D_2}{2J_2}(x_5 - \omega_0) + \omega_0\frac{P_{m_2}}{2J_2} -$$
$$-\frac{\omega_0}{2J_2}\{G_{22}x_6^2 + x_6[x_3 G_{21} sin(x_4 - x_1 - a_{21}) + x_9 G_{23} sin(x_4 - x_7 - a_{23})]$$

$$\dot{x}_6 = -\frac{1}{T'_{d_2}}x_6 + \frac{1}{T_{d_{o2}}}\frac{x_{d_2} - x'_{d_2}}{x'_{d_{\Sigma_2}}}V_s cos(x_4) + \frac{1}{T_{d_{o2}}}u_2$$

$$\dot{x}_7 = x_8 - \omega_0$$

$$\dot{x}_8 = -\frac{D_2}{2J_3}(x_8 - \omega_0) + \omega_0\frac{P_{m_3}}{2J_3} -$$
$$-\frac{\omega_0}{2J_3}\{G_{32}x_9^2 + x_9[x_3 G_{31} sin(x_7 - x_1 - a_{31}) + x_6 G_{32} sin(x_7 - x_4 - a_{32})]$$

$$\dot{x}_9 = -\frac{1}{T'_{d_3}}x_9 + \frac{1}{T_{d_{o3}}}\frac{x_{d_3} - x'_{d_3}}{x'_{d_{\Sigma_3}}}V_s cos(x_7) + \frac{1}{T_{d_{o3}}}u_3$$

2. The model of the distributed synchronous generators

The system of the interconnected generators is also written in the matrix form:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \end{pmatrix} = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \\ f_6(x) \\ f_7(x) \\ f_8(x) \\ f_9(x) \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_1(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & g_2(x) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_3(x) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

where

$$f_1(x) = x_2 - \omega_0$$

$$f_2(x) = -\frac{D_1}{2J_1}(x_2 - \omega_0) + \omega_0 \frac{P_{m_1}}{2J_1} - $$
$$-\frac{\omega_0}{2J_1}\{G_{11}x_3^2 + x_3[x_6 G_{12} sin(x_1 - x_4 - a_{12}) + x_9 G_{13} sin(x_1 - x_7 - a_{13})]$$

$$f_3(x) = -\frac{1}{T_{d_1}^{'}}x_3 + \frac{1}{T_{d_{o1}}}\frac{x_{d_1} - x_{d_1}^{'}}{x_{d_{\Sigma_1}}^{'}} V_s cos(x_1)$$

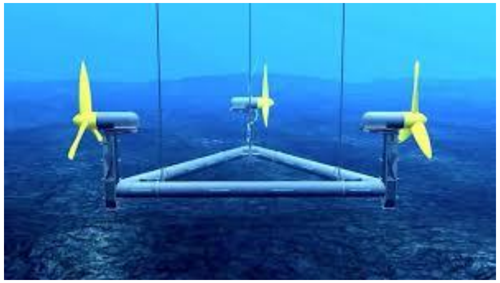
2. The model of the distributed synchronous generators

and also

$$f_4(x) = x_5 - \omega_0$$

$$f_5(x) = -\frac{D_2}{2J_2}(x_5 - \omega_0) + \omega_0 \frac{P_{m_2}}{2J_2} - \\ -\frac{\omega_0}{2J_2}\{G_{22}x_6^2 + x_6[x_3 G_{21} sin(x_4 - x_1 - a_{21}) + x_9 G_{23} sin(x_4 - x_7 - a_{23})]$$

$$f_6(x) = -\frac{1}{T'_{d_2}}x_6 + \frac{1}{T_{d_{o_2}}}\frac{x_{d_2} - x'_{d_2}}{x'_{d_{\Sigma_2}}}V_s cos(x_4)$$



$$f_7(x) = x_8 - \omega_0$$

$$f_8(x) = -\frac{D_2}{2J_3}(x_8 - \omega_0) + \omega_0 \frac{P_{m_3}}{2J_3} - \\ -\frac{\omega_0}{2J_3}\{G_{32}x_9^2 + x_9[x_3 G_{31} sin(x_7 - x_1 - a_{31}) + x_6 G_{32} sin(x_7 - x_4 - a_{32})]$$

$$f_9(x) = -\frac{1}{T'_{d_3}}x_9 + \frac{1}{T_{d_{o_3}}}\frac{x_{d_3} - x'_{d_3}}{x'_{d_{\Sigma_3}}}V_s cos(x_7)$$

$$g_1 = \frac{1}{T_{d_{o_1}}} \quad g_2 = \frac{1}{T_{d_{o_2}}} \quad g_3 = \frac{1}{T_{d_{o_3}}}u_3$$

3. Linearization of the model of the distributed synchronous generators

Local linearization is performed for the state-space model of the distributed power generators, round the operating point (x^*, u^*) where x^* is the present value of the system's state vector and u^* is the last value of the control input that was exerted on the machine.

Thus, one obtains the linearized description

$$\dot{x} = Ax + Bu + \tilde{d}$$

For the previous description of the distributed power generators' model by the state-space equation it holds that

$$A = \nabla_x[f(x) + g(x)u] = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \cdots & \frac{\partial f_1}{\partial x_9} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \cdots & \frac{\partial f_2}{\partial x_9} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \cdots & \frac{\partial f_3}{\partial x_9} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \cdots & \frac{\partial f_4}{\partial x_9} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_9}{\partial x_1} & \frac{\partial f_9}{\partial x_2} & \frac{\partial f_9}{\partial x_3} & \cdots & \frac{\partial f_9}{\partial x_9} \end{pmatrix}$$

Next, one computes the partial derivatives of the Jacobian matrix

3. Linearization of the model of the distributed synchronous generators

For the first row of the Jacobian matrix $A=\nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_1}{\partial x_1}=0$, $\frac{\partial f_1}{\partial x_2}=1$, $\frac{\partial f_1}{\partial x_3}=0$, $\frac{\partial f_1}{\partial x_4}=0$, $\frac{\partial f_1}{\partial x_5}=0$, $\frac{\partial f_1}{\partial x_6}=0$, $\frac{\partial f_1}{\partial x_7}=0$, $\frac{\partial f_1}{\partial x_8}=0$, $\frac{\partial f_1}{\partial x_9}=0$.

For the second row of the Jacobian matrix $A=\nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_2}{\partial x_1}=-\frac{\omega_0}{2J_1}x_3[x_6G_{12}cos(x_1-x_4-a_{13})]+x_9G_{13}cos(x_1-x_7-a_{13})$, $\frac{\partial f_2}{\partial x_2}=-\frac{D_1}{2J_1}$, $\frac{\partial f_2}{\partial x_3}=\frac{-\omega_0}{2J_1}\{G_{11}2x_3+[x_6]G_{12}sin(x_1-x_4-a_{12})+x_9G_{13}sin(x_1-x_7-a_{13})]\}$, $\frac{\partial f_2}{\partial x_4}=\frac{\omega_0}{2J_1}x_3x_6G_{12}cos(x_1-x_4-a_{12})$, $\frac{\partial f_2}{\partial x_5}=0$, $\frac{\partial f_2}{\partial x_6}=-\frac{\omega_0}{2J_1}x_3G_{12}sin(x_1-x_4-a_{12})$, $\frac{\partial f_2}{\partial x_7}=\frac{\omega_0}{2J_1}x_3x_9G_{13}cos(x_1-x_7-a_{13})$, $\frac{\partial f_2}{\partial x_8}=0$, $\frac{\partial f_2}{\partial x_9}=-\frac{\omega_0}{2J_1}x_3G_{13}sin(x_1-x_7-a_{13})$.

For the third row of the Jacobian matrix $A=\nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_3}{\partial x_1}=-\frac{1}{T_{do_1}}\frac{x_{d_1}-x'_{d_1}}{x'_{d\Sigma_1}}V_s sin(x_1)$, $\frac{\partial f_3}{\partial x_2}=0$, $\frac{\partial f_3}{\partial x_3}=-\frac{1}{T'_{d_1}}$, $\frac{\partial f_3}{\partial x_4}=0$, $\frac{\partial f_3}{\partial x_5}=0$, $\frac{\partial f_3}{\partial x_6}=0$, $\frac{\partial f_3}{\partial x_7}=0$, $\frac{\partial f_3}{\partial x_8}=0$, $\frac{\partial f_3}{\partial x_9}=0$.

For the fourth row of the Jacobian matrix $A=\nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_4}{\partial x_1}=0$, $\frac{\partial f_4}{\partial x_2}=0$, $\frac{\partial f_4}{\partial x_3}=0$, $\frac{\partial f_4}{\partial x_4}=0$, $\frac{\partial f_4}{\partial x_5}=1$, $\frac{\partial f_4}{\partial x_6}=0$, $\frac{\partial f_1}{\partial x_7}=0$, $\frac{\partial f_1}{\partial x_8}=0$, $\frac{\partial f_1}{\partial x_9}=0$.

3. Linearization of the model of the distributed synchronous generators

For the fifth row of the Jacobian matrix $A = \nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_5}{\partial x_1} = \frac{\omega_0}{2J2}x_6[x_3G_{21}cos(x_4-x_1-a_{21})+x_3G_{23}cos(x_4-x_7-a_{23})]$, $\frac{\partial f_5}{\partial x_2}=0$, $\frac{\partial f_5}{\partial x_3} = -\frac{\omega_0}{2J_2}x_6G_{21}sin(x_4-x_1-a_{21})$, $\frac{\partial f_5}{\partial x_4} = -\frac{\omega_0}{2J_2}x_6[x_3G_{21}cos(x_4-x_1-a_{21})+x_9G_{23}cos(x_4-x_7-a_{23})]$, $\frac{\partial f_5}{\partial x_5} = -\frac{D_2}{2J_2}$, $\frac{\partial f_5}{\partial x_6} = -\frac{\omega_0}{2J_2}\{G_{22}2x_6+[x_3G_{21}sin(x_4-x_1-a_{21})+x_9G_{23}sin(x_4-x_7-a_{23})]\}$, $\frac{\partial f_5}{\partial x_7} = \frac{\omega_0}{2J_2}x_6x_9G_{23}cos(x_4-x_7-a_{23})$, $\frac{\partial f_5}{\partial x_8}=0$, $\frac{\partial f_5}{\partial x_9} = -\frac{\omega_0}{2J_2}x_6G_{23}sin(x_4-x_7-a_{23})$.

For the sixth row of the Jacobian matrix $A = \nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_6}{\partial x_1}=0$, $\frac{\partial f_6}{\partial x_2}=0$, $\frac{\partial f_6}{\partial x_3}=$, $\frac{\partial f_6}{\partial x_4} = -\frac{1}{T_{do_2}}\frac{x_{d_2}-x'_{d_2}}{x'_{d_{\Sigma_2}}}V_s sin(x_4)$, $\frac{\partial f_6}{\partial x_5}=0$, $\frac{\partial f_6}{\partial x_6} = -\frac{1}{T'_{d_2}}$, $\frac{\partial f_6}{\partial x_7}=0$, $\frac{\partial f_6}{\partial x_8}=0$, $\frac{\partial f_6}{\partial x_9}=0$.

For the seventh row of the Jacobian matrix $A = \nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_7}{\partial x_1}=0$, $\frac{\partial f_7}{\partial x_2}=0$, $\frac{\partial f_7}{\partial x_3}=0$, $\frac{\partial f_7}{\partial x_4}=0$, $\frac{\partial f_7}{\partial x_5}=0$, $\frac{\partial f_7}{\partial x_6}=0$, $\frac{\partial f_7}{\partial x_7}=0$, $\frac{\partial f_7}{\partial x_8}=1$, $\frac{\partial f_7}{\partial x_9}=0$.

3. Linearization of the model of the distributed synchronous generators

For the eight row of the Jacobian matrix $A=\nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_8}{\partial x_1} = \frac{\omega_0}{2J_3}x_9[x_3G_{31}cos(x_7-x_1-a_{31})+x_6G_{32}cos(x_7-x_4-a_{32})]$, $\frac{\partial f_8}{\partial x_2}=0$, $\frac{\partial f_8}{\partial x_3} = -\frac{\omega_0}{2J_3}x_9G_{31}sin(x_7-x_1-a_{31})$, $\frac{\partial f_8}{\partial x_4}=\frac{\omega_0}{2J_3}x_9x_6G_{32}cos(x_7-x_4-a_{32})$, $\frac{\partial f_8}{\partial x_5}=0$, $\frac{\partial f_8}{\partial x_6} = -\frac{\omega_0}{2J_3}x_9G_{23}sin(x_7-x_4-a_{32})$, $\frac{\partial f_8}{\partial x_7}=-\frac{\omega_0}{2J_3}x_9[x_3G_{31}cos(x_7-x_1-a_{31})+x_6G_{32}cos(x_7-x_4-a_{32})]$, $\frac{\partial f_8}{\partial x_8}=-\frac{D_3}{2J3}$, $\frac{\partial f_8}{\partial x_9}=-\frac{\omega_0}{2J_3}\{G_{33}2x_9+[x_3G_{31}sin(x_7-x_1-a_{31})+x_6G_{32}sin(x_7-x_4-a_{32})]\}$.

For the ninth row of the Jacobian matrix $A=\nabla_x[f(x)+g(x)u]$ one has: $\frac{\partial f_9}{\partial x_1}=0$, $\frac{\partial f_9}{\partial x_2}=0$, $\frac{\partial f_9}{\partial x_3}=0$, $\frac{\partial f_9}{\partial x_4}=0$, $\frac{\partial f_9}{\partial x_6}=0$, $\frac{\partial f_9}{\partial x_7}=-\frac{1}{T'_{do_3}}\frac{x_{d_3}-x'_{d_3}}{x'_{d_{\Sigma_3}}}V_s sin(x_7)$, $\frac{\partial f_9}{\partial x_8}=0$, and finally $\frac{\partial f_9}{\partial x_9}=-\frac{1}{T'_{d_3}}$.

Moreover, it holds that

$$B=\nabla_u[f(x)+g(x)u]=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_1(x) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & g_2(x) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g_3(x) \end{pmatrix}$$

3. Linearization of the model of the distributed synchronous generators

Parameter d_1 stands for the **linearization error** in the distributed generators' model

$$\dot{x} = Ax + Bu + d_1 \qquad \text{A}$$

The **desirable trajectory** of the multi-generators system is denoted by

$$x_d = [x_{d_1}, x_{d_2}, x_{d_3}, \ldots, x_{d_7}, x_{d_8}, x_{d_9}]^T$$

Tracking of this trajectory is succeeded after applying the control input u^*

At every time instant the control input u^* is assumed to differ from the control input u appearing in A by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \qquad \text{B}$$

The dynamics of the system of Eq. A can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \qquad \text{C}$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \qquad \text{D}$$

3. Linearization of the model of the distributed synchronous generators

By subtracting Eq. D from Eq. A one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + \tilde{d}_3 - \tilde{d}_2 \quad \text{E}$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = \tilde{d}_3 - \tilde{d}_2$ the **tracking error dynamics** becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad \text{F}$$

The initial nonlinear system is assumed to be in the form

$$\dot{x} = f(x, u) \quad x \in R^n, \ u \in R^m$$

Linearization of the system is performed at each iteration of the control algorithm round its present operating point

$$(x^*, u^*) = (x(t), u(t - T_s)).$$

The **linearized equivalent of the system** is described by

$$\dot{x} = Ax + Bu + L\tilde{d} \quad x \in R^n, \ u \in R^m, \ \tilde{d} \in R^q$$

5. The nonlinear H-infinity control

where matrices A and B are obtained from the **computation of the Jacobians**

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Big|_{(x^*,u^*)} \qquad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \cdots & \frac{\partial f_2}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \cdots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Big|_{(x^*,u^*)}$$

and vector d denotes disturbance terms due to linearization errors.

The problem of **disturbance rejection** for the linearized model that is described by

$$\dot{x} = Ax + Bu + Ld$$
$$y = Cx$$

where $x \in R^n$, $u \in R^m$, $d \in R^q$ and $y \in R^p$ cannot be handled efficiently if the classical LQR control scheme is applied. This because of the existence of the perturbation term d.

In the $H\infty$ control approach, a **feedback control scheme** is designed for **trajectory tracking** by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner

5. The nonlinear H-infinity control

The disturbances' effect are incorporated in the following **quadratic cost function**

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + \\ + ru^T(t)u(t) - \rho^2 d^T(t)d(t)]dt, \quad r, \rho > 0$$

The coefficient r determines **the penalization of the control input** and the weight coefficient ρ determines the **reward of the disturbances**' effects. It is assumed that

Then, the **optimal feedback control** law is given by

$$u(t) = -Kx(t) \qquad \text{with} \qquad K = \frac{1}{r}B^T P$$

where ***P* is a positive semi-definite symmetric matrix** which is obtained from the solution of the **Riccati equation**

$$A^T P + PA + Q - P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P = 0$$

where Q is also a positive definite symmetric matrix.

The parameter ρ in Eq. (15), is an **indication of the closed-loop system robustness**. If the values of ρ> 0 are excessively decreased with respect to r, then the solution of the Riccati equation is no longer a positive definite matrix. Consequently, there is a lower bound ρ_{min} of for which the H-infinity control problem has a solution.

6. Lyapunov stability analysis

The **tracking error dynamics** for the distributed power generation system is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d}$$

where in the distributed generators' application example $L = I \in R^2$ with I being the identity matrix. The following **Lyapunov function** is considered

$$V = \frac{1}{2}e^T Pe$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}eP\dot{e} \Rightarrow$$

$$\dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow$$

$$\dot{V} = \frac{1}{2}e^T A^T Pe + \frac{1}{2}u^T B^T Pe + \frac{1}{2}\tilde{d}^T L^T Pe + \\ \frac{1}{2}e^T PAe + \frac{1}{2}e^T PBu + \frac{1}{2}e^T PL\tilde{d}$$

6. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(A^TP + PA)e + (\frac{1}{2}u^TB^TPe + \frac{1}{2}e^TPBu) + \\ +(\frac{1}{2}\tilde{d}^TL^TPe + \frac{1}{2}e^TPL\tilde{d})$$

Assumption: For given positive definite matrix Q *and coefficients* r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^TP + PA = -Q + P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P \qquad \text{(G)}$$

Moreover, the following **feedback control law** is applied to the system

$$u = -\frac{1}{r}B^TPe \qquad \text{(H)}$$

By substituting Eq. (H) and Eq. (G) one obtains

$$\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T)P]e + \\ +e^TPB(-\frac{1}{r}B^TPe + e^TPL\tilde{d} \Rightarrow$$

6. Lyapunov stability analysis

Continuing with computations one obtains

$$\dot{V}=-\frac{1}{2}e^TQe+\left(\frac{1}{r}PBB^TPe-\frac{1}{2\rho^2}e^TPLL^T\right)Pe$$
$$-\frac{1}{r}e^TPBB^TPe+e^TPL\tilde{d}$$

which next gives

$$\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPLL^TPe+e^TPL\tilde{d}$$

or equivalently

$$\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPLL^TPe+$$
$$+\frac{1}{2}e^TPL\tilde{d}+\frac{1}{2}\tilde{d}^TL^TPe$$

I

Lemma: The following inequality holds

$$\frac{1}{2}e^TL\tilde{d}+\frac{1}{2}\tilde{d}L^TPe-\frac{1}{2\rho^2}e^TPLL^TPe\leq\frac{1}{2}\rho^2\tilde{d}^T\tilde{d}$$

6. Lyapunov stability analysis

Proof : *The binomial* $(\rho a-\frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2a^2+\frac{1}{\rho^2}b^2-2ab\geq 0\Rightarrow \frac{1}{2}\rho^2a^2+\frac{1}{2\rho^2}b^2-ab\geq 0\Rightarrow$$
$$ab-\frac{1}{2\rho^2}b^2\leq\frac{1}{2}\rho^2a^2\Rightarrow\frac{1}{2}ab+\frac{1}{2}ab-\frac{1}{2\rho^2}b^2\leq\frac{1}{2}\rho^2a^2$$

The following substitutions are carried out: $a=\tilde{d}$ and $b=e^TPL$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^TL^TPe+\frac{1}{2}e^TPL\tilde{d}-\frac{1}{2\rho^2}e^TPLL^TPe\leq\frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \qquad \text{J}$$

Eq. J is substituted in Eq. I and the inequality is enforced, thus giving

$$\dot{V}\leq-\frac{1}{2}e^TQe+\frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \qquad \text{K}$$

Eq. K shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T\dot{V}(t)dt\leq-\frac{1}{2}\int_0^T||e||_Q^2dt+\frac{1}{2}\rho^2\int_0^T||\tilde{d}||^2dt\Rightarrow$$
$$2V(T)+\int_0^T||e||_Q^2dt\leq 2V(0)+\rho^2\int_0^T||\tilde{d}||^2dt$$

6. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} ||\tilde{d}||^2 dt \leq M_d$$

then one gets

$$\int_0^{\infty} ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} ||e||_Q^2 dt$ is bounded.

Moreover, V(T) is bounded and from the definition of the Lyapunov function V it becomes clear that **e(t) will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}.$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$lim_{t \to \infty} e(t) = 0.$$

This completes the stability proof.

7. Simulation tests

- The nonlinear **H-nfinity control scheme** is tested through simulation examples

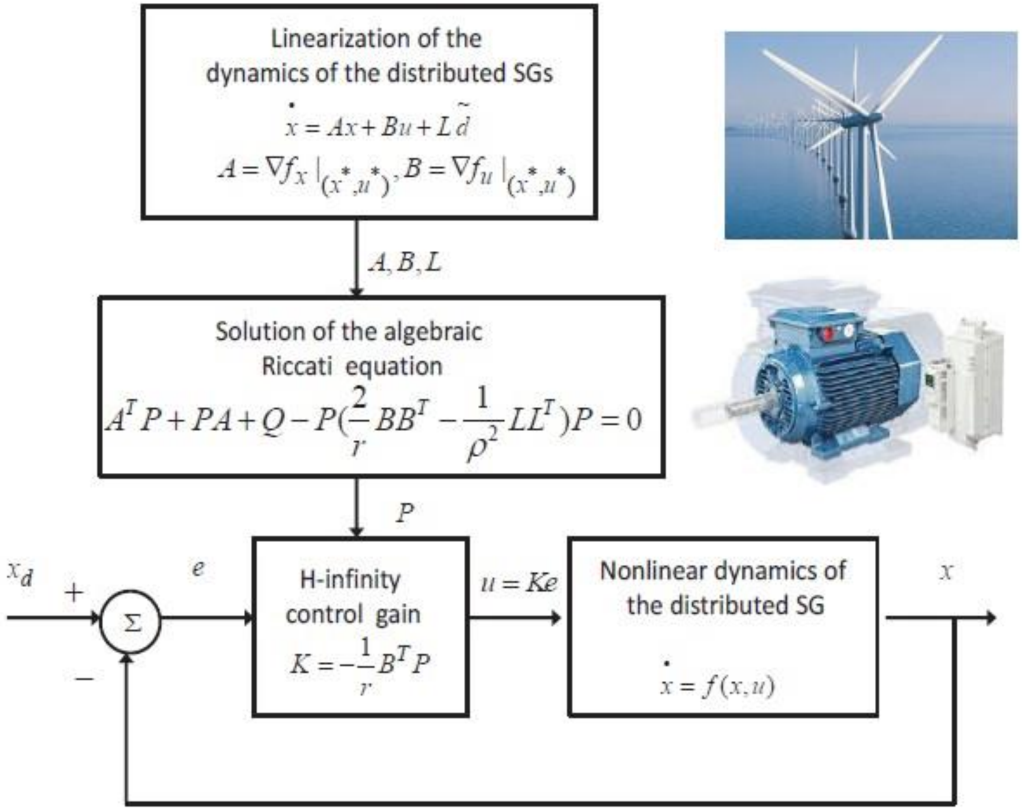
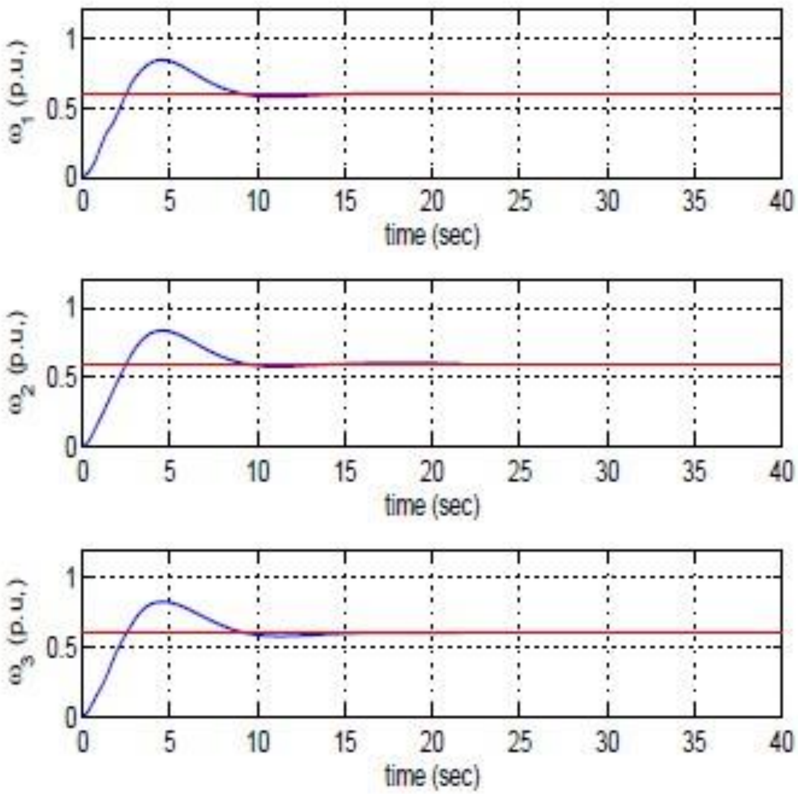


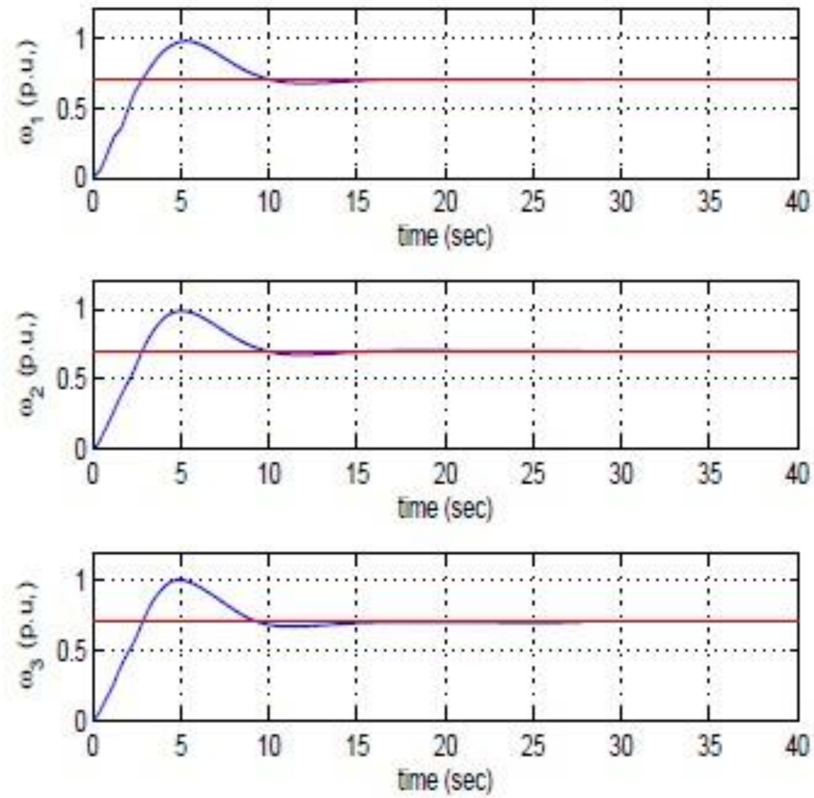
Fig. 2: Diagram of the control scheme for the distributed synchronous generators

It can be noted that the H-infinity algorithm exhibited remarkable robustness to uncertainty in the model of the distributed power generators which was to approximate linearization.

7. Simulation tests

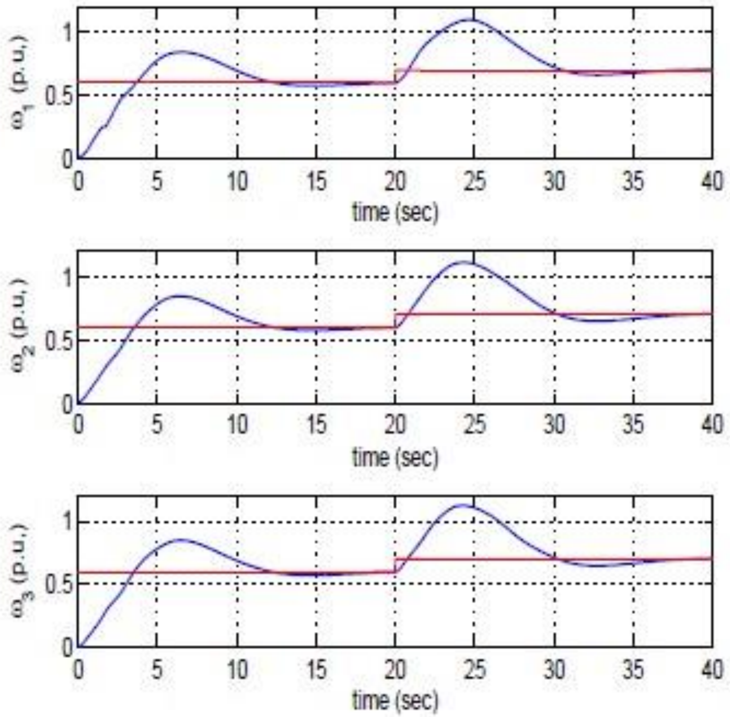


Tracking of setpoint no 1 by the rotational speeds of the distributed synchronous generators

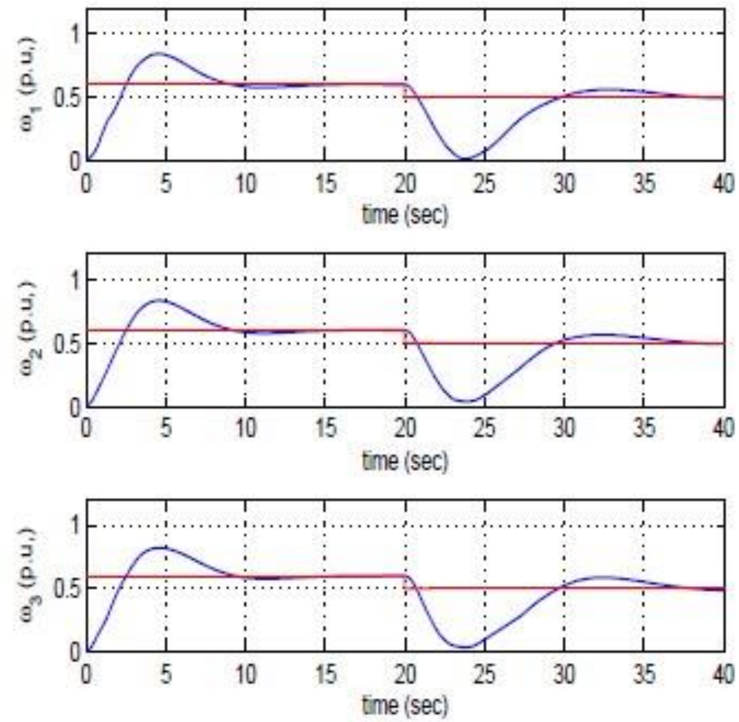


Tracking of setpoint no 2 by the rotational speeds of the distributed synchronous generators

7. Simulation tests

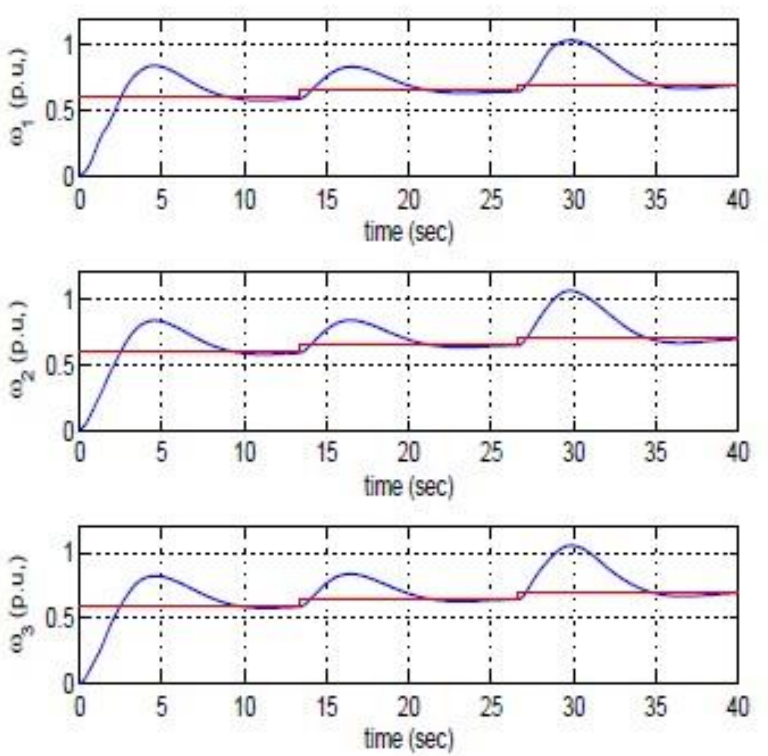


Tracking of setpoint no 3 by the rotational speeds of the distributed synchronous generators

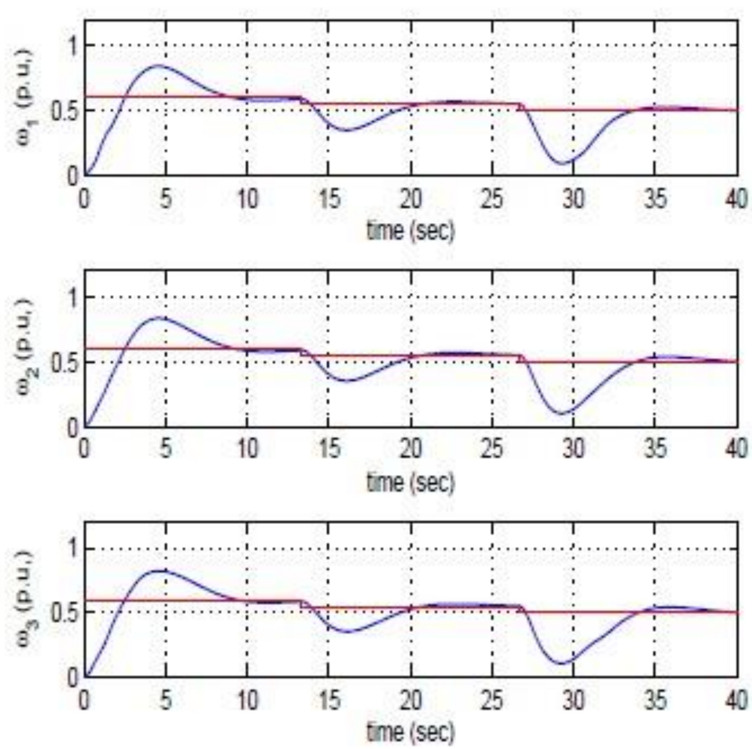


Tracking of setpoint no 4 by the rotational speeds of the distributed synchronous generators

7. Simulation tests



Tracking of setpoint no 5 by the rotational speeds of the distributed synchronous generators



Tracking of setpoint no 6 by the rotational speeds of the distributed synchronous generators

8. Conclusions

• The problem of **synchronization and stabilizati**on of the **distributed synchronous generators** has been solved with the application of a nonlinear H-infinity (optimal) control method.

• A **new nonlinear feedback control method for distributed generators** has been developed based on approximate linearization and the use of ***H*-infinity control and stability theory**.

• The first stage of the proposed control method is the **linearization of the distributed power generators' model** using first order Taylor series expansion and the computation of the associated Jacobian matrices.

• The errors due to the **approximative linearization** have been considered as disturbances that affect, together with external perturbations, the distributed power generators' model.

• Using the **linearized model of the disitrubeted generators** an **H-infinity feedback co**ntrol law is computed at each iteration of the control algorithm, after previously solving an **algebraic Riccati equation**.

• The known **robustness features of H-infinity control** enable to compensate for the errors of the approximative linearization, as well as to eliminate the effects of external perturbations.

• The **efficiency of the proposed control scheme** is shown analytically and is confirmed through simulation experiments.