

Lecture on

**New approaches to nonlinear control
of robotic systems:**

Approximate linearization methods

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1. Outline

- A new **nonlinear H-infinity control method** for stabilization and synchronization of **underactuated surface vessels**.
- At first stage **local linearization** of the model of the underactuated vessels is performed round its present operating point.
- The **approximation error** that is introduced to the linearized model is due to **truncation of higher-order terms in the performed Taylor series expansion** and is represented as a disturbance.
- The control problem is now formulated as a **mini-max differential game** in which the control input tries to minimize the state vector's tracking error while the disturbances affecting the model try to maximize it.
- Using the linearized description of the distributed generators' dynamics an **H-infinity feedback controller** is designed through the solution of a **Riccati equation** at each step of the control algorithm.
- The **inherent robustness properties of H-infinity control** assure that the disturbance effects will be eliminated and the state variables of the underactuated surface vessel will converge to the desirable setpoints.
- The proposed method, stands for a reliable solution to the problem of **nonlinear control** and stabilization for **unmanned surface vessels** exhibiting underactuation..



2. Model of the underactuated vessel

- The underactuated vessel's model stems from the generic ship's model, after setting specific values for the elements of the inertia and Coriolis matrix and after reducing the number of the available control inputs.

$$\begin{aligned}\dot{x} &= u \cos(\psi) - v \sin(\psi) \\ \dot{y} &= u \sin(\psi) + v \cos(\psi) \\ \dot{\psi} &= r \\ \dot{u} &= v \cdot r + \tau_u \\ \dot{v} &= -u \cdot r - \beta v \\ \dot{r} &= \tau_r\end{aligned}$$

ψ is the orientation angle

u is the surge velocity

v is the sway velocity

r is the yaw rate



The control inputs are the surge force τ_u and the yaw torque τ_r

The underactuated vessel's model is also written in the matrix form

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{u} \\ \dot{v} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} u \cos(\psi) - v \sin(\psi) \\ u \sin(\psi) + v \cos(\psi) \\ r \\ v r \\ -u r - \beta v \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tau_u \\ \tau_r \end{pmatrix}$$

2. Model of the underactuated vessel

or equivalently, one has the description $\dot{\tilde{x}} = \tilde{f}(\tilde{x}) + \tilde{g}(\tilde{x})\tilde{v}$

The system's state vector is denoted as $\tilde{x} = [x, y, \psi, u, v, r]^T$

while $f(\tilde{x}) \in R^{6 \times 1}$ and $\tilde{g}(\tilde{x}) = [\tilde{g}_a, \tilde{g}_b] \in R^{6 \times 2}$

while the control input is the vector $\tilde{v} = [\tau_u, \tau_r]^T$

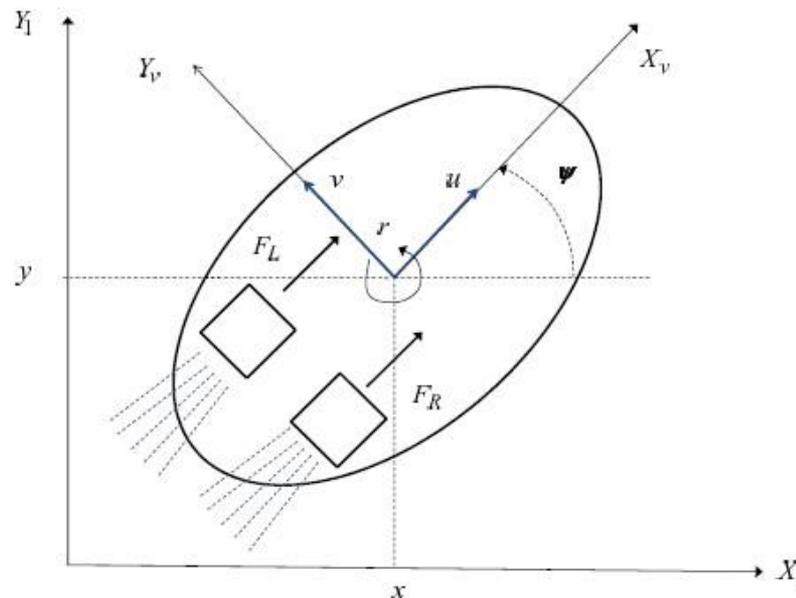


Fig. 1. Diagram of the underactuated hovercraft's kinematic model

2. Model of the underactuated vessel

The system's state vector can be extended by including as additional state variables the control input τ_u and its first derivative $\dot{\tau}_u$.

The extended state-space description of the system becomes

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{u} \\ \dot{v} \\ \dot{r} \\ \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} u \cos(\psi) - v \sin(\psi) \\ u \sin(\psi) + v \cos(\psi) \\ r \\ vr + z_1 \\ -ur - \beta v \\ 0 \\ z_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \ddot{\tau}_u \\ \tau_r \end{pmatrix}$$

or equivalently, one has the description $\dot{z} = f(z) + g(z)\tilde{v}$

The extended system's state vector is denoted as $z = [x, y, \psi, u, v, r, z_1, z_2]^T$.

Moreover, one has $f(z) \in R^{8 \times 1}$ and $g(z) = [g_a, g_b] \in R^{8 \times 2}$,

while the control input is the vector is $\tilde{v} = [\ddot{\tau}_u, \tau_r]^T$.



3. Linearization of the model of the underactuated vessel

Local linearization is performed for the state-space model of the underactuated vessel, around the operating point (x^*, u^*) where x^* is the present value of the system's state vector and u^* is the last value of the control input that was exerted on the machine.

The joint kinematics and dynamics model is written in the form: $\dot{x} = f(x) + g(x)u$

where the state vector is: $x = [x_1, x_2, x_3, x_4, x_5, x_6]^T = [x, y, \psi, u, v, r]^T$; and

$$f(x) = \begin{pmatrix} v\cos(\psi) - v\sin(\psi) \\ u\sin(\psi) + v\cos(\psi) \\ r \\ v \cdot r \\ -ur - \beta v \\ 0 \end{pmatrix} \quad g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$



and using the state variables notation one gets the description

$$f(x) = \begin{pmatrix} x_4\cos(x_3) - x_5\sin(x_3) \\ x_4\sin(x_3) + x_5\cos(x_3) \\ x_6 \\ x_5x_6 \\ -x_4x_6 - \beta x_5 \\ 0 \end{pmatrix} \quad g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$



3. Linearization of the model of the underactuated vessel

The linearization of the vessel's model round the temporary equilibrium gives

$$\dot{x} = Ax + Bu$$

where

$$A = \nabla_x [f(x) + g(x)u] |_{(x^*, u^*)} \Rightarrow A = \nabla_x f(x) |_{(x^*, u^*)}$$

$$B = \nabla_u [f(x) + g(x)u] |_{(x^*, u^*)} \Rightarrow B = g(x) |_{(x^*, u^*)}$$

For the Jacobian matrix $A = \nabla_x [f(x) + g(x)u] |_{(x^*, u^*)} =$

$$= \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \dots & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \dots & \frac{\partial f_2}{\partial x_6} \\ \dots & \dots & \dots & \dots & \dots \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \frac{\partial f_6}{\partial x_3} & \dots & \frac{\partial f_6}{\partial x_6} \end{pmatrix}$$



For the first row of the aforementioned Jacobian matrix one has:

$$\frac{\partial f_1}{\partial x_1} = 0, \frac{\partial f_1}{\partial x_2} = 0, \frac{\partial f_1}{\partial x_3} = -x_4 \sin(x_3) - x_5 \cos(x_3), \frac{\partial f_1}{\partial x_4} = \cos(x_3), \frac{\partial f_1}{\partial x_5} = -\sin(x_3), \frac{\partial f_1}{\partial x_6} = 0.$$

3. Linearization of the model of the underactuated vessel

For the second row of the aforementioned Jacobian matrix one has:

$$\frac{\partial f_2}{\partial x_1} = 0, \frac{\partial f_2}{\partial x_2} = 0, \frac{\partial f_2}{\partial x_3} = x_4 \cos(x_3) - x_5 \sin(x_3), \frac{\partial f_2}{\partial x_4} = \sin(x_3), \frac{\partial f_2}{\partial x_5} = \cos(x_3), \frac{\partial f_2}{\partial x_6} = 0.$$

For the third row of the aforementioned Jacobian matrix one has:

$$\frac{\partial f_3}{\partial x_1} = 0, \frac{\partial f_3}{\partial x_2} = 0, \frac{\partial f_3}{\partial x_3} = 0, \frac{\partial f_3}{\partial x_4} = 0, \frac{\partial f_3}{\partial x_5} = 0, \frac{\partial f_3}{\partial x_6} = 1.$$

For the fourth row of the aforementioned Jacobian matrix one has:

$$\frac{\partial f_4}{\partial x_1} = 0, \frac{\partial f_4}{\partial x_2} = 0, \frac{\partial f_4}{\partial x_3} = 0, \frac{\partial f_4}{\partial x_4} = 0, \frac{\partial f_4}{\partial x_5} = x_6, \frac{\partial f_4}{\partial x_6} = x_5.$$

For the fifth row of the aforementioned Jacobian matrix one has:

$$\frac{\partial f_5}{\partial x_1} = 0, \frac{\partial f_5}{\partial x_2} = 0, \frac{\partial f_5}{\partial x_3} = 0, \frac{\partial f_5}{\partial x_4} = -x_6, \frac{\partial f_5}{\partial x_5} = -\beta, \frac{\partial f_5}{\partial x_6} = -x_6.$$

For the sixth row of the aforementioned Jacobian matrix one has:

$$\frac{\partial f_6}{\partial x_1} = 0, \frac{\partial f_6}{\partial x_2} = 0, \frac{\partial f_6}{\partial x_3} = 0, \frac{\partial f_6}{\partial x_4} = 0, \frac{\partial f_6}{\partial x_5} = 0, \frac{\partial f_6}{\partial x_6} = 0.$$



3. Linearization of the model of the underactuated vessel

Parameter d_1 stands for the **linearization error** in the underactuated vessels' model

$$\dot{x} = Ax + Bu + d_1 \quad \text{(A)}$$

The **desirable trajectory** of the underactuated vessel is denoted by

$$x_d = [x_{d_1}, x_{d_2}, x_{d_3}, \dots, x_{d_7}, x_{d_8}, x_{d_9}]^T$$

Tracking of this trajectory is succeeded after applying the control input u^*

At every time instant the control input u^* is assumed to differ from the control input u appearing in (A) by an amount equal to Δu , that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \quad \text{(B)}$$

The dynamics of the system of Eq. (A) can be also written in the form

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad \text{(C)}$$

and by denoting $d_3 = -Bu^* + d_1$ as an **aggregate disturbance** term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad \text{(D)}$$



5. The nonlinear H-infinity control

where matrices A and B are obtained from the **computation of the Jacobians**

$$A = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{pmatrix} \Big|_{(x^*, u^*)} \quad B = \begin{pmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} & \dots & \frac{\partial f_1}{\partial u_m} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} & \dots & \frac{\partial f_2}{\partial u_m} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f_n}{\partial u_1} & \frac{\partial f_n}{\partial u_2} & \dots & \frac{\partial f_n}{\partial u_m} \end{pmatrix} \Big|_{(x^*, u^*)}$$

and vector d denotes disturbance terms due to linearization errors.

The problem of **disturbance rejection** for the linearized model that is described by

$$\begin{aligned} \dot{x} &= Ax + Bu + Ld \\ y &= Cx \end{aligned}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $d \in \mathbb{R}^q$ and $y \in \mathbb{R}^p$, cannot be handled efficiently if the classical LQR control scheme is applied. This because of the existence of the perturbation term d .

In the H^∞ control approach, a **feedback control scheme** is designed for **trajectory tracking** by the system's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner



5. The nonlinear H-infinity control

The disturbances' effect are incorporated in the following **quadratic cost function**

$$J(t) = \frac{1}{2} \int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2 d^T(t)d(t)] dt, \quad r, \rho > 0$$

The coefficient r determines the **penalization of the control input** and the weight coefficient ρ determines the **reward of the disturbances' effects**. It is assumed that

Then, the **optimal feedback control law** is given by

$$u(t) = -Kx(t) \quad \text{with} \quad K = \frac{1}{r} B^T P$$

where P is a **positive semi-definite symmetric matrix** which is obtained from the solution of the **Riccati equation**

$$A^T P + PA + Q - P \left(\frac{1}{r} BB^T - \frac{1}{2\rho^2} LL^T \right) P = 0$$

where Q is also a positive definite symmetric matrix.

The parameter ρ in Eq. (15), is an **indication of the closed-loop system robustness**. If the values of $\rho > 0$ are excessively decreased with respect to r , then the solution of the Riccati equation is no longer a positive definite matrix. Consequently, there is a lower bound ρ_{\min} of for which the H-infinity control problem has a solution.



6. Lyapunov stability analysis

The **tracking error dynamics** for the distributed power generation System is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d}$$



where in the underactuated vessel's application example $L = I \in \mathbb{R}^2$ with I being the identity matrix. The following **Lyapunov function** is considered

$$V = \frac{1}{2}e^T P e$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{e}^T P e + \frac{1}{2}e^T P \dot{e} \Rightarrow \\ \dot{V} &= \frac{1}{2}[Ae + Bu + L\tilde{d}]^T P + \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow \end{aligned}$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T] P e + \\ &+ \frac{1}{2}e^T P [Ae + Bu + L\tilde{d}] \Rightarrow \end{aligned}$$

$$\begin{aligned} \dot{V} &= \frac{1}{2}e^T A^T P e + \frac{1}{2}u^T B^T P e + \frac{1}{2}\tilde{d}^T L^T P e + \\ &+ \frac{1}{2}e^T P A e + \frac{1}{2}e^T P B u + \frac{1}{2}e^T P L \tilde{d} \end{aligned}$$



6. Lyapunov stability analysis

The previous equation is rewritten as

$$\dot{V} = \frac{1}{2}e^T(A^T P + PA)e + \left(\frac{1}{2}u^T B^T P e + \frac{1}{2}e^T P B u\right) + \left(\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d}\right)$$



Assumption: For given positive definite matrix Q and coefficients r and ρ there exists a positive definite matrix P , which is the solution of the following matrix equation

$$A^T P + PA = -Q + P\left(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T\right)P \quad \text{(G)}$$

Moreover, the following **feedback control law** is applied to the system

$$u = -\frac{1}{r}B^T P e \quad \text{(H)}$$

By substituting Eq. (H) and Eq. (G) one obtains

$$\dot{V} = \frac{1}{2}e^T\left[-Q + P\left(\frac{1}{r}BB^T - \frac{1}{2\rho^2}LL^T\right)P\right]e + e^T P B \left(-\frac{1}{r}B^T P e + e^T P L \tilde{d}\right) \Rightarrow$$



6. Lyapunov stability analysis

Continuing with computations one obtains

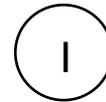
$$\dot{V} = -\frac{1}{2}e^T Q e + \left(\frac{1}{r} P B B^T P e - \frac{1}{2\rho^2} e^T P L L^T\right) P e - \frac{1}{r} e^T P B B^T P e + e^T P L \tilde{d}$$

which next gives

$$\dot{V} = -\frac{1}{2}e^T Q e - \frac{1}{2\rho^2} e^T P L L^T P e + e^T P L \tilde{d}$$

or equivalently

$$\dot{V} = -\frac{1}{2}e^T Q e - \frac{1}{2\rho^2} e^T P L L^T P e + \frac{1}{2}e^T P L \tilde{d} + \frac{1}{2}\tilde{d}^T L^T P e$$



Lemma: The following inequality holds

$$\frac{1}{2}e^T L \tilde{d} + \frac{1}{2}\tilde{d}^T L^T P e - \frac{1}{2\rho^2} e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d}$$



6. Lyapunov stability analysis

Proof : The binomial $(\rho a - \frac{1}{\rho} b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2} b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2} b^2 - ab \geq 0 \Rightarrow$$

$$ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2} b^2 \leq \frac{1}{2}\rho^2 a^2$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T PL$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T P e + \frac{1}{2}e^T P L \tilde{d} - \frac{1}{2\rho^2}e^T P L L^T P e \leq \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \tag{J}$$

Eq. (J) is substituted in Eq. (I) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Q e + \frac{1}{2}\rho^2 \tilde{d}^T \tilde{d} \tag{K}$$

Eq. (K) shows that the **H-infinity tracking performance criterion** is satisfied.

The integration of \dot{V} from 0 to T gives

$$\int_0^T \dot{V}(t) dt \leq -\frac{1}{2} \int_0^T \|e\|_Q^2 dt + \frac{1}{2}\rho^2 \int_0^T \|\tilde{d}\|^2 dt \Rightarrow$$

$$2V(T) + \int_0^T \|e\|_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T \|\tilde{d}\|^2 dt$$



6. Lyapunov stability analysis

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^{\infty} \|\bar{d}\|^2 dt \leq M_d$$

then one gets

$$\int_0^{\infty} \|e\|_Q^2 dt \leq 2V(0) + \rho^2 M_d$$

Thus, the integral $\int_0^{\infty} \|e\|_Q^2 dt$ is bounded.

Moreover, $V(t)$ is bounded and from the definition of the Lyapunov function V it becomes clear that **$e(t)$ will be also bounded** since

$$e(t) \in \Omega_e = \{e | e^T P e \leq 2V(0) + \rho^2 M_d\}.$$

According to the above and with the use of **Barbalat's Lemma** one obtains:

$$\lim_{t \rightarrow \infty} e(t) = 0.$$

This completes the stability proof.



7. Robust state estimation with the use of the H-infinity Kalman Filter

A discrete-time description of the linearized state-space model of the vessel is assumed. The **H-infinity Kalman Filter**, for the **model of the underactuated vessel**, can be formulated in terms of a **measurement update** and a **time update part**

Measurement update:

$$D(k) = [I - \theta W(k)P^-(k) + C^T(k)R(k)^{-1}C(k)P^-(k)]^{-1}$$

$$K(k) = P^-(k)D(k)C^T(k)R(k)^{-1}$$

$$\hat{x}(k) = \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)]$$

Time update:

$$\hat{x}^-(k+1) = A(k)x(k) + B(k)u(k)$$

$$P^-(k+1) = A(k)P^-(k)D(k)A^T(k) + Q(k)$$

where θ is sufficiently small to assure positive definiteness for the covariance matrix

$$P^-(k) - \theta W(k) + C^T(k)R(k)^{-1}C(k)$$

One can measure only a subset of the state variables of the vessel's model (e.g. cartesian coordinates) and can **estimate through filtering** the rest of the state vector elements.

Besides the filter can be used for **sensor fusion** purposes.

8. Simulation tests

- The nonlinear **H-infinity control scheme** is tested through simulation examples

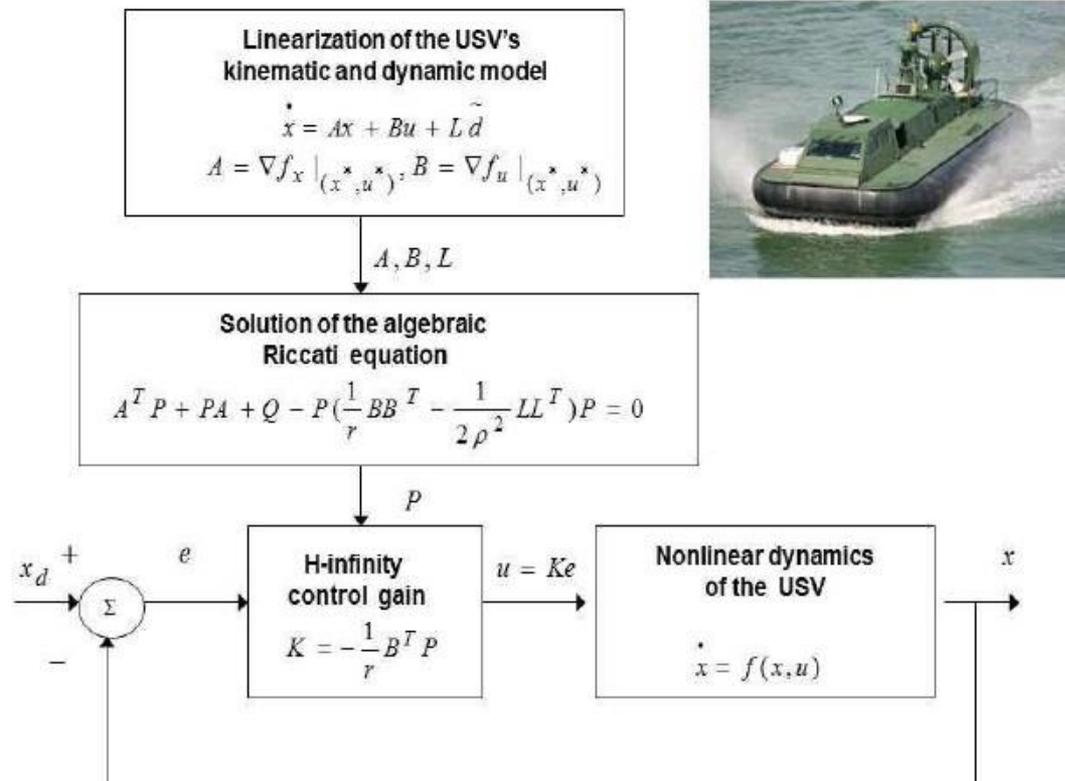
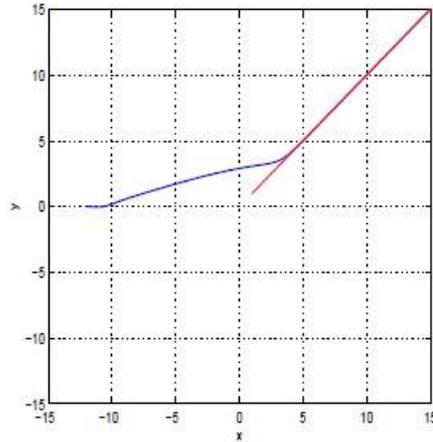


Fig. 2: Diagram of the control scheme for the underactuated vessel

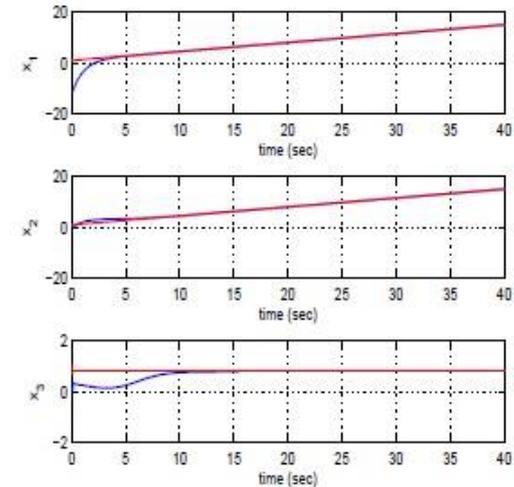
It can be noted that the H-infinity algorithm exhibited remarkable robustness to uncertainty in the model of the distributed power generators which was to approximate linearization.

8. Simulation tests

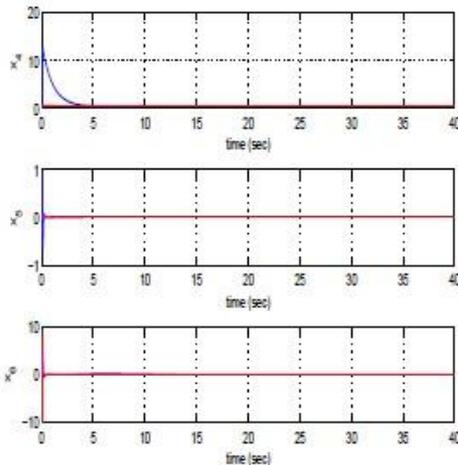
Path 1



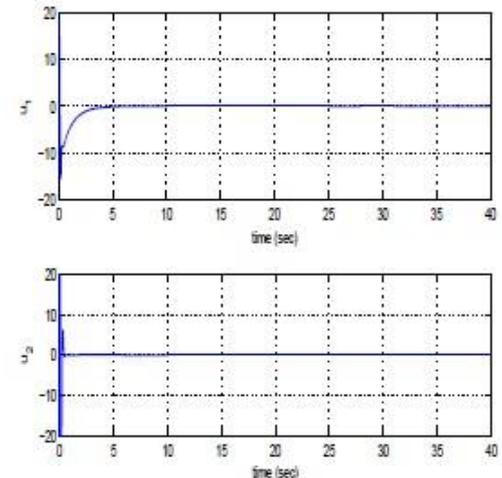
Tracking of the reference trajectory (red line) in the $x - y$ plane by the unmanned surface vessel (blue line),



Convergence of the state variables $x_1 = x$, $x_2 = y$ and $x_3 = \psi$ to the reference values



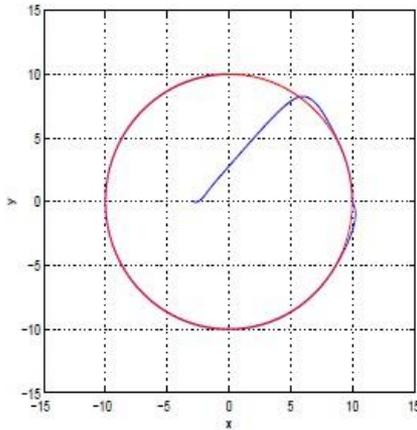
Convergence of the state variables of the vessel $x_4 = u$, $x_5 = v$ and $x_6 = r$ to the reference values



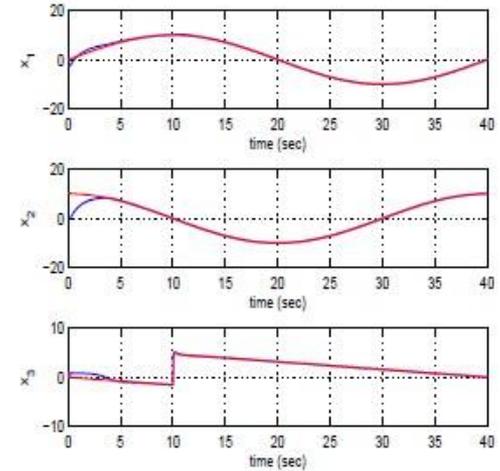
Control inputs u_1 and u_2 exerted on vessel

8. Simulation tests

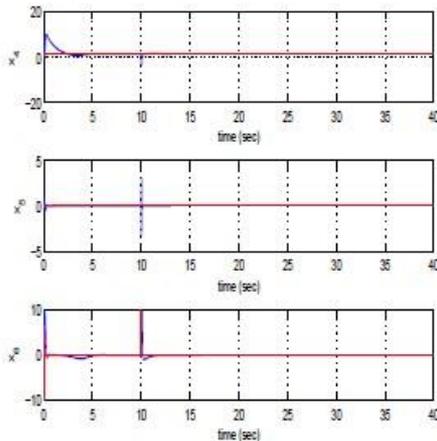
Path 2



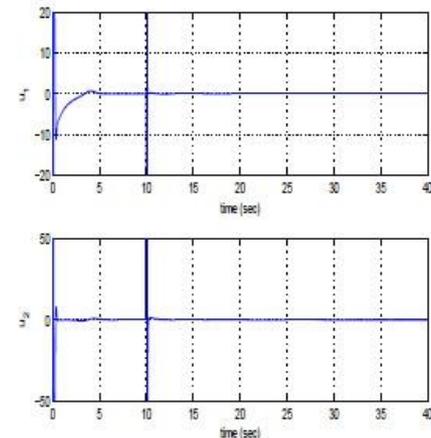
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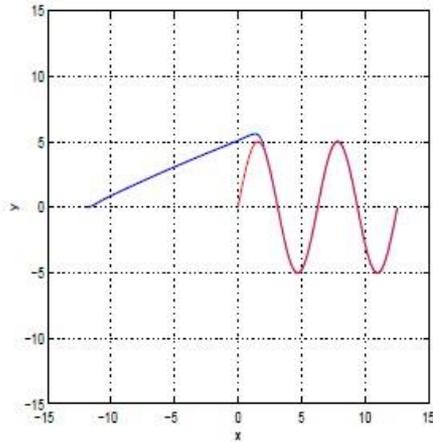
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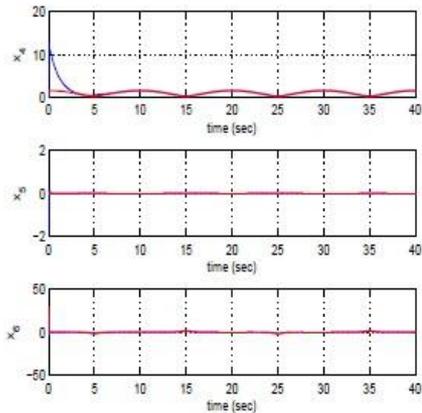
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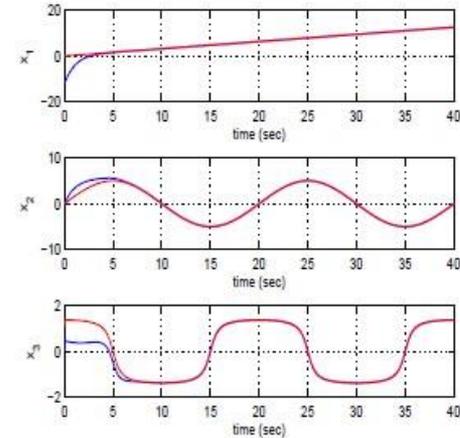
Path 3



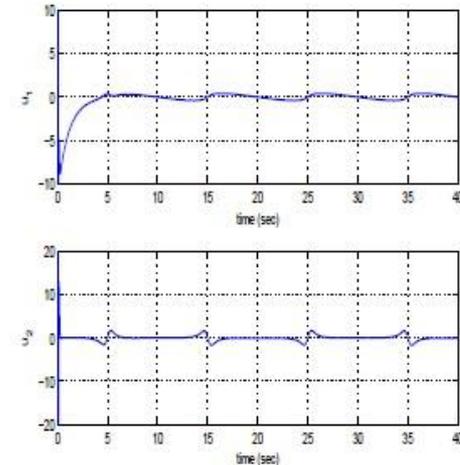
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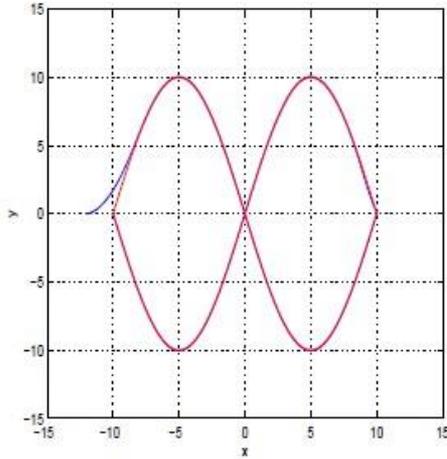
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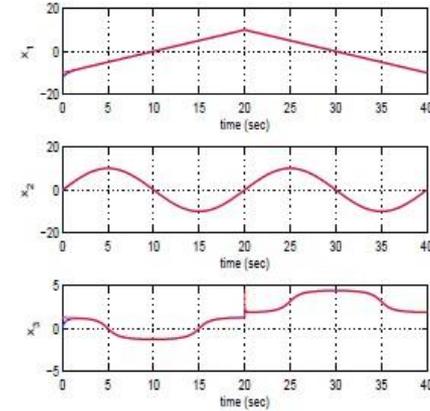
Control inputs u_1 and u_2 exerted on vessel

8. Simulation tests

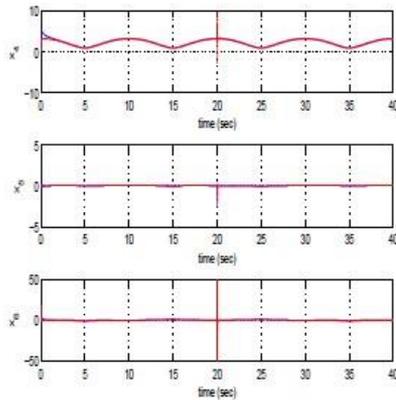
Path 4



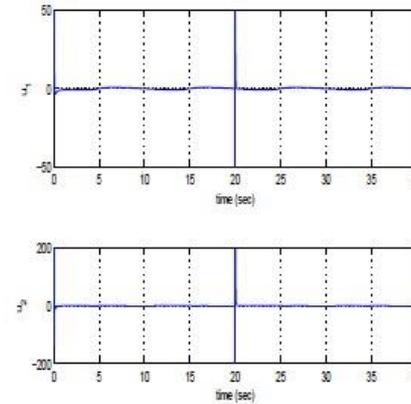
Tracking of the reference trajectory (red line) in the $x - y$ plane by the unmanned surface vessel (blue line),



Convergence of the state variables $x_1 = x$, $x_2 = y$ and $x_3 = \psi$ to the reference values



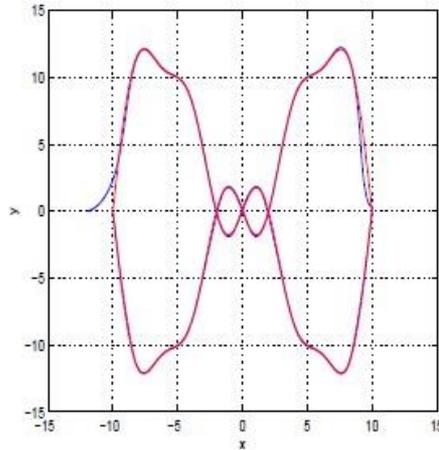
Convergence of the state variables of the vessel $x_4 = u$, $x_5 = v$ and $x_6 = r$ to the reference values



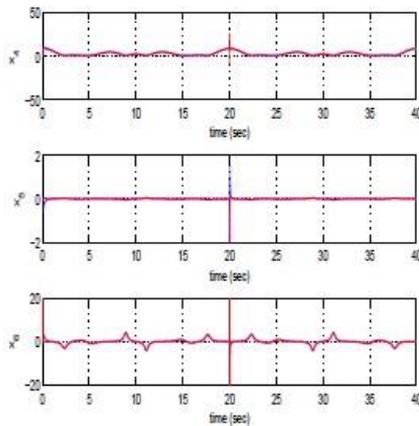
Control inputs u_1 and u_2 exerted on vessel

8. Simulation tests

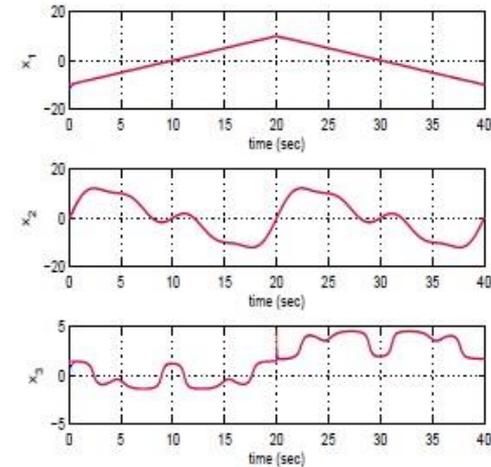
Path 5



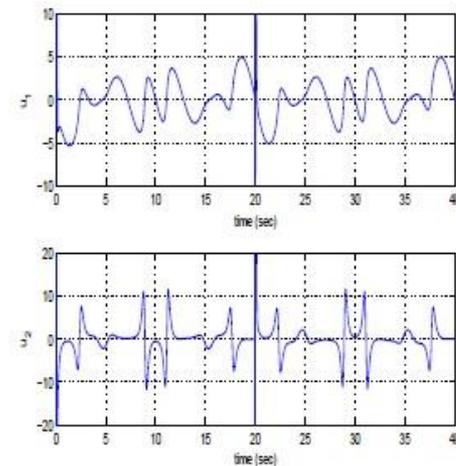
Tracking of the reference trajectory (red line) in the $x - y$ plane by the unmanned surface vessel (blue line),



Convergence of the state variables of the vessel $x_4 = u$, $x_5 = v$ and $x_6 = r$ to the reference values



Convergence of the state variables $x_1 = x$, $x_2 = y$ and $x_3 = \psi$ to the reference values



Control inputs u_1 and u_2 exerted on vessel

9. Conclusions

- The problem of **trajectory tracking control** of underactuated **unmanned surface vessels** has been solved with the application of a nonlinear H -infinity (optimal) control method.
- A **new nonlinear feedback control method for underactuated vessels** has been developed based on approximate linearization and the use of **H -infinity control and stability theory**.
- The first stage of the proposed control method is the **linearization of the distributed power generators' model** using first order Taylor series expansion and the computation of the associated Jacobian matrices.
- The errors due to the **approximative linearization** have been considered as disturbances that affect, together with external perturbations, the distributed power generators' model.
- At a second stage the implementation of H -infinity feedback control has been proposed. Using the **linearized model of the vehicle** an **H -infinity feedback control** law is computed at each iteration of the control algorithm, after previously solving an **algebraic Riccati equation**.
- The known **robustness features of H -infinity control** enable to compensate for the errors of the approximative linearization, as well as to eliminate the effects of external perturbations.
- The **efficiency of the proposed control scheme** is shown analytically and is confirmed through simulation experiments.

